ON THE FORMULATION OF FINITE ELEMENT MODELS OF CAM MECHANISMS.

By

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SYNOPSIS

This paper describes a general procedure of kineto elasto-dynamic analysis of cam mechanisms based on the finite element approach. The present discrete technique can be utilized to provide various versions of finite element models of planar or spatial more complicated cam mechanisms. The procedure is introduced by utilizing the cam operated transfer mechanism found in Koster's Work[1].

1 - INTRODUCTION

The dynamic analysis of cam mechanisms as prefectly rigid systems has become increasingly inadequate, since the necessary prerequisities for setting up the vibration would not be satisfied. To improve the representation of dynamic behavior of a cam mechanism, various methods have been performed such as for example the methods by Matlhew and Tesar[3], Eiss[4], Bloom and Radcliffe[5]. There is however a common criticism to the previously mentioned analysis concerning the indiscrminate modelling technique. In reference (1), Koster gave a Kineto-elasto-dynamic analysis method for which the common drawbacks of the mentioned methods can be avoided, but still Koster's technique is sufficiently applicable for those simple cam mechanisms with low degrees of freedom and still many questions on the modelling of the cam shaft remain.

To permit a closer simulation of the dynamic behaviour of actual complicated cam mechanisms than was possible with simple models, the finite element approach has been utilized in the present analysis. Since this method (7,8) is an efficient tool for

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cam operated transfer mechanism of figure (2) represented by 15 DF finite element model consists of a 12-DF simulated follower set and of a 3-DF simulated cam set as shown.

The follower set is modelled by connecting a series of links and each link may be simulated by one element performing a typical type of motion such as longitudnal, torsional, and or fleuxeral. With regard to the topology of the set, the individual elements are meeting at either pin or rigid joints. It is of interest to note that a series of dynamic models with lower degrees of freedom may be generated by eliminating the selected number of nodal generalized forces or of nodal generalized coordinates concerned the simulated follower set. Equivalently the sizes of mass and stiffness matrices are reduced either by the condensation of matrices or by the elimination of the particular number of rows and columns. These concepts will be visualized through the reduction of the simulated follower set to be as derived by Koster (1,2).

In the modelling of the flexible cam set, the camshaft is represented by an assembly of flexural torsional beams interconnected at a rigid joint where as the cam element, (inertial element), is assumed to be lumped. Using the conditions of invariance of the kinetic and potential energies under coordinate transformation β the mass matrix m_e and the stiffness matrix K_e of the e th element shown in figure 1, can be easly formulated.

The formulation of characteristic M_q and K_q matrices of the entire mechanism are then built up by adjoing the characteristic matrices of the cam and of the follower sets. The adjoing process is carried out by pre and post multiplication of the element characteristic matrices by the coupling matrix which represents the compatibility conditions through the nodal displacements. In that view the presence of coupling may be represented schematatically by a kinematic coupling set, (governed by the cam curve slope) as found in reference (1).

The practical use of the proposed method is usually in need to digital computers, since the higher order of the characteristic matrices, the better will be the accuracy of the analysis.

3 - THE MODELLING OF THE CAM TRANSFER MECHANISM:

The simulated cam mechanism is considered as a combination of the follower set formed from five structural elements and of the cam set formed from three elements. Both sets are coupled by means of the coupler set which is simulated by a kinematic mechanism (1,2) as shown in figure (3).

In the uncoupled position of the follower set, the independent parameters q_j^f $(j = 1,2,\dots,12)$ are selected as the generalized coordinates. In the presence of coupling an auxiliarly dependent coordinate q^* may be also utilized for simulating the cam action, (the motion machined in the cam).

The generalized coordinates q_j^h $(j = 1, \dots,4)$ are utilized for describing the configuration of the rigidly supported cam set. Hereby the configuration of the simulated mechanism can be completely described by employing q_j $(j = 1,2,\dots,16)$ generalized coordinates.

Figure (3) shows that the model of the entire mechanism consists of three submodels: follower, cam and coupling sets. In the modelling process the typical element is regarded as an elastic element, lumped masses and or rigid massless elements.

3-1: Modelling of the Follower Set:

Follower set represented by that 12-DF model consists of the finite elements (1) up to (5) interconnected at two active pin joints (I, II) and two active rigid joints (III, IV). Taking into account the compatibility conditions, the location of the set of generalized coordinates q_1^f .... q_{12}^f are selected as shown in figure (4). In the coupled position of the follower set, The auxiliarly generalized coordinate q^*, (which is utilized to simulate the cam action) is located at the proper distance (a) from the passive joint O.

A typical element (e) of mass ρ is regarded an elastic element of longitudnal rigidity EA, torsional rigidity GJ and fleuxeral rigidity EI, having a uniform cross-sectional area A across its length l. The element mass and stiffness matrices can be shown (7,8) to be

$$\underline{m}_e = \frac{\rho}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}, \quad \underline{K}_e = \frac{KA}{l}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \text{for } e = 2 \quad \ldots.(1)$$

$$\underline{m}_e = \frac{I_p}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}, \quad \underline{K}_e = GJ/l\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \text{for } e = 4 \quad \ldots.(2)$$

$$\underline{m}_e = \frac{\rho}{420}\begin{bmatrix} 156 & 22l & 54 & -13l \\ & 4l^2 & 13l & -3l^2 \\ \text{symmetric} & & 156 & -22l \\ & & & 4l^2 \end{bmatrix}, \quad \underline{K}_e = \frac{EI}{l^3}\begin{bmatrix} 12 & 6l & -12 & 6l \\ & 4l^2 & -6l & 2l^2 \\ \text{symmetric} & & 12 & -6l \\ & & & 4l^2 \end{bmatrix} \text{for } e=3,5 \quad \ldots..(3)$$

In the coupled position of the follower set it may be convenient to subdivide the input link, (at the location of the auxiary coordinate q*), into two fleuxeral elements as shown in figure (5). The mass and stiffness matrices measured in a local system (δ_1^2, δ_2^2, δ_3^2, δ_4^2) can be synthesized by adjoing the characteristic matrices of two elements, using the elimination and condensation techniques, here as.

$$\underline{m}_1 = \rho A\begin{bmatrix} \frac{13}{35}l + \frac{a}{1680} & {}^{11}/_{210}(b^2-a^2) - \frac{a^2}{560} & 9b/70 & -13b^2/420 \\ & a^3/560 + \frac{a^3+b^3}{105} & 13b^2/420 & -b^3/140 \\ \text{symmetric} & & 13b/35 & -11b^2/420 \\ & & & b^3/105 \end{bmatrix}_{4\times4}$$

$$\underline{K}_1 = \begin{bmatrix} 12EI/a^3+12EI/b^3 & 6EI/b^2-9EI/a^2 & -12EI/b^3 & 6EI/b^2 \\ & 5EI/a + 4EI/b & -6EI/b^2 & 2EI/b \\ \text{symmetric} & & 12I/b^3 & -6EI/b^2 \\ & & & 4EI/b \end{bmatrix}_{4\times4} \quad \ldots(4)$$

In figure (5) the elements of the follower set are shown seperated. Appropriate displacments are labelled on each, measured in the local and in the gobal coordinate systems. With $\lambda = \mathrm{Cos}\,\phi$ and $\mu = \sin\phi$, a transformation matrix R_e of the typical element (e), figure (1), may be defined (6,7).

$$R_e = \begin{bmatrix} \lambda & \mu & 0 & 0 \\ 0 & 0 & \lambda & \mu \end{bmatrix}_e \qquad \text{for } e = 2,4 ,$$

and

$$R_e = \begin{bmatrix} -\mu & \lambda & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\mu & \lambda & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}_e \qquad \text{for } e = 1,3,5 \quad .....(5)$$

The following holds

$$\delta_e = R_e \; U_e$$

and .....(6)

$$\dot{\delta}_e = R_e \; \dot{U}_e$$

Where δ_e and U_e are sets of displacements at the two ends of the element in local and global coordinate systems as shown in figures (1) and (5). The invariance of kinetic and strain energies under coordinate transformation, such as:

$$T_e = \frac{1}{2} \; \dot{\delta}_e^T \; \underline{m}_e \; \dot{\delta}_e = \frac{1}{2} \; \dot{U}_e^T \; m_e \; \dot{U}_e \qquad ....(7)$$

and

$$V_e = \frac{1}{2} \; \delta_e^T \, \underline{K}_e \; \delta_e = \frac{1}{2} \; U_e^T \, K_e U_e, \qquad ....(8)$$

facilitate the expressing the elemnt mass and stiffness matrices of the e <u>th</u> element:

$$\underset{(S\times S)}{m_e} = R_e^T \underline{m}_e R_e = \begin{bmatrix} m_{11}^e & m_{12}^e & \cdots\cdots & m_{1S}^e \\ & & \ddots & \vdots \\ \text{symmetric} & & & m_{SS}^e \end{bmatrix}$$

and

$$K_{e(S\times S)} = R_e^T \underline{K}_e R_e = \begin{bmatrix} K_{11}^e & K_{12}^e & \cdots\cdots & K_{1S}^e \\ & \ddots & & \vdots \\ \text{symmetric} & & & K_{SS}^e \end{bmatrix} \quad \ldots\ldots(9),$$

Where S is the number of nodal displacements of the $e^{\underline{th}}$ element. By the help of the previous equations, the symmetric element mass and stiffness matrices measured in a global system for the all isolated elements shown in figure (5) may be expressed as.

$$\left[m_2\right] = \frac{\rho}{6}\left[N_2\right] \quad , \quad \left[K_2\right] = \frac{EA}{l^2}\left[S_2\right],$$

$$\left[m_3\right] = \frac{\rho}{420}\left[N_3\right] \quad , \quad \left[K_3\right] = EI/l_3^3\left[S_3\right],$$

$$\left[m_4\right] = \frac{I_\rho}{6}\left[N_4\right] \quad , \quad \left[K_4\right] = \frac{GJ}{l}\left[S_4\right],$$

$$\left[m_5\right] = \rho/420\left[N_5\right] \quad , \quad \left[K_5\right] = EI/l_5^3\left[S_5\right], \quad \ldots\ldots(10)$$

