MEASURING OF THE BASE CIRCLE RADIUS AND INVOLUTE PROFILE ERRORS OF GEARS USING OPTICAL METHOD

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ABSTRACT

It is,in principle,impossible to determine by measuring a gear either the module,the pressure angle or the pitch circle radius.It remains possible to determine the various dimensions of a gear when the base radius is known,the measurement of same if for this reason essential and we will now describe an optical acurate method which permit to find the base radius, also it can be continued to give the correctness of the shape of the involute profile.

INTRODUCTION

During the hobbing process the combination hob/gear forms a transmission analogous to that of rack and gear.Module m and pressure angle ϕ of this transmission are determined by flank angle ϕ and pitch P of the hob (or rack),and since the radius r_p of the pitch circle has been defined as $r_p = 0.5\ m\ N$, where m is the module and N is the number of teeth,it results from this that the base radius $r_b = 0.5\ m\ N\ \cos\phi$.In the course of the hobbing process the flank of the hob contacts the gear on the pitch circle,which therefore is the tangent circle of the transmission hob/gear.The pitch(and consequently also the module) of the hob is naturally equal to that of the gear measured along the pitch circle.The involute is, mathematically,determined by the base radius;the number of teeth is a functional datum and because $r_p = 0.5\ m\ N$ this quantity is a constant value,belonging to the gear and being dependent on the hob.

As soon as the gear forms a transmission with another gear, the situation alters.The notation " pressure angle " only holds,when a transmission is concerned and the pressure angle of a transmission gear 1/gear 2 need not necessarily be equal to that of gear/hob.As it is,we find that

$$\cos \phi = \frac{r_{b1} + r_{b2}}{C} \qquad (1)$$

see fig. 1,so that the pressure angle ϕ of a transmission gear 1/gear 2 depends on the centre distance and the base radii,while the pressure angle of the cobination hob/gear is exelusivly dependent on the flank angle of the rack.In fig. 1.the gears contact one another in p,the circles that pass through p and the centres of which are O_1 and O_2 are the tangent circles,and in the nature of things the pitch measured along the tangent circle of gear 1 must be equal to the one measured along the tangent circle of gear 2.Although the pitch circles are often identical with the tangent circles,this is no necessity,for the pitch circle radii are exclusively dependent on the invariable dimensions of the hob, where as the tangent circle radii are determined by the distance between the centers of the two gears.

From the above it follows that both module and pressure angle of a transmission may differ from those of a transmission hob/gear.Furthermore it is,at least throritically,possible to make a single gear by means of hobs whose pressure angles (flank angles)are differnt.Since in that event the gears are likewise contacted by the hobs on different circles,the hob modules too,will be differnt from each other. The ratio i of a pair of gears is not only determined by the number of teeth but also by the base radii, the pitch circle radii do not determine this ratio. In consequence, the basic magnitude of a gear are the base radius and the number of teeth and it is, in principle, impossible to determine by measuring a gear either the module, the pressure angle or the pitch circle radius, because of the fact that these magnitudes depend on one of the hobs with which the gear could have been made. For the purpose of limiting the number of hobs the flank angle "pressure angle" and the pitch (module) of the hob are standerdized.

Owing to this it remains possible to determine the various dimensions of a gear when the base radius is known. The measurements of same is for this reason essential.

ANALYTICAL INVESTIGATION

When an involute acts against a straight line, we have the condition shown in figs. 2 and 3. The straight line, is tangent to the involute curve and is always perpindicular to its line of action. When it is constrained to move only in the direction of the line of action, it will be moved at a uniform rate to that of the end of the generating line.

The gear is fixed upon the rotary table of the Universal Measuring Microscope (Carl Zeiss Jena). The gear is centrically positioned using double image occular, then this occular will be replaced by the geniometer occular while the gear is still in its centric position. The geniometer occular should be adjusted so that its cross hair will be parallel to the X-Y directional measurements of the universal measuring microscope.

