OUTPUT FEEDBACK STABILIZATION OF DISCRETE SYSTEMS

HISHAM M. SOLIMAH
Electrical Engineering Department, Faculty of Engineering, Cairo University, Cairo, EGYPT. Currently with the Electrical Engineering Department, Garyounis University, Bengazi, LIBYA-P.O.Box 9476.

(الموازنة با لتغذ ية المرتدة للخرج للانظمة المنفصله الزمن) .

د . هشام سليمان

الخلاصـــة:
تـم عرض طريقة لحل مشكلة موازنه النظم الخطية المنفصلة الزمن بأستخدام تغذ ية الخرج . والطريقة تعتمد على تحويل المسأله الى دالة (تمثل شرط كافى للموازنة) يجب اقلالها باستخدام الطرق الغير خطية لاقلال الدوال . وقد أعطى مثـال لايضاح الدراسة النظريـــة . .

ABSTRACT : An algorithm is presented to solve the problem of stabilizing linear time invariant discrete systems using output feedback. The algorithm is based on transforming the problem into a functional, a derived sufficient condition for stability, to be minimized using nonlinear optimization techniques. An illustrative example is given to support theoretical investigations.

I. INTRODUCTION:

The regulator or feedback stabilization problem is the basic problem that control theory attempts to solve. Many design procedures can only be initiated after a nominal stabilizing controller has been found. Recently, there has been a lot of activity in the area of state feedback stabilization of discrete time control system [1].

If not all state variables are available, as is usually the case in practice, because either some of them are not accessible or the cost makes it impractical for the designer to utilize measuring devices for every state variable. This is an output feedback stabilization problem. In this case a prediction estimator, or a current estimator [1] is used to reconstruct the state vector to implement a feedback control law. Such estimators, however, are dynamic in nature and usually of high order, thus their use is not practical when the designer deals with a high dimensional system. However, except for very special cases, there are no analytic solution available to solve the stabilization problem when the controllers order is limited to a low fixed value. Existing solutions to stabilization problem can only generate controllers that are of high enough order that arbitrary pole placement becomes possible [2]. In general, all these design methods generate unnecessarily high order controllers if stabilization is the only requirement. Extensive research in the problem of designing low order controllers has been done by many researchers [3,4,5]. The last approaches [4,5] fall into the class of model reduction problems.

The algorithm that is given depends on the parameterization of the output feedback stabilization problem. In section II, the output feedback stabilization problem is formulated as an optimization problem, and section III describes how the chosen objective function can be minimized iteratively using a gradient type minimization algorithm . In section IV an illustration example is given.

II. OUTPUT FEEDBACK CONTROLLERS

Consider the linear time invariant plant

$$X(t+1)= A\ X(t)+ B\ U(t) \quad X \in R^n, U \in R^m, Y \in R^l \qquad (1)$$
$$Y(t)= C\ X(t)$$

cascaded with static output feedback controller

$$U(t)= F\ Y(t) \qquad (2)$$

Notice that generalization to dynamic compensators of order p can be easily cast into a static output feedback problem.
The closed loop system is given by

$$X(t+1)= (A+ BFC)\ X(t)= G\ x(t) \qquad (3)$$

Let Λ denotes a symmetric set of n complex numbers(i.e. complex numbers occur in complex conjugate pairs) and let

$$F(\Lambda) = \{ F/F \in R^{mxl}, \lambda(G) \in \Lambda \} \qquad (4)$$

where $\lambda(.)$ indicates a set of eigen values. The stabilization problem is that the specification of Λ to be simply a connected region $\Omega \in D$ (the unit circle of center origin). In other words determine the family

$$F(\Omega) = \{F/ F \in R^{mxl}, \lambda(G) \subset \Omega \subset D\} \qquad (5)$$

This stabilization problem has a solution if and only if

$$\rho(G) < 1 \qquad (6)$$

where $\rho(.)$ is the spectral radius of (.). The problem is to find the matrix F such that (6) is satisfied. One way of doing this is to minimize

$$J= \rho(G) -1 \qquad (7)$$

by letting $\lambda(G)$ range over the region $\Omega \subset D$ .A solution to the stabilization problem is achieved if and only if J becomes <0 . Minimizing J can be carried out by any standard nongradient type optimization subroutine .For reasons of speed and accuracy of solution of this minimization problem, it is preferable to use a gradient based algorithm. For this purpose, the closed form expression of the gradient of the objective function J with respect to the variable matrix F is evaluated . These details are given next.

III. STABILIZATION ALGORITHM

The gradient $\partial J/\partial F$,where J is given in (7),is difficult to obtain. An easy expression can be derived if J is replaced by another objective function

$$J^* = \| G \|^2 -1 \qquad (8)$$

when J^* becomes <0 ,this represents only a sufficient condition for stability . Where $\| (.) \|$ is the Euclidean norm of (.).

Theorem: Given the objective function J^* in (8) ,the gradient of J^* with respect to the independent matrix F is

$$\partial J^* / \partial F = 2\ B^T(A+ BFC)\ C^T \qquad (9)$$

Proof:
From (8) ,we have

$$J^* = tr\ (G\ G^T) -1 \qquad (10)$$

where tr(.) denotes the trace of (.) and the superscript T is the matrix transpose. To compute $\partial J^* / \partial F$, G is changed to $G+\Delta G$ and the first order change is

$$\Delta J^* = tr(B.\Delta F.C)\ G^T + tr(G)(B.\Delta F.C)^T$$

$$= 2tr(G)^T (B.\Delta F.C) = 2\ trC.G^T B.\Delta F$$

Then the gradient matrix is given by

$$\partial J^* / \partial F = 2B^T (A+BFC)C^T \qquad Q.E.D \qquad (11)$$

The negativity of the functional (8) is a sufficient condition for stability , consequently it gives unnecessarily high gains for the feedback matrix F. To overcome this difficulty, the functional (8) is reduced iteratively and in each iteration ,stability check (7) is carried out.

IV. AN ILLUSTRATIVE EXAMPLE

Stabilize the system

$$X(t+1) = AX(t)+BU(t), Y(t)=CX(t)$$

where

$$A=\begin{bmatrix}0 & 1\\ 2.5 & 1\end{bmatrix} \quad B=\begin{bmatrix}0\\ 1\end{bmatrix}$$

$$C=[1 \quad 0]$$

From (8) , $J^*_* = 8.25 + 5\ f + f^2 - 1$

From (9) , $\partial J^* / \partial f = 2.5 + f = 0$

$\therefore f = -2.5$

V. CONCLUSIONS

The results given here are algorithmic in nature .The algorithm is very simple and attractive to solve the problem of output feedback stabilization for discrete systems.As any other optimization technique the solution depends on the number of iterations and the initial conditions started with.

VI. REFERENCES

1.K.Ogata"Discrete time control systems" Englwood Cliffs,N.J.Prentice Hall,1987

2.F.M.Brasch and J.B.Pearson"Pole placement using dynamic compensator" IEEE Trans. Autom. Contr.,Vol. AC-15,p34-43,1970

3.H.M.Soliman and R.J.Fleming"Minimal order decentralized stabilizers for interconnected power systems" Int.J.Systems Science,Vol. 14,No.7,p765-76,1983

4.A.Yousuff and R.E.Skelton "Controller reduction by component cost analysis" IEEE Trans. Autom. Contr. Vol. AC-29,p520-30,1984

5.E.A.Jonckheereand L.M.Silverman " A new set of ninvarients for linear systems-Application to reduced order compensator design" ibid,Vol. 28,p953-64,1983