Where the symmetric matrices $[N]$ and $[S]$ are defined:

$$\left[N_e\right] = \begin{bmatrix} 2\lambda^2 & 2\lambda\mu & \lambda^2 & \lambda\mu \\ & 2\mu^2 & \mu\lambda & \mu^2 \\ \text{symmetric} & & 2\lambda^2 & 2\lambda\mu \\ & & & 2\mu^2 \end{bmatrix}_e , \quad \left[S_e\right] = \begin{bmatrix} \lambda^2 & \lambda\mu & -\lambda^2 & -\lambda\mu \\ & \mu^2 & -\lambda\mu & -\mu^2 \\ \text{symmetric} & & \lambda^2 & \lambda\mu \\ & & & \mu^2 \end{bmatrix}_e \quad \text{for } e=2,4,$$

and

$$\left[N_e\right] = \begin{bmatrix} 156M^2 & -156\lambda M & -22M\lambda & 54M^2 & -54Ml_e & 13Ml_e \\ & 156\lambda^2 & 22\lambda l_e & -54\lambda M & 54\lambda^2 & -13\lambda l_e \\ & & 4l_e^2 & -13Ml_e & 13\lambda l_e & -3l_e^2 \\ \text{symmetric} & & & 156M^2 & -156\lambda M & 22M\lambda \\ & & & & 156\lambda^2 & -22M\lambda \\ & & & & & 4l_e^2 \end{bmatrix}_e ; \left[S_e\right] = \begin{bmatrix} 12M^2 & -12\lambda M & -6l_e M & -12M^2 & 12\lambda M & -6Ml_e \\ & 12\lambda^2 & 6\lambda l_e & 12\lambda M & -12\lambda^2 & 6\lambda l_e \\ & & 4l_e^2 & 6Ml_e & -6\lambda l_e & 2l_e^2 \\ \text{symmetric} & & & 12M^2 & -12\lambda M & 6Ml_e \\ & & & & 12\lambda^2 & -6\lambda l_e \\ & & & & & 4l_e^2 \end{bmatrix}_e \quad \text{for } e = 3,5.$$

In that way the mass and stiffness matrices of the input link (1) can be expressed as:

$$\left[m_1\right] = \begin{bmatrix} m^{1}_{11} & m^{1}_{12} & m^{1}_{13} & m^{1}_{14} & m^{1}_{15} & m^{1}_{16} \\ & m^{1}_{22} & m^{1}_{23} & m^{1}_{24} & m^{1}_{25} & m^{1}_{26} \\ & & m^{1}_{33} & m^{1}_{34} & m^{1}_{35} & m^{1}_{36} \\ & & & m^{1}_{44} & m^{1}_{45} & m^{1}_{46} \\ \text{symmetric} & & & & m^{1}_{55} & m^{1}_{56} \\ & & & & & m^{1}_{66} \end{bmatrix}_{6\times6} , \left[K_1\right] = \begin{bmatrix} K^{1}_{11} & K^{1}_{12} & K^{1}_{13} & K^{1}_{14} & K^{1}_{15} & K^{1}_{16} \\ & K^{1}_{22} & K^{1}_{23} & K^{1}_{24} & K^{1}_{25} & K^{1}_{26} \\ & & K^{1}_{33} & K^{1}_{34} & K^{1}_{35} & K^{1}_{36} \\ & & & K^{1}_{44} & K^{1}_{45} & K^{1}_{46} \\ \text{symmetric} & & & & K^{1}_{55} & K^{1}_{56} \\ & & & & & K^{1}_{66} \end{bmatrix}_{6\times6} \quad \ldots\ldots\ldots\ldots\ldots(11)$$

where the inertial coefficients $\overset{1}{m}_{ij}$ and the elastic coefficients $\overset{1}{K}_{ij}$ are given in the app endix.

Superimposing the kinetic and strain energies of the five elements shown in figure (5), the total energies may be expressed, using equations (7) and (8), as

$$T = \sum_{e=1}^{5} T_e = \frac{1}{2} \dot{U}^T M_u \dot{U} \quad \text{.................(12)}$$

and

$$V = \sum_{e=1}^{5} V_e = \frac{1}{2} U^T K_u U \quad \text{.................(13)}$$

Here the mass M_u and stiffness K_u matrices of all elements comprising the follower set are derived by locating the element matrices of the five elements along the diagonal in the respective order (6). here has

$$M_u = \begin{bmatrix} [m_1] & . & . & . & [m_5] \end{bmatrix}_{24x24} \quad \text{...............(14)}$$

and

$$K_u = \begin{bmatrix} [K_1] & . & . & . & [K_5] \end{bmatrix}_{24x24} \quad \text{...............(15)}$$

The relationships between the generalized coordinates q^*, q_1^f .... q_{12}^f and the nodal displacements $U_1, U_2,$ ...... U_{24} of all elements of the follower set may be expressed as

$$U = B\,q \quad \text{.................(16)}$$

and

$$\dot{U} = B\,\dot{q} \quad \text{.................(17)}$$

where B is the connecting matrix of order (24 x 13), which can be easly deduced according to the compatibility conditions through out the follower set.

Therefore the mass and stiffness matrices of the follower set in the coupled position may be expressed, using equations: (14) + (17):

$$M_{fc} = B^T M_u B$$

$$M_{fc} = \left[\begin{array}{c:cccccccccccc}
\overset{1}{m}_{11} & \overset{1}{m}_{14} & \overset{1}{m}_{15} & \overset{1}{m}_{16} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hdashline
 & \overset{1}{m}_{44}+\overset{2}{m}_{11} & \overset{1}{m}_{54}+\overset{2}{m}_{12} & \overset{1}{m}_{46} & \overset{2}{m}_{13} & \overset{2}{m}_{14} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & & \overset{1}{m}_{55}+\overset{2}{m}_{22} & \overset{1}{m}_{56} & \overset{2}{m}_{23} & \overset{2}{m}_{24} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & & & \overset{1}{m}_{66} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & & & & \overset{2}{m}_{33}+\overset{3}{m}_{11} & \overset{2}{m}_{34} & \overset{3}{m}_{12} & \overset{3}{m}_{13} & \overset{3}{m}_{16} & 0 & 0 & 0 & 0 \\
 & & & & & \overset{2}{m}_{44} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & & & & & & \overset{3}{m}_{22} & \overset{3}{m}_{23} & \overset{3}{m}_{26} & 0 & 0 & 0 & 0 \\
 & & & & & & & \overset{3}{m}_{33} & \overset{3}{m}_{36} & 0 & 0 & 0 & 0 \\
 & & & & & & & & \overset{3}{m}_{66}+\overset{4}{m}_{11} & \overset{4}{m}_{12} & 0 & 0 & 0 \\
 & \text{Symmetric} & & & & & & & & \overset{4}{m}_{22}+\overset{5}{m}_{33} & \overset{5}{m}_{34} & \overset{5}{m}_{35} & \overset{5}{m}_{36} \\
 & & & & & & & & & & \overset{5}{m}_{44} & \overset{5}{m}_{45} & \overset{5}{m}_{46} \\
 & & & & & & & & & & & \overset{5}{m}_{55} & \overset{5}{m}_{56} \\
 & & & & & & & & & & & & \overset{5}{m}_{66}
\end{array}\right]_{13\times 13} \quad \text{.....(18)}$$

Similarly the stiffness matrix

$$K_{fc} = B^T K_u B \quad \text{..............................(19)}$$

The expand form equation (19) is simillar to that given in equation (18) where the inertial coefficient m_{ij}^{e} and the elastic coefficient K_{ij}^{e} are given in the appendix.

In what follows, the derivation of relationship will be limited to the inertia properties, where as the elastic properties can be deduced similarly.

The (12 x 12) mass and stiffness matrices M_{fun} and K_{fun} of the follower set in the uncoupled position can be deduced by eliminating the first row and column, (corresponding to q^{*}), from the matrices M_{fc} and K_{fc} as indicated by dashed lines in equation (18).

For many reasons it may be required to simulate the actual system with lower degrees of freedom model such as the shown 5-DF model of the follower set. In this case the (5 x 5) mass and stiffness matrices may be obtained by eliminating, the rows and columns which correspond to q^{*}, q_2^{f}, q_3^{f}, q_5^{f}, q_6^{f}, q_7^{f}, q_{11}^{f} and q_{12}^{f}, figure (6), from the (13x13) M_{fc} and K_{fc} matrices.

The resulting matrices are:

$$M_f = \begin{bmatrix} \overset{1}{m}_{44} + \overset{2}{m}_{11} & \overset{2}{m}_{13} & 0 & 0 & 0 \\ & \overset{2}{m}_{33} + \overset{3}{m}_{11} & \overset{3}{m}_{16} & 0 & 0 \\ & & \overset{3}{m}_{66} + \overset{4}{m}_{11} & \overset{4}{m}_{12} & 0 \\ \text{symmetric} & & & \overset{4}{m}_{22} + \overset{5}{m}_{33} & \overset{5}{m}_{34} \\ & & & & \overset{5}{m}_{44} \end{bmatrix} \quad \ldots\ldots(20)$$

N.B. The (5x5) stiffness matrix K_f has a similar form given in equation (20).

The comparison between the matrices given in equation (20) and that derived in reference (1) shows that the developed matrices are more accurate for expressing the inertial and elastic properties, since the mutual effects of various types of deformations are still maintained.