A movement along the X-axis and the Y-axis causes the horizontal cross line to contact a tooth flank as shown in fig. 2. for instance at A_1, from involumetry the vertical cross line passes through A_1 must be a tangent to the base circle of the gear. Theoritically, the base radius of the gear r_b now equal x_1, but

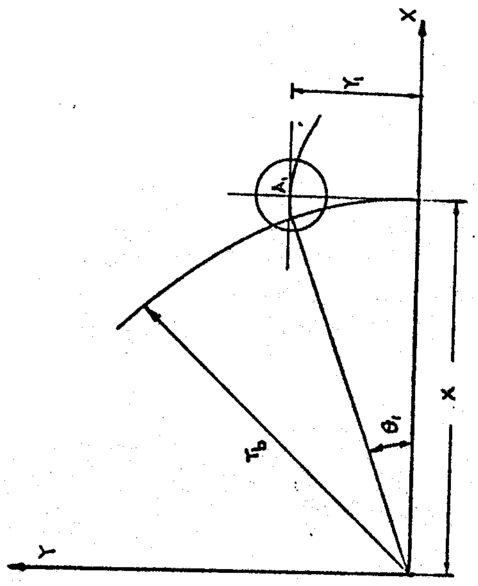


Fig. 2

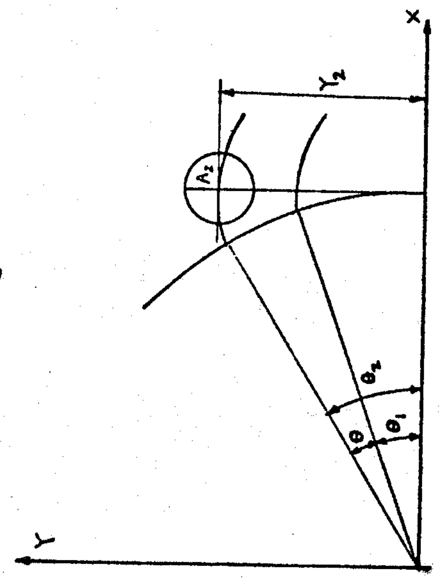


Fig. 3

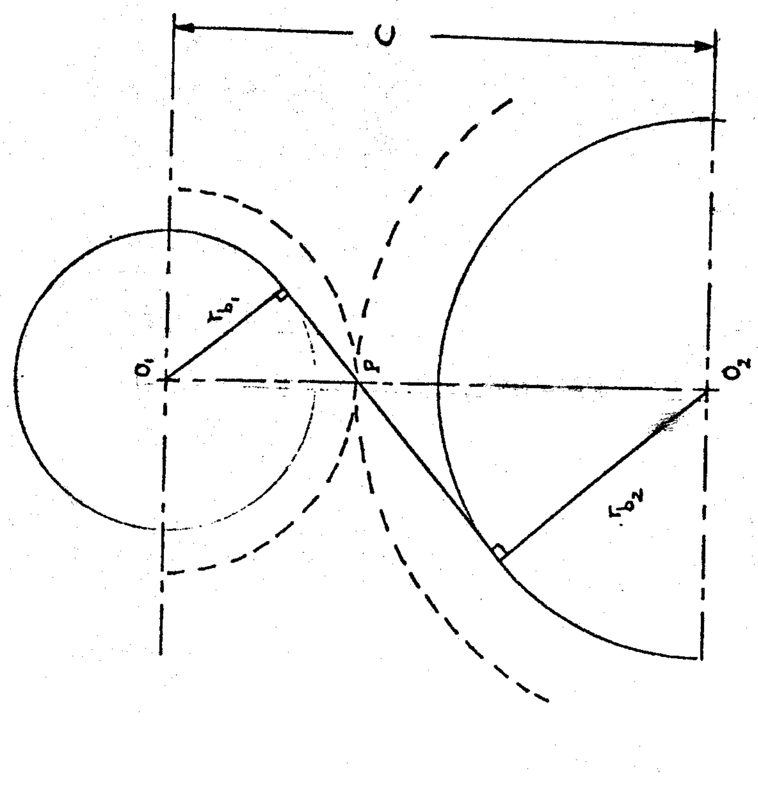


Fig. 1

for more accuracy,we shall proceed in a different way,which is being described herinafter.

