Menoufia University

Faculty of Engineering, Shebin El kom

Mechanical Power Engineering Department

First semester Examination,2013-2014

Date of Exam: 5/1/2014

Subjec : Engineering Matrhematics

Year: 2^{nd} year Mech.(ثانية قوى حديث)

Time allowed: 3 hours

Total marks: 100 marks

ATTEMPT TO SOLVE ALL QUESTIONS.

Q.1

(A) If $\bar{F} = 2x^2z\bar{i} - (xy^2z)\bar{j} + 3yz^2\bar{k}$ find,

(i) $div\bar{F}$ **(ii)** $grad(div\bar{F})$ **(iii)** $Laplacian(div\bar{F})$

(B) Evaluate using Green's theorem $\int_C (3x+4y)dx + (x-3y)dy$ where C is a circle of radius a.

Q.2

(A) For any Scalar function ϕ, prove that $curl(\phi\, grad\, \phi) = 0$

(B) Is $\bar{F} = (y + z\cos(xz))\bar{i} + x\bar{j} + x\cos(xz)\bar{k}$ is conservative?

(i) If it is conservative find its scalar potential.

(ii) Evaluate $\oint_C \bar{F}\cdot d\bar{r}$ along any simple closed curve.

(iii) Evaluate $\int_C \bar{F}\cdot d\bar{r}$ along the line segment from (0,0,0) to $(1,1,\pi)$.

Q.3

(A) For any two vectors $\bar{A}$ and $\bar{B}$ prove that

(i) $|\bar{A}\times\bar{B}|^2 = A^2B^2 - |\bar{A}\cdot\bar{B}|^2$

(ii) The projection of $\bar{A}$ on $\bar{B}$ is equal to $\bar{A}\cdot b$, where b is a unit vector in the direction of $\bar{B}$

(iii) Evaluate $\oint (x^2y\cos x + 2xy\sin x - y^2e^x)\,dx + (x^2\sin x - 2ye^x)\,dy$

around the hypocycloid $x^{2/3} + y^{2/3} = a^{2/3}$

	(B)	Evaluate $\iint_S \nabla \times \bar{F} \cdot \bar{n}\, ds$ (by **Stoke's** theorem) for $\bar{F} = (x^2 + y + 2)\bar{i} + 2xy\bar{j} - (3xyz + z^3)\bar{k}$ Where S is upper half surface of sphere $x^2 + y^2 + z^2 = 9$ above the xy-plane.
Q.4	**(A)**	Evaluate the following integrals: i) $\Gamma(-5/2)$ ii) $\int_0^1 \frac{dx}{\sqrt{-\ln x}}$ iii) $\int_0^{\pi/2} \sin^6 \theta\, d\theta$ iv) $\int_0^{\infty} \sqrt{y}\, .\, e^{-y^3}\, dy$
	(B)	Prove that i) $\beta(m,n) = 2\int_0^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta\, d\theta$ ii) $\int_0^{2\pi} \sin^8 \theta\, d\theta = \frac{35\pi}{64}$
	(C)	Using divergence theorem to evaluate the surface integral $\iint_S \bar{F} \cdot \bar{n} ds$ where $\bar{F} = x^2\bar{i} + y^2\bar{j} + z^2\bar{k}$ and the surface S is the surface of the cube $0 \le x \le 1,\ 0 \le y \le 1,\ 0 \le z \le 1$.
Q.5	**(A)**	Discuss each of the following: **(i)** Standard form of LP problem. **(ii)** Unbounded solution. **(iii)** Artificial variable.
	(B)	Solve the following LP problem using **simplex** method, $Max\ Z = 12x_1 + 8x_2$ S.t. $5x_1 + 2x_2 \le 150$, $2x_1 + 3x_2 \le 100$, $4x_1 + 2x_2 \le 80$, $x_1, x_2 \ge 0$. then check your answer using graphical method.