A more precise representation of the inertial and elastical properties of the simulated follower set can be obtained by using the condensation technique (6) for the mass and stiffness matrices, since the kinematic compatibility conditions are still remain. For example the elimination of the nodal generalized forces Q^*, Q_2, Q_3, Q_5, Q_6, Q_7, Q_{11} and Q_{12} is equivalent to the reduction of the (13 x 13) M_{fc} and K_{fc} matrices as:

$$\underset{(5x5)}{M_r} = m_{11} - m_{12} \; m_{22}^{-1} \; m_{21} \, ,$$

and

$$\underset{(5x5)}{K_r} = K_{11} - K_{12} \; K_{22}^{-1} \; K_{21} \qquad \text{.........(21)}$$

where

$$M_{fc} = \left[\begin{array}{c|c} \underset{(5x5)}{m_{11}} & \underset{(5x8)}{m_{12}} \\ \hline \underset{(8x5)}{m_{21}} & \underset{(8x8)}{m_{22}} \end{array}\right], \quad K_{fc} = \left[\begin{array}{c|c} \underset{(5x5)}{K_{11}} & \underset{(5x8)}{K_{12}} \\ \hline \underset{(8x5)}{K_{21}} & \underset{(8x8)}{K_{22}} \end{array}\right]$$

The expand forms of the reduced matrices may be expressed, using equations (18) (19) and (21):

$$M_r = \begin{bmatrix} {}^{1}m_{44} + {}^{2}m_{11} - A_{22} & {}^{2}m_{13} - A_{23} & 0 & 0 & 0 \\ & {}^{2}m_{33} + {}^{3}m_{11} - A_{33} & {}^{3}m_{16} - A_{34} & 0 & 0 \\ & & {}^{3}m_{66} + {}^{4}m_{11} - A_{44} & {}^{4}m_{12} & 0 \\ \text{Symmetric} & & & {}^{4}m_{22} + {}^{5}m_{33} - A_{55} & {}^{5}m_{34} - A_{56} \\ & & & & {}^{5}m_{44} - A_{66} \end{bmatrix}_{5\times5}$$

$$, K_r = \begin{bmatrix} {}^{1}K_{44} + {}^{2}K_{11} - B_{22} & {}^{2}K_{13} - B_{23} & 0 & 0 & 0 \\ & {}^{2}K_{33} + {}^{3}K_{11} - B_{33} & {}^{3}K_{16} - B_{34} & 0 & 0 \\ & & {}^{3}K_{66} + {}^{4}K_{41} - B_{44} & {}^{4}K_{12} & 0 \\ \text{Symmetric} & & & {}^{4}K_{22} + {}^{5}K_{33} - B_{55} & {}^{5}K_{34} - B_{56} \\ & & & & K_{44} - B_{66} \end{bmatrix}_{5\times5}$$

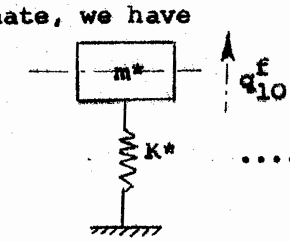
....................(22)

Where A_{ij} and B_{ij} are functions of the mass and elastic coefficients of the individual elements in global coordinate system as shown in the appendix.

In that way, a more simplified dynamic model can be derived by further reducing the characteristic matrices. For example a single degree of freedom simulated follower set may be derived by recondensation of the M_r and K_r matrices. For example if q^f_{10} is selected to be the generalized coordinate, we have

$$m^* = R_{22} - \frac{R_{21}R_{12}}{R_{11}}$$

$$K^* = S_{22} - \frac{S_{21}S_{12}}{S_{11}}$$



....(23)

It is seen that the form of equation (23) is more convenient for the dynamic investigation, since the parameters R_{11} .... S_{22} contain the inertial and elastical parameters of the elements comprising the follower set; (See the appendix).

3-2 Modelling of the Cam Set:

The rigidly supported cam set represented by that 4-DF model consists of three structural elements interconnected at three active nodes as shown in figure (7).

The camshaft is discretized into two fleuxeral-tersional elements 1 and 3 interconnected at the rigid joint II. The disk cam is idealized as nonelastic element lumped at the nodal point II as shown in figure (8).

Assuming uniform cross-section fleuxeral-torsional structural element. The characteristic matrices can be shown (6,7) to be

$$m_e = \rho \begin{bmatrix} 13/35 & 0 & 0 & -13/35 & 0 & 0 \\ & 13/35 & 0 & 0 & -13/35 & 0 \\ & & I_p/3A & 0 & 0 & -I_p/3A \\ \text{Symmetric} & & & 13/35 & 0 & 0 \\ & & & & 13/35 & 0 \\ & & & & & I_p/3A \end{bmatrix}, \quad K_e = \frac{EI}{l^3} \begin{bmatrix} 12 & 0 & 0 & -12 & 0 & 0 \\ & 12 & 0 & 0 & -12 & 0 \\ & & \psi l^2 & 0 & 0 & -\psi l^2 \\ \text{Sym.} & & & 12 & 0 & 0 \\ & & & & 12 & 0 \\ & & & & & \psi l^2 \end{bmatrix} \quad \text{..................(24)}$$

Where ψ is a ratio GJ/EI.

Assuming the cam element of mass γ and of mass moment of inertia $\mathcal{J}$ the mass matrix may be shown to be

$$m_2 = \lceil \gamma \quad \gamma \quad \mathcal{J} \quad 0 \quad 0 \quad 0 \rfloor \quad \text{...........(25)}$$

With $\lambda_I = \cos\phi_I$ and $\mu_I = \sin\phi_I$ at the end I, the (6 x 6) transformation matrix may be defined as

$$R_e = \left[\begin{array}{c|c} R_I & 0 \\ \hline 0 & R_I \end{array}\right], \quad R_I = \begin{bmatrix} \lambda_I & \mu_I & 0 \\ -\mu_I & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \ldots\ldots\ldots\ldots(26)$$

Therefore the (6 x 6) characteristic matrices of the structural element may be expressed as:

$$m_e = \left[\begin{array}{ccc|ccc} 13/35 & 0 & 0 & -\frac{13}{35}P & -\frac{13}{35}v & 0 \\ & 13/35 & 0 & -\frac{13}{35}v & -\frac{13}{35}P & 0 \\ & & I_p/3A & 0 & 0 & -I_p/3A \\ \hline & & & 13/35 & 0 & 0 \\ \text{Symmetric} & & & & 13/35 & 0 \\ & & & & & I_p/3A \end{array}\right], \quad K_e = \frac{EI}{l^3}\left[\begin{array}{ccc|ccc} 12 & 0 & 0 & -12P & -12v & 0 \\ & 12 & 0 & -12v & -12P & 0 \\ & & \psi l^2 & 0 & 0 & \psi l^2 \\ \hline & & & 12 & 0 & 0 \\ \text{Symmetric} & & & & 12 & 0 \\ & & & & & \psi l^2 \end{array}\right]$$

$$\ldots\ldots\ldots\ldots(27)$$

Where $P = \lambda_I\lambda_{II} + \mu_I\mu_{II} = \cos(\phi_{II} - \phi_I)$, $v = \lambda_I\mu_{II} - \mu_I\lambda_{II} = \sin(\phi_{II} - \phi_I)$

With the help of equations (12) to (15), the kinetic and strain engeries of the cam set are then given by

$$T = T_1 + T_2 + T_3 \quad , \quad V = V_1 + V_3 \ ,$$

and the mass and stiffness matrices of isolated elements comprising the cam set are

$$M_u = \begin{bmatrix} [m_1] & [m_2] & [m_3] \end{bmatrix}, \quad K_u = \begin{bmatrix} [K_1] & [0] & [K_2] \end{bmatrix} \ldots\ldots\ldots\ldots(28)$$

The relationships between the generalized coordinates q_1^h ... q_4^h and the nodal displacements U_1 ....U_{18}, figure(8); may be expressed with the same forms given in equations (16) and (17). Therefore the mass and stiffness matrices of the cam set are then given by

$$M_h = \left[\begin{array}{ccc|c} {}^{1}m_{44}+{}^{2}m_{11}+\gamma & 0 & 0 & 0 \\ & {}^{1}m_{55}+{}^{2}m_{22}+\gamma & 0 & 0 \\ \text{Symmetric} & & {}^{1}m_{66}+{}^{2}m_{33}+\mathcal{J} & {}^{2}m_{16} \\ \hline & & & {}^{2}m_{66} \end{array}\right] = \left[\begin{array}{ccc|c} m_1 & 0 & 0 & 0 \\ & m_2 & 0 & 0 \\ \text{Sym.} & & m_3 & m_{34} \\ \hline & & & m_4 \end{array}\right]_{4x4},$$

$$K_h = \left[\begin{array}{ccc|c} {}^{1}K_{44}+{}^{3}K_{11} & 0 & 0 & 0 \\ & {}^{1}K_{55}+{}^{3}K_{22} & 0 & 0 \\ \text{Symmetric} & & {}^{1}K_{66}+{}^{3}K_{33} & {}^{3}K_{16} \\ \hline & & & {}^{3}K_{66} \end{array}\right] = \left[\begin{array}{ccc|c} K_1 & 0 & 0 & 0 \\ & K_2 & 0 & 0 \\ \text{Symmetric} & & K_3 & K_{34} \\ \hline & & & K_4 \end{array}\right]_{4x4} \quad .....(29)$$

Where the inertial and elastic coefficients are given in the appendix. If the inertia couple concerned with q_4^h is neglected, one get a statically and dynamically decoupling model similar to that derived in reference (1). The (3 x 3) mass and stiffness matrices are then obtained, as indicated by dashed lines in equation (29).

Refer to (1,2), the coupling between the follower and the cam sets is modeled kinematically a plane submechanism formed from a set of massless rigid links. The topology of the coupling set shown in figure (9) is governed by the transmission ratio(i) which is continuously variable and depends mainly on the cam curve slope (2).

The simple model derived in references (1, 2, 3, 4, 5) can be obtained as special cases of the simulated model developed here, such as for example the model derived by Bloom (5) can be deduced by neglecting the mass and flexability of the cam set in the vertical and tangential directions, whilst the neglection of the inertia and elastic parameters concerning the tangential and torsional direction leads to the Eiss's model (4) as shown in figure (10-a) and (10-b) respectively.

It may be if interest to note that the inertial and elastical parameters (m_1, m_3, K_1, K_3) are expressed in terms of the inertial and elestical parameters of the elements comprising the cam set. Therefore the effect of local modifications on the dynamic characteristics of the system can be easly investigated.