The rotary table and the gear fixes upon it are turned through an angle θ and once again we cause the horizontal cross line to come in contact with the tooth flank at A_2 , as shown in fig. 3. this time only by movement in the direction of the axis of ordination.The basr radius r_b equals arc A_1A_2 divided by angle θ,and arc A_1A_2 equals y_2-y_1 = y in consequence of which:-

$$r_b = \frac{y_2 - y_1}{\theta_2 - \theta_1} = \frac{y}{\theta}$$

$$y = r_b \theta \qquad (2)$$

The relation between the variation in linear dispacement y and angular rotation θ is a straight line relation.The limitation of this relation to be a straight line depends on the involute error exists.Since y is not only vary due to the rotational angle θ but also due to the involute profile errors. Applying the above analysis both the base radius and the error in involute profile can be detected.

EXPERIMENTAL WORK

The above procedure for measuring the base radius and determining the involute prifile errors is performed on a new ground gear with a 42 teeth,and a hobbing flank angle 20°.Measurements of variation in y displacement corresponding to the angular position θ were taken over 12 position for every tooth.The measurements were taken for 8 different teeth.

Results of The Experimental Work

Figures (4 - 19) shows the results of the experimental measurements. The relations between the displacement y and the angular rotation θ are linear, and these are shown in figres (4,5,8,9,12,13,16,17), the involute profile error for each measured tooth are shown in figures (6,7,10,11,14,15,18,19)

Determination of Base Radius

The base radius is determined from equation 2, according to the slope of y and θ relation, from the regression analysis these relation can be considered linear with a high confidence limit.

we are by way of example giving the description of a measurement performed on the new ground gear with 42 teeth, the measurements were taken upon the first teeth; its results are shown in figures (4,6), the following ordinate values were found:-
0.593-0.601-0.605-0.601-0.605-0.605-0.601-0.609-0.601-0.603
0.599-0.603, for $\theta = 1^\circ$ or 0.017455 rad.

The average value of y = 0.603 (y being the average of 12 readings), so the base radius from equation 2

$$r_b = \frac{0.603}{0.017455} = 34.546 \quad \text{mm}$$

$\phi = 20^\circ$, we find that:-

$$r_p = 0.5\ m\ N = \frac{r_b}{\cos 20} = \frac{34.546}{0.9397} = 36.763\ \text{mm}$$

$$r_p = 36.763 = 0.5\ m\ N$$

That is to say $m = \frac{73.526}{42} = 1.7506$ mm

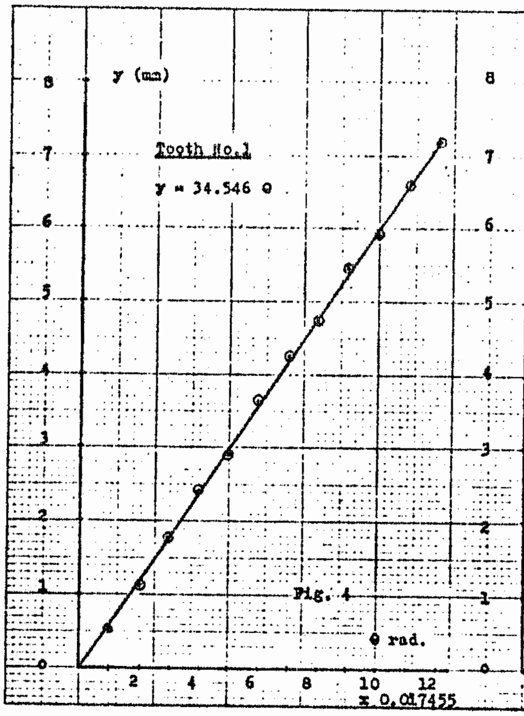


Fig. 4