3-3 Synthesis of the characteristic matrices of the simulated Cam-operated transfer Mechanism:

The characteristic matriqes M_q and K_q of the entire mechanism are synthesized by the respective, pre and post multiplication of the characteristic matrices of the follower and cam sets in the uncoupled positions by a transformation matrix A. The latter matrix which specifies the compatibility condition between the two sets, relates the (16 x 1) q_{un} uncoupled vector with the (15 x 1) q_c coupled vector of the entire mechanism here as

$$q_{un} = A \quad q_c \qquad \ldots\ldots\ldots\ldots\ldots\ldots(30)$$

where from definitions, we have

$$q_{un}^T = \begin{bmatrix} q^* & q_f^T & q_h^T \end{bmatrix}^T,$$

and

$$q_c^T = \begin{bmatrix} q_f^T & & q_h^T \end{bmatrix}^T,$$

The auxiliarly coordinate q^*, (which simulates the motion machined in the cam), and the q_1^h, q_2^h and q_3^h, (concerned with the vertical, tangential and torsional deformations of the cam set as shown in figure 9), are related (1) in the following matrix form

$$q^* = D \ q_h \qquad \ldots\ldots\ldots\ldots\ldots\ldots(31)$$

Where D is the meshing vector given by,

$$D = \lfloor -1 \quad i \quad i \rfloor \qquad \ldots\ldots\ldots\ldots\ldots\ldots(32)$$

The through inspection of eqns (30) + (32) reveals that the expanded forms of the transformation matrix may be given in the following partitioned scheme

$$A = \left[\begin{array}{c|c} O & D \\ \hline I & O \\ \hline O & I \end{array}\right]_{16 \times 15} \quad \ldots\ldots\ldots\ldots\ldots\ldots(33)$$

With the help of equations (18) and (19) the both matrices may be partitioned in conformance with equation (33). Thus

$$M_{fc} = \left[\begin{array}{c|c|c} M_{11} & M_{12} & O \\ \hline M_{12}^T & M_{fun} & O \\ \hline O & O & M_h \end{array}\right] ; K_{fc} = \left[\begin{array}{c|c|c} K_{11} & K_{12} & O \\ \hline K_{12}^+ & K_{fun} & O \\ \hline O & O & K_h \end{array}\right] \quad \ldots\ldots\ldots(34)$$

Where M_{11} and K_{11} are the inertial and elastical parameters corresponding to q^*. Hereby the mass and stiffness matrices of the simulated entire mechanism may be expressed, using equatiosn (30) + (34), as

$$M_q = A^T M_{fc} A = \left[\begin{array}{c|c} M_{fun} & M_{12}^T D \\ \hline D^T M_{12} & D^T M_{11} D + M_h \end{array}\right] \quad \ldots\ldots\ldots(35)$$

Similarly

$$K_q = A^T K_{fc} A = \left[\begin{array}{c|c} K_{fun} & K_{12}^T D \\ \hline D^T K_{12} & D^T K_{11} D + K_h \end{array}\right] \quad \ldots\ldots\ldots(36)$$

Where from definitions, we have

1) M_{fun} and K_{fun} are the (12 x 12) characteristic matrices of the follower set in the uncoupled position,

2) M_h and K_h are the (3x3) characteristic matrices of the cam set in the uncoupled position.

3) $D^T M_{11} D = M_{11} D^T D$ is the diagonal submatrix of order (3x3), (since M_{11} is a scaler quantily).

4) $D^T M_{12}$ is the (3 x 12) off diagonal submatrix, here as

$$D^T M_{12} = \left[\begin{array}{c:c:c:c} B & 0 & 0 & 0 \end{array} \right] \quad \text{....(37)}$$

where

$$B = \begin{bmatrix} -\frac{1}{m_{12}} & -\frac{1}{m_{13}} & -\frac{1}{m_{14}} \\ i\frac{1}{m_{12}} & i\frac{1}{m_{13}} & i\frac{1}{m_{14}} \\ i\frac{1}{m_{12}} & i\frac{1}{m_{13}} & i\frac{1}{m_{14}} \end{bmatrix} \quad \text{.....(38)}$$

The developed partitioned scheme factitates markedly the formulation effort for synthesizing the characteristic matrix of the entire cam mechanism. Since the partitioned matrices given in equations 35 and 36 can be built up successively and therefore the required size of computer is considerably reduced.

For the sake of comparison between the present method and Koster's, the foundamental frequancy is computed (See appendix 2) for the various versions of simulated models of the cam operate transfer mechanism by using the bound formula (9). From the calculation, it is shown that the error, introduced by Koster technique relative to the present work lie within 5.56 to 6.83 percent.

CONCLUSION

The present approximate method attempts to provide a sufficiently simple and powerful tool for generating various versions in the modelling of the rigidly supported cam mechanism based on the finite element approach.

The proposed procedure avoids the drawbacks which arise in the applications of many classical methods such as methods mentioned here. The simple methods given in references (1, 2, 3, 4, 5) can be derived as special cases and as crude approximations of the developed method.

The formulation of the mass and stiffness matrices are introduced in such a way to render the problem tackable by limited capacity computer, which affect sharply economically in the computation effort for the machine design. This is because the characteristic matrices even in the condensed forms include the inertial and elastic parameters of all elements comprising the original system in the deterministic forms. However the computed result of the fundamental frequancy shows that present modelling technique is sufficient.

To integrate this work, further study and investigation concerning the variational effects of inertial and elastic parameters of actual constructive values of the system may be carried out in the future.

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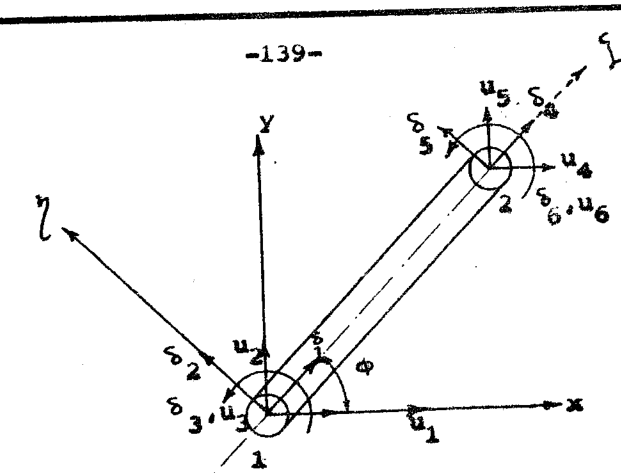

FIGURE (1): GENERALIZED DISPLACEMENTS OF BEAM ELEMENT IN TWO COORDINATE SYSTEMS.

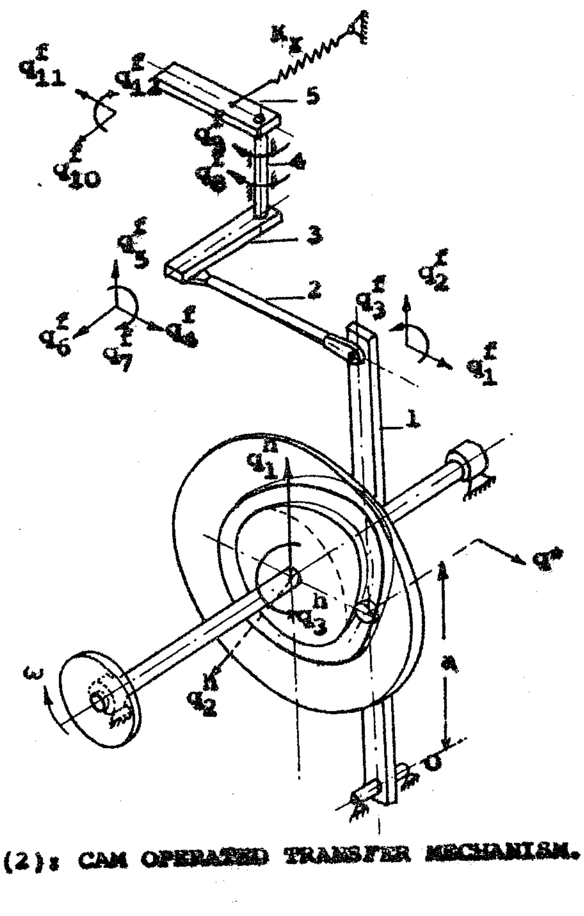

FIGURE (2): CAM OPERATED TRANSFER MECHANISM.

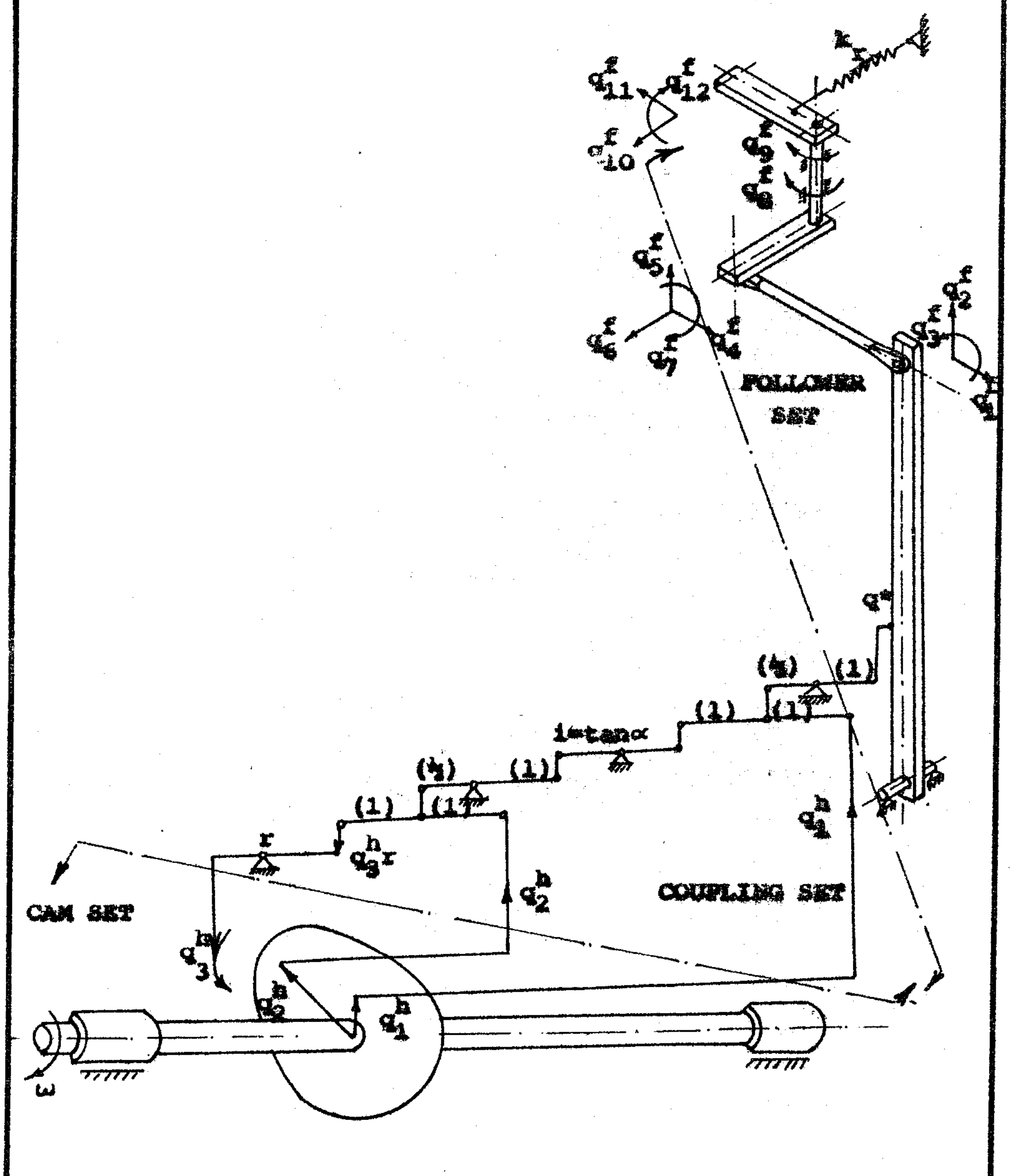

FIGURE (3): THE FOLLOWER, CAM AND COUPLING SETS COMPRISING THE CAM-OPERATED TRANSFER MECHANISM.

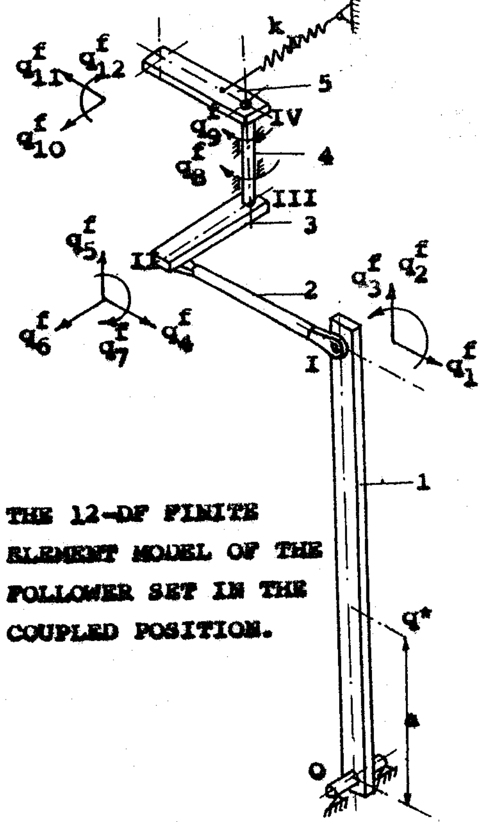

FIGURE (4): THE 12-DF FINITE ELEMENT MODEL OF THE FOLLOWER SET IN THE COUPLED POSITION.

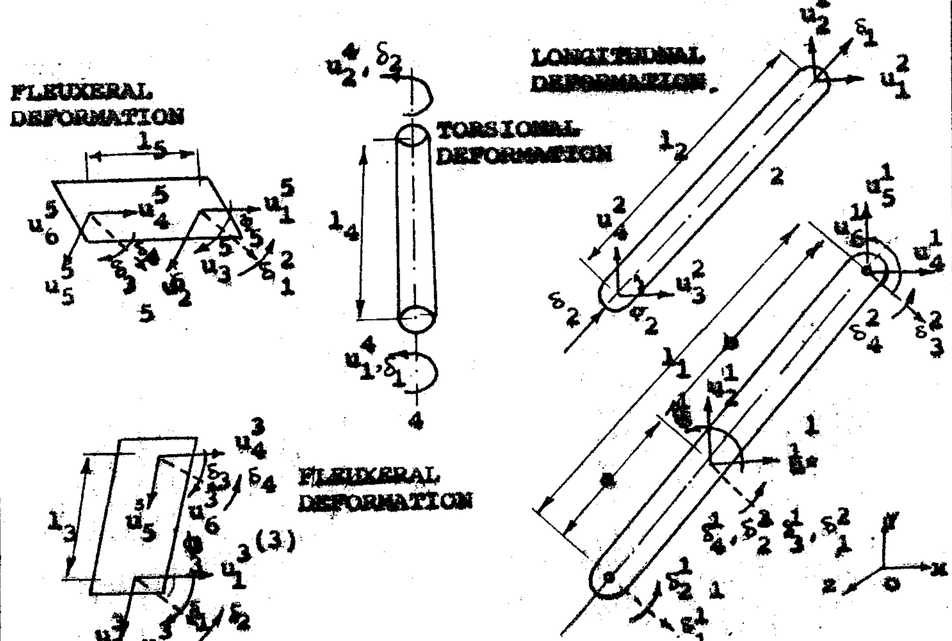

FIGURE (5): NODAL DISPLACEMENTS OF INDIVIDUAL ELEMENTS OF THE FOLLOWER SET.

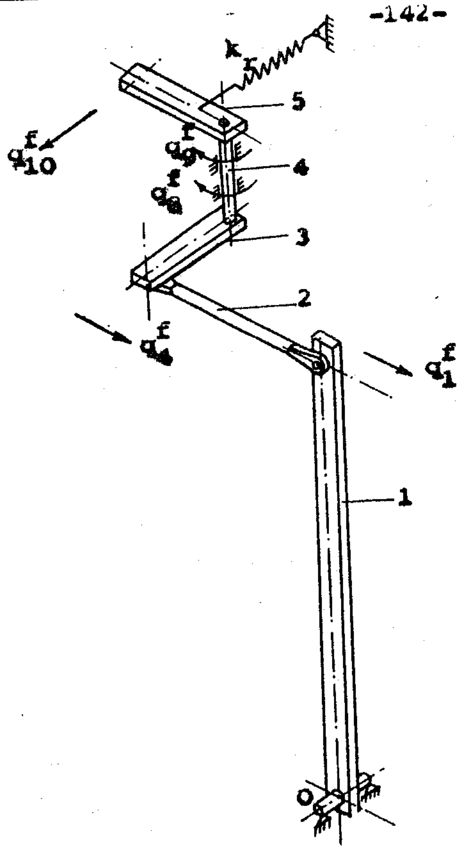

FIGURE (6): THE 5-DF FINITE ELEMENT MODEL OF THE FOLLOWER SET IN THE UNCOUPLED POSITION.

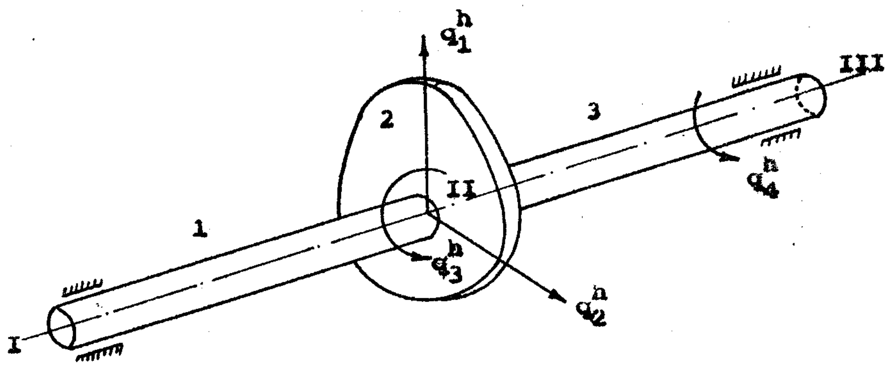

FIGURE (7): THE 4-DF FINITE ELEMENT MODEL OF A RIGIDLY SUPPORTED CAM SET IN THE UNCOUPLED POSITION.

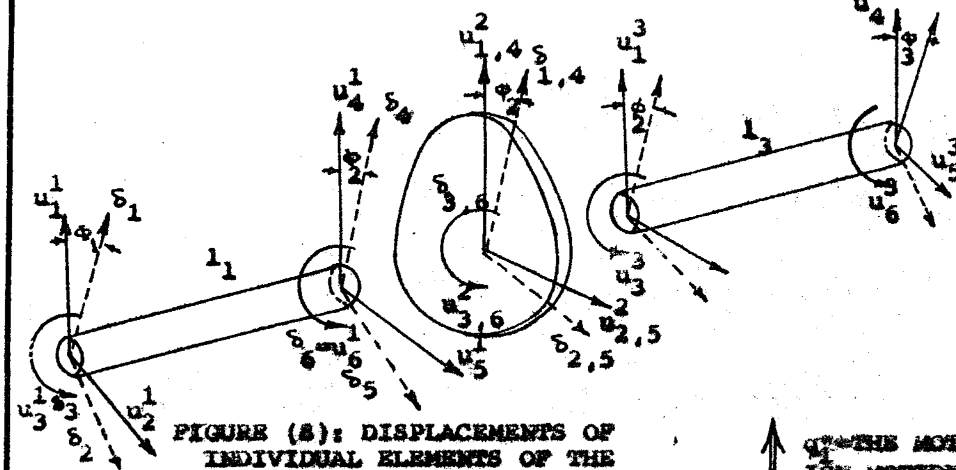

FIGURE (8): DISPLACEMENTS OF INDIVIDUAL ELEMENTS OF THE CAM SET IN LOCAL AND GLOBAL COORDINATE SYSTEMS.

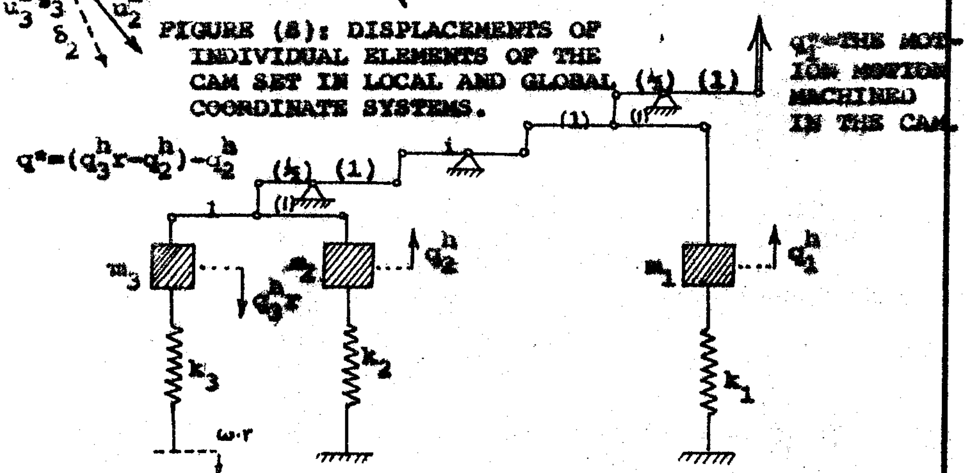

FIGURE (9): THE 3-DF FINITE ELEMENT MODEL OF THE CAM SET IN THE COUPLED POSITION.