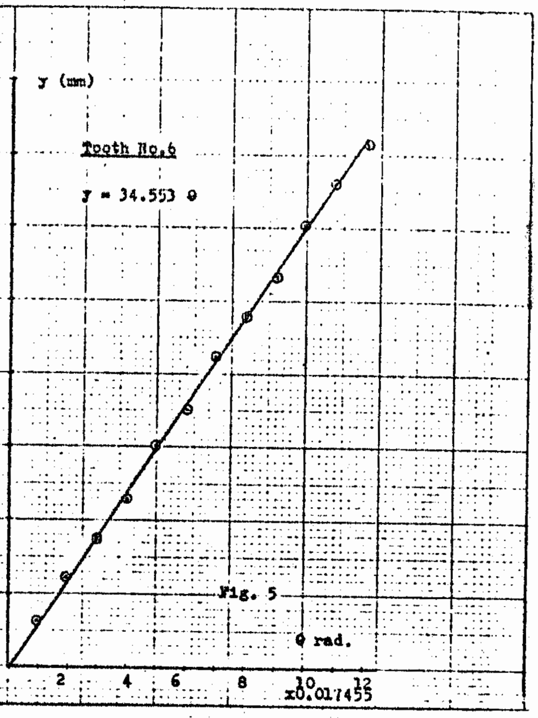


Fig. 5

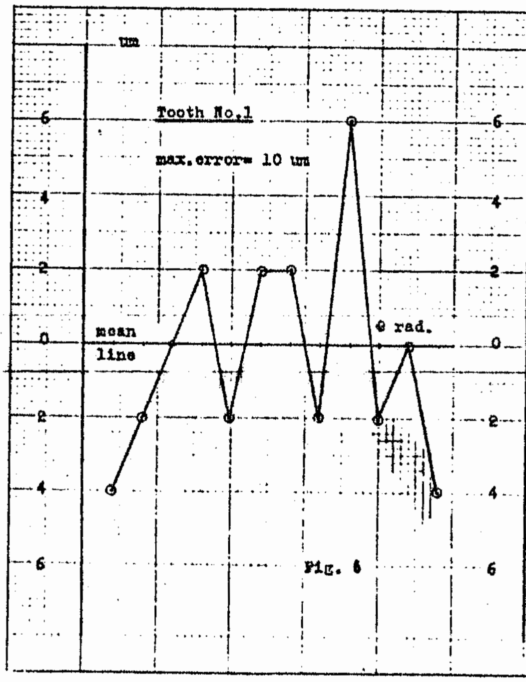


Fig. 6

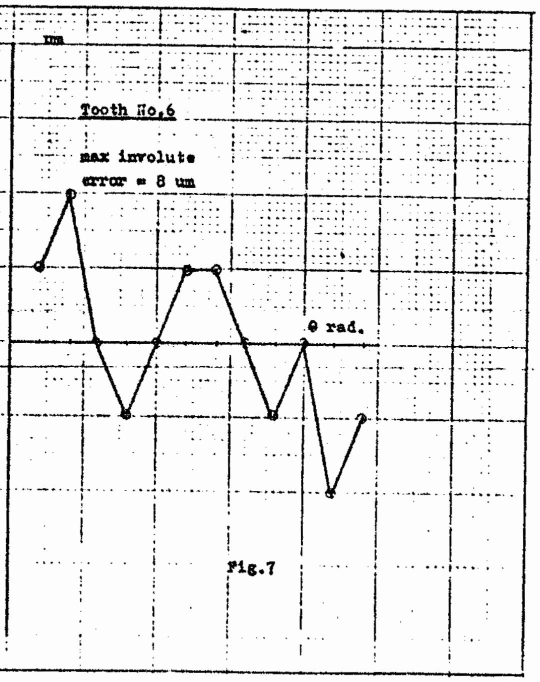