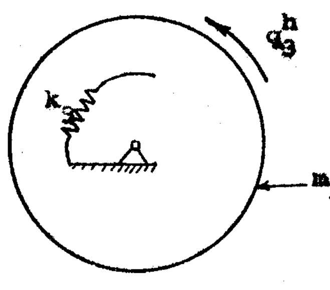

FIGURE (10-a)

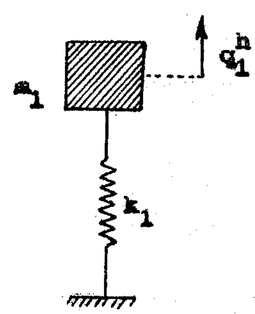

FIGURE (10-b)

APPENDIX (1)

The mass coefficients of the m_1 matrix given in equation (11) are:

$\overset{1}{m}_{11}=\rho A(\frac{13}{35}l+\frac{a}{1680})\mu^2;\overset{1}{m}_{12}=-\rho A(\frac{13}{35}l+\frac{a}{1680})\quad ;\overset{1}{m}_{13}=\ A(\frac{11}{210}(b^2-a^2)-\frac{a}{1680})$

$;\ \overset{1}{m}_{14}=\frac{9}{70}\rho Ab\mu^2;\ \overset{1}{m}_{15}=-\frac{9\rho Ab}{70}\mu\lambda;\ \overset{1}{m}_{16}=\frac{13\rho Ab^2}{420}\mu^2,\ \overset{1}{m}_{22}=\rho A(\frac{13}{35}l+\frac{a}{1680})\lambda^2$

$;\ \overset{1}{m}_{23}=\rho A(\frac{11}{210}(b^2-a^2)-\frac{a^2}{560})\ 2;\ \overset{1}{m}_{24}=-\frac{9\rho Ab}{70}\mu\lambda;\overset{1}{m}_{25}=\frac{9\rho Ab}{70}\lambda^2;\ \overset{1}{m}_{26}=\frac{13\rho Ab^3}{420}\lambda$

$,\ \overset{1}{m}_{33}=\rho A(\frac{a^3}{560}+\frac{a^3+b^3}{105});\ \overset{1}{m}_{34}=\frac{-13\rho Ab^2}{420}\mu;\overset{1}{m}_{35}=\frac{13\rho Ab^2}{420}\mu\quad ,\overset{1}{m}_{36}=\frac{-\rho Ab^3}{140};$

$;\ \overset{1}{m}_{44}=\frac{13\rho Ab}{35}\mu^2,\overset{1}{m}_{45}=-\frac{13\rho Ab}{35}\mu\lambda;\ \overset{1}{m}_{46}=\frac{11\rho Ab^2}{420}\mu;\overset{1}{m}_{55}=\frac{13\rho Ab}{35}\lambda^2,\overset{1}{m}_{56}=-\frac{11\rho Ab}{420}\lambda$

; and $\overset{1}{m}_{66}=\frac{\rho Ab^2}{105}$; **where $a+b=l_1$, Figure (5).**

The elastic coefficients of the K_1 matrix given in equation (11) are:

$\overset{1}{k}_{11}=(\frac{12EI}{b^3}+\frac{21EI}{a^3})\mu^2,\overset{1}{k}_{12}=-(\frac{12EI}{b^3}+\frac{21EI}{a^3})\mu\lambda;\ \overset{1}{k}_{13}=(\frac{9EI}{a^2}-\frac{6EI}{b^2})\mu;\ \overset{1}{k}_{14}=-(\frac{12EI}{b^3})\mu^2$

$;\overset{1}{k}_{15}=(\frac{12EI}{b^3})\mu\lambda;\overset{1}{k}_{16}=-(\frac{6EI}{6^2})\mu^2;\ \overset{1}{k}_{22}=(\frac{12EI}{b^3}+\frac{21EI}{a^3})\lambda^2\ ;\ \overset{1}{k}_{23}=(\frac{6EI}{b^2}-\frac{9EI}{a^2})\lambda$

$\overset{1}{k}_{24}=-(\frac{12EI}{b^3})\mu;\overset{1}{k}_{25}=-(\frac{12EI}{b^3})\lambda^2;\ \overset{1}{k}_{26}=\frac{6EI}{b^2}\lambda;\overset{1}{k}_{33}=(\frac{4EI}{b}+\frac{5EI}{a}),\overset{1}{k}_{34}=\frac{6EI}{b^2}\mu$

$;\overset{1}{k}_{35}=-(\frac{6EI}{b^2})\lambda\ ;\overset{1}{k}_{36}=(\frac{2EI}{b});\ \overset{1}{k}_{44}=(\frac{12EI}{b^3})\mu^2;\ \overset{1}{k}_{45}=-(\frac{12EI}{b^3})\mu\lambda$

$;\overset{1}{k}_{46}=-(\frac{6EI}{b^2})\mu;\ \overset{1}{k}_{55}=\frac{12EI}{b^3}\lambda^2;\ \overset{1}{k}_{56}=-(\frac{6EI}{b^2})\lambda$; and $\overset{1}{k}_{66}=\frac{4EI}{b}$

The mass coefficients of the M_{fc} matrix given in equation (18) are:

$\overset{2}{m}_{11}=\overset{2}{m}_{33}=2\overset{2}{m}_{13}=2\rho Al\lambda^2\ ;\ \overset{2}{m}_{12}=\overset{2}{m}_{34}=2\overset{2}{m}_{14}=2\overset{2}{m}_{23}=2\rho Al\mu\lambda$

$\overset{2}{m}_{22}=\overset{2}{m}_{44}=2\overset{2}{m}_{24}=2\rho Al\lambda^2;\ \overset{3}{m}_{11}=\frac{13}{35}\rho Al\mu^2;\ \overset{3}{m}_{12}=-\frac{13}{35}\rho Al\mu\lambda;\overset{3}{m}_{13}=\frac{-11\rho Al^2}{210}\mu$

$; {}^{3}m_{16} = \frac{13\rho A l^2}{420}\mu ; {}^{3}m_{22} = \frac{13\rho A l}{35}\lambda^2 ; {}^{3}m_{23} = \frac{11\rho A l^2}{210}\lambda ; {}^{3}m_{26} = \frac{13\rho A l^2}{420}\lambda ; {}^{3}m_{33} = {}^{3}m_{66} = \frac{\rho A l^3}{105}$

$, {}^{3}m_{36} = \frac{-\rho A l^3}{140} ; {}^{4}m_{11} = {}^{4}m_{22} = 2\,{}^{4}m_{12} = \frac{\rho I_p l}{3} ; {}^{5}m_{33} = {}^{5}m_{66} = \frac{\rho A l^3}{105} ;$

$; {}^{5}m_{34} = -\frac{13\rho A l^2}{420}\mu ; {}^{5}m_{35} = \frac{13\rho A l^2}{420}\lambda ; {}^{5}m_{36} = -\rho A l^3/140 ; {}^{5}m_{44} = \frac{13\rho A l}{105}\mu^2$

${}^{5}m_{45} = -\frac{13\rho A l}{35}\mu\lambda ; {}^{5}m_{46} = \frac{11\rho A l^2}{210}\mu ; {}^{5}m_{55} = \frac{13\rho A l}{35}\lambda^2 ;$ and ${}^{5}m_{56} = -\frac{11\rho A l^2}{210}\lambda .$

The elastic coefficients of K_{fc} matrix given in equation (19) are:

${}^{2}k_{11} = {}^{2}k_{33} = -{}^{2}k_{13} = \frac{EA}{l}\lambda^2 ; {}^{2}k_{12} = -{}^{2}k_{14} = -{}^{2}k_{23} = {}^{2}k_{34} = \frac{EA}{l}\mu\lambda ;$

${}^{2}k_{22} = {}^{2}k_{44} = -{}^{2}k_{24} = \frac{EA}{l}\mu^2 ; {}^{3}k_{11} = (\frac{12EI}{l^3}\mu^2) ; {}^{3}k_{12} = (\frac{-12EI}{l^3})\mu\lambda ;$

${}^{3}k_{13} = {}^{3}k_{16} = -\frac{6EI}{l^2}\mu ; {}^{3}k_{22} = (\frac{12EI}{l^3})\lambda^2 ; {}^{3}k_{23} = {}^{3}k_{26} = \frac{6EI}{l^2}\lambda ; {}^{3}k_{33} = {}^{3}k_{66} = \frac{4EI}{l} ; {}^{3}k_{36} = \frac{2EI}{l^2}$

$; {}^{4}k_{11} = {}^{4}k_{22} = -{}^{4}k_{12} = GJ_p/l ; {}^{5}k_{33} = 2\,{}^{5}k_{36} = {}^{5}k_{66} = \frac{4EI}{l} ; {}^{5}k_{34} = {}^{5}k_{46} = \frac{6EI}{l^2}\mu$

${}^{5}k_{35} = {}^{5}k_{56} = -\frac{6EI}{l^2}\lambda ; {}^{5}k_{44} = (\frac{12EI}{l^3} + K_r)\mu^2 ; {}^{5}k_{45} = -(\frac{12EI}{l^3} + K_r)\mu\lambda$

and ${}^{5}k_{55} = (\frac{12EI}{l^3} + k_r)\lambda^2 .$

The mass coefficients of the M_r matrix given in equation (22) are:-

$$A_{22} = \Big[[({}^{1}m_{46} + {}^{2}m_{12}) F_{11} + {}^{1}m_{46}F_{21} + {}^{2}m_{14}F_{31}]({}^{1}m_{45} + {}^{2}m_{12}) + [({}^{1}m_{45} + {}^{2}m_{12})F_{12} + {}^{1}m_{46}F_{22} + {}^{2}m_{14}F_{32}]\,{}^{1}m_{46} + [({}^{1}m_{45} + {}^{2}m_{12})F_{13} + {}^{1}m_{46}F_{23} + {}^{2}m_{14}F_{33}]\,{}^{2}m_{14} \Big] \cdot \frac{1}{\Delta}$$

$$; A_{23} = \Big[[({}^{1}m_{45} + {}^{1}m_{12})F_{11} + {}^{1}m_{46}F_{21} + {}^{2}m_{14}F_{31}]\;{}^{2}m_{23} + ({}^{2}m_{23}F_{13} + {}^{2}m_{34}F_{33}) \Big] \frac{1}{\Delta}$$

$$; A_{33} = \Big[({}^{2}m_{23}F_{11} + {}^{2}m_{34}F_{31}){}^{2}m_{23} + ({}^{2}m_{23}F_{13} + {}^{2}m_{34}F_{33}){}^{2}m_{34} + ({}^{3}m_{12}F_{44} + {}^{3}m_{13}F_{45}){}^{3}m_{12} + ({}^{3}m_{12}F_{45} + {}^{3}m_{13}F_{55})\,{}^{3}m_{13} \Big] \cdot \frac{1}{\Delta}$$

$$; A_{34} = \Big[({}^{3}m_{12} \cdot F_{44} + {}^{3}m_{13}F_{45}){}^{3}m_{26} + ({}^{3}m_{12}F_{45} + {}^{3}m_{13}F_{55})\,{}^{3}m_{36} \Big] \cdot \frac{1}{\Delta}$$

$$; A_{44} = \Big[({}^{3}m_{26}F_{44} + {}^{3}m_{36}F_{45})\,{}^{3}m_{26} + ({}^{3}m_{26}F_{45} + {}^{3}m_{36}F_{55})\,{}^{3}m_{36} \Big] \frac{1}{\Delta}$$

$$;A_{55} = \left[(\overset{5}{m}_{35}F_{66}+\overset{5}{m}_{36}F_{67})\ \overset{5}{m}_{35} + (\overset{5}{m}_{36}\ F_{67}+\overset{5}{m}_{36}F_{77})\ \overset{5}{m}_{36} \right] \cdot \frac{1}{\triangle}$$

$$;A_{56} = \left[(\overset{5}{m}_{35}F_{66} + \overset{5}{m}_{36}F_{67})\overset{5}{m}_{46} + (\overset{5}{m}_{35}\ F_{67} + \overset{5}{m}_{36}F_{77})\ \ \overset{5}{m}_{46} \right] \cdot \frac{1}{\triangle}$$

and

$$A_{66} = \left[(\overset{5}{m}_{45}F_{66}+\overset{5}{m}_{46}F_{67})\ \overset{5}{m}_{46} + (\overset{5}{m}_{45}F_{67} + \overset{5}{m}_{46}\ F_{77})\ \overset{5}{m}_{46} \right] \cdot \frac{1}{\triangle}$$

Where;

$$\triangle = (\overset{1}{m}_{55} + \overset{3}{m}_{22})\ F_{11} + \overset{1}{m}_{56}\ F_{12} + \overset{2}{m}_{24}\ F_{13}$$

$$;F_{11} = \overset{1}{m}_{66} \cdot \overset{2}{m}_{44} \left[\overset{3}{m}_{22}\ \overset{3}{m}_{33} - (\overset{3}{m}_{23})^2 \right] \left[\overset{3}{m}_{66} \cdot \overset{5}{m}_{55} - (\overset{5}{m}_{56})^2 \right]; F_{12} = -\overset{1}{m}_{56}\ \frac{F_{11}}{\overset{1}{m}_{66}}$$

$$;F_{13} = -\overset{2}{m}_{24}\ \frac{F_{11}}{\overset{2}{m}_{44}}\ ;\ F_{22} = (F_{11}/\overset{1}{m}_{66} \cdot \overset{2}{m}_{44}) \left[\overset{2}{m}_{44}(\overset{1}{m}_{55}+\overset{2}{m}_{22})\ - (\overset{2}{m}_{24})^2 \right]$$

$$;F_{23} = (F_{11}/\overset{1}{m}_{66}\overset{2}{m}_{44})(\overset{1}{m}_{56})^2,\ F_{33} = (F_{11}/\overset{1}{m}_{66}\overset{2}{m}_{44}) \left[(\overset{1}{m}_{55}+\overset{2}{m}_{22})\overset{1}{m}_{66} - (\overset{1}{m}_{56})^2 \right]$$

$$;F_{44} = \overset{3}{m}_{33} \left[\left[(\overset{1}{m}_{55}+\overset{2}{m}_{22})\ \overset{1}{m}_{66}\overset{2}{m}_{44} - (\overset{2}{m}_{24})^2(\overset{1}{m}_{66}) - (\overset{1}{m}_{56})^2\ \overset{2}{m}_{44} \right] \left[\overset{5}{m}_{66}\overset{5}{m}_{55} - (\overset{5}{m}_{56})^2 \right] \right]$$

$$;F_{45} = -\overset{3}{m}_{23}\ \frac{F_{44}}{\overset{3}{m}_{33}}\ ,\ F_{55} = \overset{2}{m}_{22}\ \frac{F_{44}}{\overset{3}{m}_{33}}\ ;F_{66} = \overset{5}{m}_{66} \left[\left[(\overset{1}{m}_{55}+\overset{2}{m}_{22})\overset{1}{m}_{66}\overset{2}{m}_{44} - (\overset{2}{m}_{24})^2 \right. \right.$$

$$\left. \left. \overset{1}{m}_{66} - (\overset{1}{m}_{56})^2\ \overset{2}{m}_{44} \right] \left[\overset{3}{m}_{33}\ \overset{3}{m}_{22}\ - (\overset{3}{m}_{33})^2 \right] \right]$$

$$;F_{67} = \overset{5}{m}_{56}\ \frac{F_{66}}{\overset{5}{m}_{66}} \quad \text{and} \quad F_{77} = \overset{5}{m}_{55} \cdot \frac{F_{66}}{\overset{5}{m}_{66}}$$

The elastic coefficients of the K_r matrix given in equation (22) are:-

$$B_{22} = \left[\left[(\overset{1}{k}_{46}+\overset{2}{k}_{12})L_{11}+\overset{1}{k}_{46}L_{21}+\overset{2}{k}_{14}L_{31} \right] \ (\overset{1}{k}_{45}+\overset{2}{k}_{12}) + \overset{1}{K}_{46} \left[(\overset{1}{k}_{45}+\overset{2}{k}_{12})L_{12}+\overset{1}{k}_{46}L_{22}+\overset{2}{k}_{14}L_{32} \right] \right.$$

$$\left. + \left[(\overset{1}{k}_{45} + \overset{2}{k}_{12})\ L_{13} + K_{46}\ L_{23} + \overset{2}{k}_{14}\ L_{33} \right] \ (\overset{2}{k}_{14}) \right] \cdot \frac{1}{\triangle}$$

$$;B_{23} = \left[\left[(\overset{1}{k}_{45}+\overset{1}{k}_{12})L_{11}+\overset{1}{k}_{46}L_{21}+\overset{2}{k}_{14}L_{31} \right] \overset{2}{k}_{23} + \left[\overset{2}{k}_{23}L_{13} + \overset{2}{k}_{34}L_{33} \right] \right] \cdot \frac{1}{\triangle}$$

$$;B_{33} = \left[(\overset{2}{k}_{23}L_{11}+\overset{2}{k}_{34}L_{31})\ \overset{2}{k}_{23} + (\overset{2}{k}_{23}L_{12}+\overset{2}{k}_{34}L_{33})\ \overset{2}{k}_{34} + (\overset{3}{k}_{12}L_{44}+\overset{3}{k}_{13}L_{45})\overset{3}{k}_{12} \right.$$

$$\left. + (\overset{3}{k}_{12}L_{45} + \overset{3}{k}_{13}L_{55})\ \ \overset{3}{k}_{13} \right] \cdot \frac{1}{\triangle}$$

$$;B_{34} = \left[(\overset{3}{K}_{12}L_{44}+\overset{3}{K}_{13}L_{45})\ \overset{3}{K}_{26} + (\overset{3}{K}_{12}L_{45} + \overset{3}{K}_{13}\ L_{55})\ \overset{3}{K}_{36}\right] \cdot \frac{1}{\acute{\Delta}}$$

$$;B_{44} = \left[(\overset{3}{K}_{26}L_{44}+\overset{3}{K}_{36}L_{45})\overset{3}{K}_{26} + (\overset{3}{K}_{26}L_{45}+\overset{3}{K}_{36}L_{55})\ \overset{3}{K}_{36}\right] \cdot \frac{1}{\acute{\Delta}}$$

$$;B_{55} = \left[(\overset{5}{K}_{35}L_{66}+\overset{5}{K}_{36}L_{67})\overset{5}{K}_{35} + (\overset{5}{K}_{35}L_{67}+ \overset{5}{K}_{36}L_{77})\ \overset{5}{K}_{36}\right] \cdot \frac{1}{\acute{\Delta}}$$

$$;B_{56} = \left[(\overset{5}{K}_{54}L_{66}+ \overset{5}{K}_{46}L_{67})\ \overset{5}{K}_{35}+(\overset{5}{K}_{45}L_{67} + \overset{5}{K}_{46}L_{77})\ \overset{5}{K}_{46}\right] \cdot \frac{1}{\acute{\Delta}}$$

and

$$B_{66} = \left[(\overset{5}{K}_{45}L_{66} + \overset{5}{K}_{46}L_{67})\ \overset{5}{K}_{46}+ (\overset{5}{K}_{45}L_{67}+\overset{5}{K}_{46}L_{77})\ \overset{5}{K}_{46}\right] \cdot \frac{1}{\acute{\Delta}}$$

where;

$$\acute{\Delta} = (\overset{1}{K}_{55} + \overset{2}{K}_{22})\ L_{11} + \overset{1}{K}_{56}L_{12} + K_{24}L_{13}$$

$$;L_{11} = \overset{1}{K}_{66}\cdot\ \overset{2}{K}_{44}\left[\overset{3}{K}_{22}\ \overset{3}{K}_{33} - (\overset{3}{K}_{23})^2\right]\left[\overset{5}{K}_{66}\ \overset{5}{K}_{55} - (\overset{5}{K}_{56})^2\right]\ ;L_{12} = -\overset{1}{K}_{56}\ \frac{L_{11}}{\overset{1}{K}_{66}}$$

$$;L_{13} = -\overset{2}{K}_{24}\cdot\ \frac{L_{11}}{\overset{2}{K}_{44}}\ ;\ L_{22} = (L_{11}/\overset{1}{K}_{66}\cdot\overset{2}{K}_{44})\left[\overset{2}{K}_{44}(\overset{1}{K}_{55}+\overset{2}{K}_{22}) - (\overset{2}{K}_{24})^2\right]$$

$$;L_{23} = (L_{11}/\overset{1}{K}_{66}\overset{2}{K}_{44})(\overset{1}{K}_{56})^2;\ L_{33}=(L_{11}/\overset{1}{K}_{66}\overset{2}{K}_{44})\left[(\overset{1}{K}_{55}+\overset{2}{K}_{22})\ \overset{1}{K}_{66}-(\overset{1}{K}_{56})^2\right]$$

$$;L_{44} = \overset{3}{K}_{33}\ \left[(\overset{1}{K}_{55}+\overset{2}{K}_{22})\ \overset{1}{K}_{66}\overset{2}{K}_{44}-(\overset{2}{K}_{24})^2(\overset{1}{K}_{66})-(\overset{1}{K}_{56})^2\ \overset{2}{K}_{44}\right]\left[\overset{5}{K}_{66}\overset{5}{K}_{55}-(\overset{5}{K}_{56})^2\right]$$

$$;L_{45} = -\ \frac{L_{44}\cdot K^3_{23}}{\overset{3}{K}_{33}}\ ,\ L_{55} = \frac{L_{44}}{\overset{3}{K}_{33}}\ \overset{3}{K}_{22}$$

$$L_{66}=\overset{5}{K}_{66}\left[(\overset{1}{K}_{55}+\overset{2}{K}_{22})\overset{1}{K}_{66}\overset{2}{K}_{44}-(\overset{2}{K}_{24})^2\ \overset{1}{K}_{66}-(\overset{1}{K}_{56})^2\ \overset{2}{K}_{44}\right]\left[\overset{3}{K}_{33}\overset{3}{K}_{22} - (\overset{3}{K}_{33})^2\right]$$

$$;L_{67} = \overset{5}{K}_{56}\cdot\frac{L_{66}}{\overset{5}{K}_{66}}\ \text{ and } L_{77} = \overset{5}{K}_{55}\cdot\frac{L_{66}}{\overset{5}{K}_{66}}\ .$$