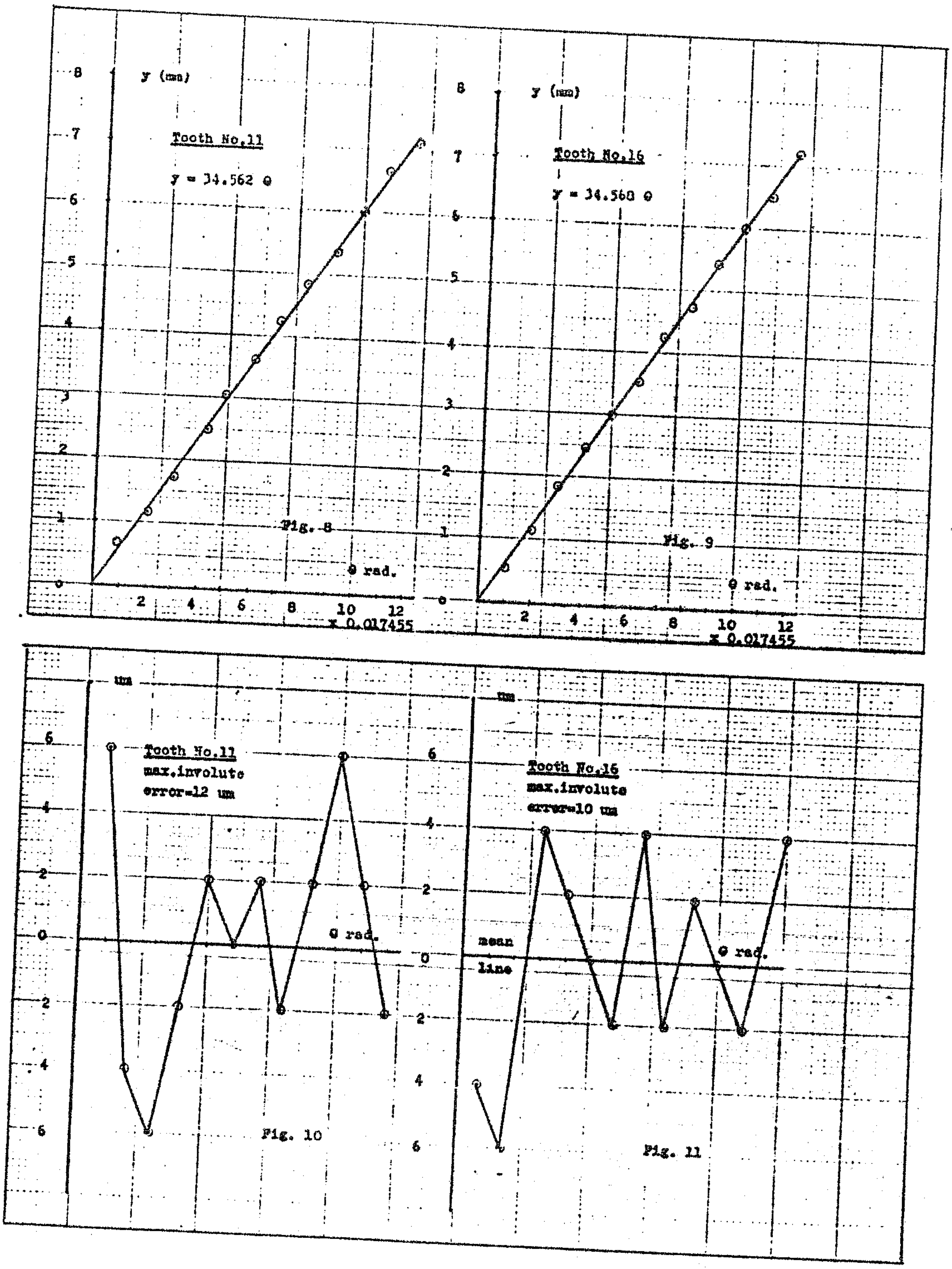
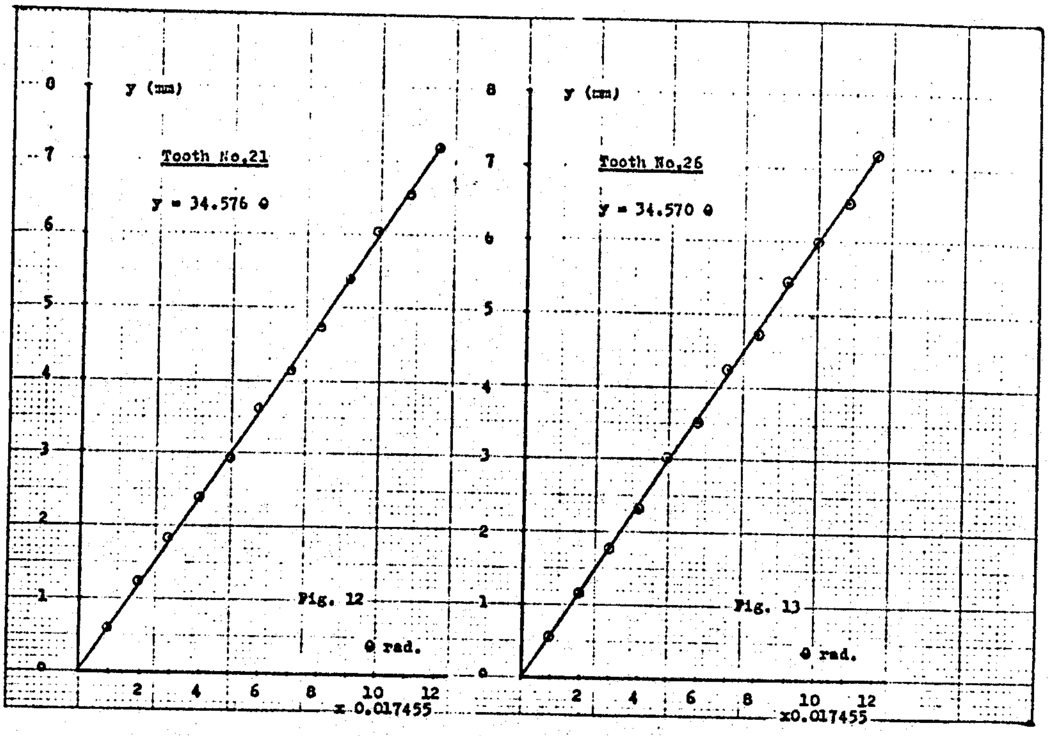
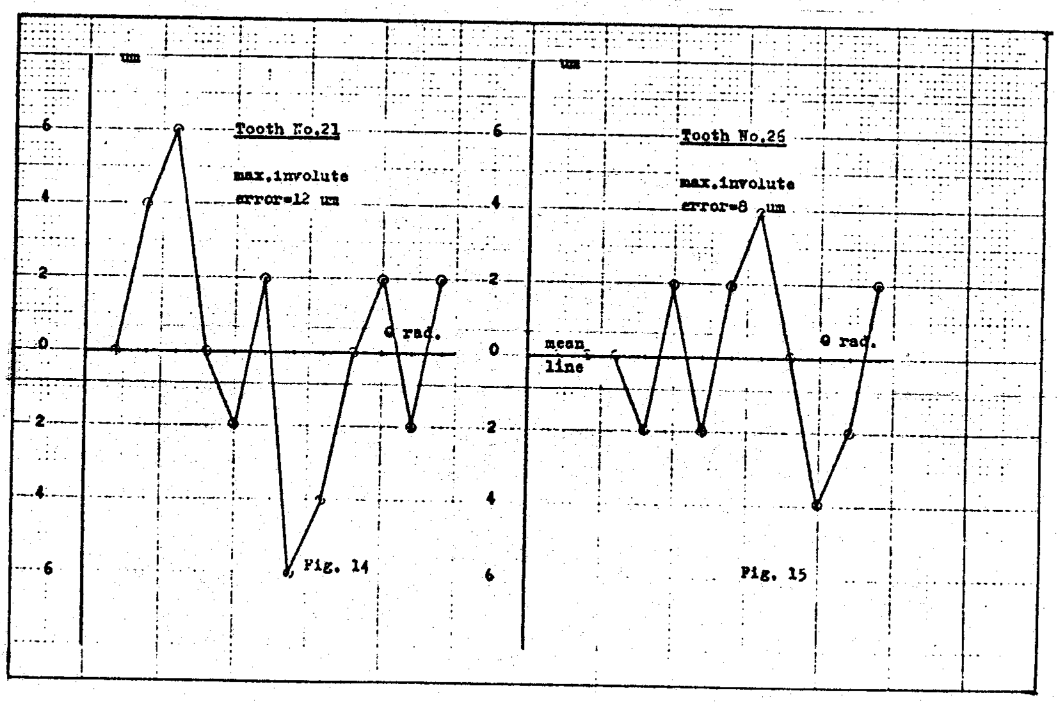


Fig.7


y (mm)
Tooth No.11
y = 34.562 θ
Fig. 8
θ rad.
x 0.017455
y (mm)
Tooth No.16
y = 34.568 θ
Fig. 9
θ rad.
x 0.017455
um
Tooth No.11
max.involute
error=12 um
θ rad.
Fig. 10
um
Tooth No.16
max.involute
error=10 um
mean
line
θ rad.
Fig. 11


y (mm)
Tooth No.21
y = 34.576 θ
Fig. 12
θ rad.
x 0.017455
y (mm)
Tooth No.26
y = 34.570 θ
Fig. 13
θ rad.
x0.017455


um
Tooth No.21
max.involute
error=12 um
θ rad.
Fig. 14
um
Tooth No.26
max.involute
error=8 um
mean
line
θ rad.
Fig. 15

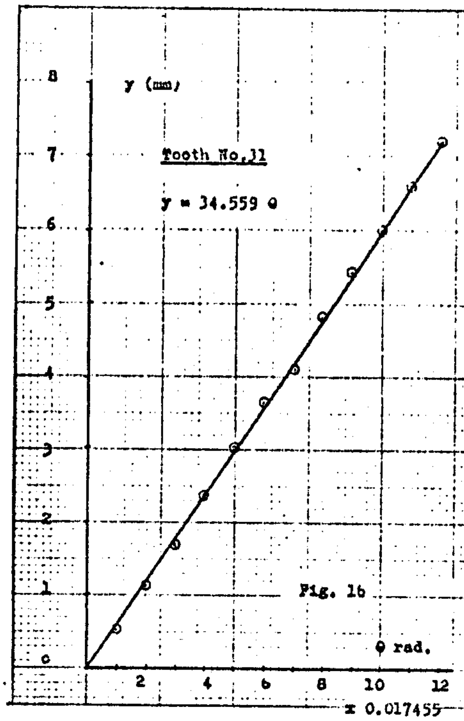


Fig. 16

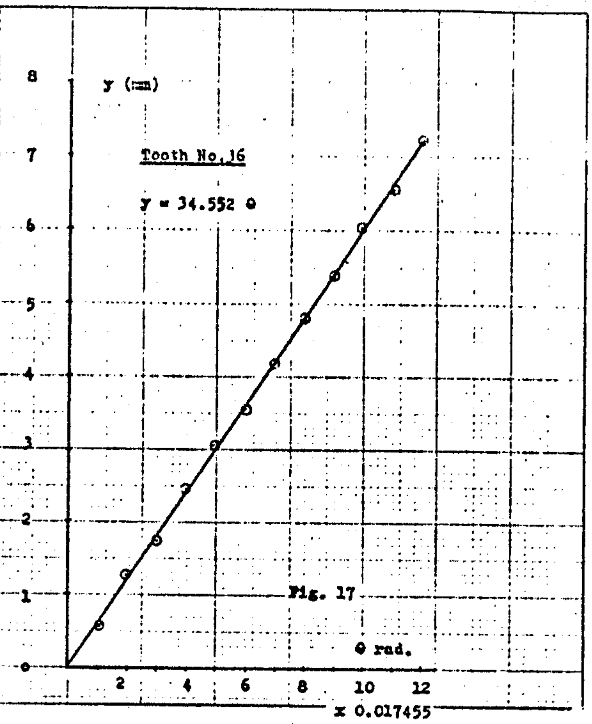


Fig. 17

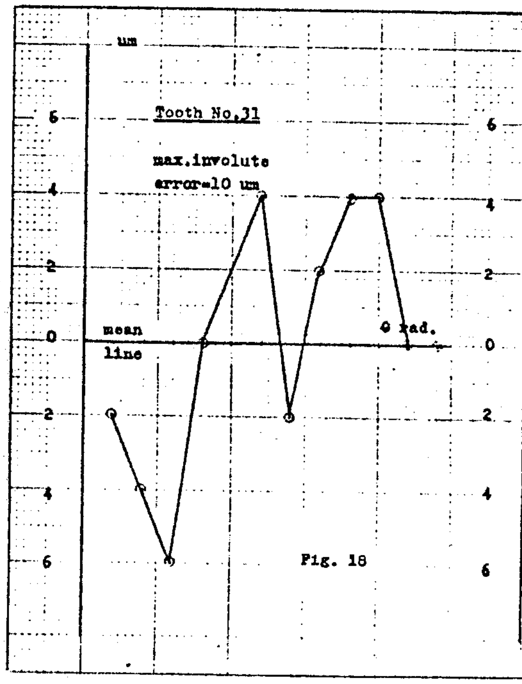


Fig. 18

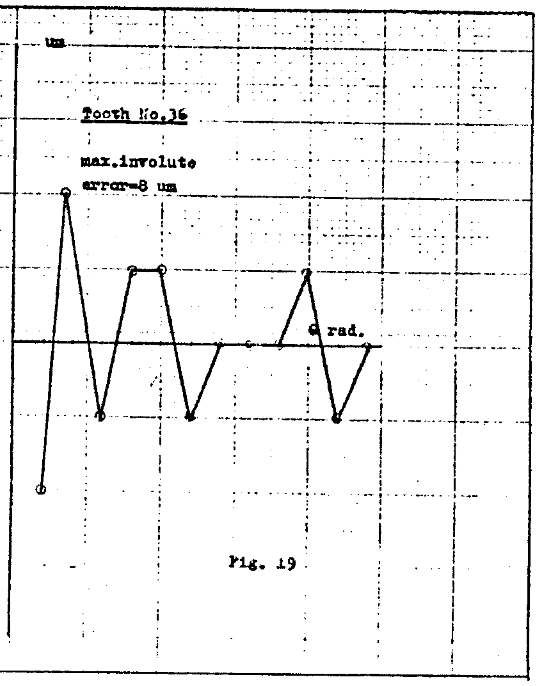


Fig. 19

A normal module is 1.75,starting from this figure we find that

$$r_b = 0.5 \text{ m N} \cos \phi$$
$$= 0.5 \times 1.75 \times 42 \times 0.9397$$
$$= 34.534 \text{ mm}$$

The value calculated is only 0.012 mm too large,so that the data regarding the gear wheel are in this way very determinable.

The non linearity in the experimental results can be referred to the involute orror exist on the measured tooth.

The variability shown in the slopes of the measured values, indicates variability in the values of the base radius,this variability can be aligned to the eccentrisity in the base circle, since measurements are taken referred to intrinsic datum (gear concentric) position with respect to the optical axis of the instrument.By analysing the variability in the base radius value the eccentricity of base circle can be detected.The eccentrisity is approximately 15 um.

Accuracy of Determination of Base Radius

Often in engineering problems the important criterion is a function of several independent varaiables.In our case base radius r_b is a function of two divided independent variables (y/θ). A definition of the ditribution of the resultant function in terms of the combination of distributions of independent variables is known as the " propagation of errors ".In our case

$$S_r^2 = \left(\frac{S_Y}{u_\theta} \right)^2 + \left(\frac{u_Y \, S_\theta}{u_\theta^2} \right)^2 \qquad (3)$$

where

S_r = resultant standard deviation of the two variables

S_y = standard deviation of y coordinates

u_y = mean value of y coordinates

u_θ = mean value of angle θ

S_θ = standard deviation of angle θ

from which it follows

$$S_r^2 = \frac{1}{u_\theta^2} \left(S_y^2 + \frac{u_y^2 \; S_\theta^2}{u_\theta^2} \right)$$

Since S_y^2 is negligible with respect to $\frac{u_y^2 \; S_\theta^2}{u_\theta^2}$, we may say

$$S_r = \frac{u_y \; S_\theta}{u_\theta^2} \qquad (4)$$

It appears that the precision of the angular measurements is decisive for the degree of precision with which the base radius is found.For the numerical example of tooth No.1 which explained before,the standard deviation S_y = 0.003 mm and let S_θ = 0.1 m rad. (approximately 10").Resultant standard deviation from equation 4

$$S_r = \frac{0.603 x 0.1}{0.0175^2} = 20 \text{ um}$$

<u>Detection of Involute Profile</u>

As mentioned before the relation between the displacement in the direction tangent to the base circle y and the angular rotation θ is linear,any non linearity in the experimintal results can be referred to the involute profile error existed on the measured tooth. For the numerical example of tooth No.1 which explained before, the mean value of the measured readings was found 0.603 mm, the deviation than that mean is the involute profile error and it was found for that tooth to be equal to 10 um, as shown in fig. 6; and so on for all the teeth.

CONCLUSION

An optical method for measuring the base radius and detecting the error in the involute profile is introduced. The method provide an accurate determination of base radius and involute error. The basic dimensions of gear can be dtermined using the measured base radius value. This method is practical, since no special instruments for gears are necessary to be used and can be applied using any optical metroscope available.

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