The mass coefficients of the m^* given in equation (23) are:

$$R_{11}=(\overset{1}{m}_{11}-A_{11})-\frac{1}{\Delta_2}\left[(\overset{1}{m}_{14}-A_{11})C_{11}-A_{13}C_{21}\right](\overset{1}{m}_{14}-A_{21})+A_{31}\left[(A_{11}-\overset{1}{K}_{14})C_{12}+A_{13}C_{22}\right]$$

where

$$A_{11}=\ \overset{1}{m}_{15}(\overset{1}{m}_{15}F_{11}+\overset{1}{m}_{16}F_{21})\ +\overset{1}{m}_{16}(\overset{1}{m}_{15}F_{12} + \overset{1}{m}_{16}\ F_{22})\ \cdot\frac{1}{\Delta}$$

$$;A_{13}=\ (\overset{1}{m}_{15}F_{11}+\overset{1}{m}_{16}F_{21})\ \overset{2}{m}_{23} + (\overset{1}{m}_{15}F_{13} + \overset{1}{m}_{16}F_{23})\ \overset{2}{m}_{34}\ \cdot\frac{1}{\Delta}$$

$$;A_{21}=\left[\left[(\overset{1}{m}_{45}+\overset{2}{m}_{12})F_{11}+\overset{1}{m}_{46}F_{21}+\overset{1}{m}_{14}F_{31}\right]\overset{1}{m}_{15}+\left[(\overset{1}{m}_{45}+\overset{2}{m}_{12})F_{12}+\overset{1}{m}_{46}F_{22}+\overset{2}{m}_{14}F_{32}\right]\overset{1}{m}_{16}\right]\cdot\frac{1}{\triangle}$$

$$;C_{11}=(\overset{3}{m}_{33}+\overset{3}{m}_{11}-A_{33})(\overset{3}{m}_{66}+\overset{4}{m}_{11}-A_{44})(\overset{5}{m}_{22}+\overset{5}{m}_{33}-A_{55})-(\overset{3}{m}_{33}+\overset{3}{m}_{11}-A_{33})(\overset{4}{m}_{14})^2$$

$$-(\overset{3}{m}_{16}-A_{34})^2(\overset{5}{m}_{22}+\overset{5}{m}_{33}-A_{55})$$

$$;C_{21}=(A_{23}-\overset{2}{m}_{13})\left[(\overset{3}{m}_{66}+\overset{4}{m}_{11}-A_{44})(\overset{5}{m}_{22}+\overset{5}{m}_{33}-A_{55})-(\overset{4}{m}_{12})^2\right]=C_{21};C_{22}=(\overset{1}{m}_{44}+\overset{2}{m}_{11}-A_{11})$$

$$\left[(\overset{5}{m}_{22}+\overset{5}{m}_{33}-A_{55})(\overset{3}{m}_{66}+\overset{4}{m}_{11}-A_{44})-(\overset{4}{m}_{12})^2\right].$$

$$R_{12}=R_{21}=(A_{56}-\overset{5}{m}_{34})\left[C_{14}(\overset{1}{m}_{14}-A_{11})-A_{13}\ C_{24}\right]\frac{1}{\Delta_2}$$

Where;

$$C_{14}=\overset{4}{m}_{12}(A_{34}-\overset{3}{m}_{16})(\overset{2}{m}_{13}-A_{23})\ ;C_{24}=(\overset{1}{m}_{44}+\overset{2}{m}_{11}-A_{11})(\overset{3}{m}_{16}-A_{34})(\overset{4}{m}_{12})$$

$$R_{22}=(\overset{5}{m}_{44}-A_{66})-\frac{1}{\Delta_2}(\overset{5}{m}_{34}-A_{56})\left[C_{44}\ (\overset{5}{m}_{34}-A_{56})\right]$$

where;

$$C_{44}=(\overset{1}{m}_{44}+\overset{2}{m}_{11}-A_{22})(\overset{3}{m}_{33}+\overset{3}{m}_{11}-A_{33})(\overset{3}{m}_{66}+\overset{4}{m}_{11}-A_{44})+(A_{22}-\overset{1}{m}_{44}+\overset{2}{m}_{11})(\overset{3}{m}_{16}-A_{34})^2$$

$$+(\overset{2}{m}_{13}-A_{23})^2(A_{44}-\overset{3}{m}_{66}-\overset{4}{m}_{11});\ \triangle_2=(\overset{1}{m}_{44}+\overset{2}{m}_{11}-A_{22})C_{11}+(A_{23}-\overset{2}{m}_{13})\ C_{12}$$

The elastic coefficients of the k* given in equation (23) are:-

$$S_{11}=(\overset{1}{k}_{11}-B_{11})-\frac{1}{\triangle_3}\left[(\overset{1}{k}_{14}-B_{11})D_{11}-B_{13}D_{21}\right](\overset{1}{k}_{14}-B_{21})+B_{31}\left[(B_{11}-\overset{1}{k}_{14})D_{12}+B_{13}D_{22}\right]$$

Where;

$$B_{11}=\left[\overset{1}{k}_{15}(\overset{1}{k}_{15}L_{11}+\overset{1}{k}_{16}L_{12})+\overset{1}{k}_{16}(\overset{1}{k}_{15}L_{12}+\overset{1}{k}_{16}L_{22})\right]\cdot\frac{1}{\triangle};$$

$$B_{13}=\left[(\overset{1}{k}_{15}L_{11}+\overset{1}{k}_{16}L_{21})\ \overset{1}{k}_{23}+(\overset{1}{k}_{15}L_{13}+\overset{1}{k}_{16}L_{23})\overset{2}{k}_{34}\right]\cdot\frac{1}{\triangle}$$

$$B_{21}=\left[\left[(\overset{1}{k}_{46}+\overset{2}{k}_{12})\ L_{11}+\overset{1}{k}_{46}\ L_{21}+\overset{2}{k}_{14}\ L_{31}\right]\overset{1}{k}_{15}+\left[(\overset{1}{k}_{45}+\overset{2}{k}_{12})L_{12}+\overset{1}{k}_{46}L_{22}\right.\right.$$

$$\left.\left.+\overset{2}{k}_{14}\ L_{32}\right]\overset{1}{k}_{16}\right]\cdot\frac{1}{\triangle}$$

$$D_{11}=(\overset{3}{k}_{33}+\overset{3}{k}_{11}-B_{33})(\overset{3}{k}_{66}+\overset{4}{k}_{14}-B_{44})(\overset{5}{k}_{22}+\overset{5}{k}_{33}-B_{55})+(B_{33}-\overset{3}{k}_{33}-\overset{3}{k}_{11})(\overset{4}{k}_{12})^2$$

$$+(\overset{3}{k}_{16}-B_{34})^2(B_{55}-\overset{5}{k}_{22}-\overset{5}{k}_{33})$$

$$D_{21} = (B_{23}-\overset{2}{k}_{13})\left[(\overset{3}{k}_{66}+\overset{4}{k}_{11}-B_{44})(\overset{5}{k}_{22}+\overset{5}{k}_{33}-B_{55})-(\overset{4}{k}_{12})^2\right] = D_{21}; \text{ and}$$

$$D_{22} = (\overset{1}{k}_{44}+\overset{2}{k}_{11} - B_{11})\left[(\overset{5}{k}_{22}+\overset{5}{k}_{33}- B_{55})(\overset{3}{k}_{66}+\overset{4}{k}_{11}-B_{44})-(\overset{4}{k}_{12})^2\right]$$

$$;S_{12} = S_{21} = (B_{56}-\overset{5}{k}_{34})\left[D_{14}(\overset{1}{k}_{14}-B_{11})- B_{13}D_{24}\right]\frac{1}{\Delta_3}$$

where;

$$D_{14} = \overset{4}{k}_{12}(B_{34}-\overset{3}{k}_{16})(\overset{2}{k}_{13}-B_{23}); \text{and } D_{24} = (\overset{1}{k}_{44}+\overset{2}{k}_{11}-B_{11})(\overset{3}{k}_{16}-B_{34})(\overset{4}{k}_{12})$$

and

$$S_{22} = (\overset{5}{k}_{44} - B_{66}) + \frac{1}{\Delta_3}(B_{56} - \overset{5}{k}_{34})\left[D_{44}\ (\overset{4}{k}_{34} - B_{56})\right]$$

where;

$$D_{44} = (\overset{1}{k}_{44}+\overset{2}{k}_{11}-B_{22})(\overset{3}{k}_{33}+\overset{3}{k}_{11}-B_{33})(\overset{3}{k}_{66}+\overset{4}{k}_{11}-B_{44})+(B_{22}-\overset{1}{k}_{44}-\overset{2}{k}_{11})(\overset{3}{k}_{16}-B_{34})^2$$

$$+(\overset{2}{k}_{13}-B_{23})^2(B_{44}-\overset{3}{k}_{66}-\overset{4}{k}_{11}) \text{ and } \Delta_3=(\overset{1}{k}_{44}+\overset{2}{k}_{11}-B_{22})D_{11}+(B_{23}-\overset{2}{k}_{13})\ D_{12}.$$

The mass coefficients of the M_h given in equation (29) are:

$$m_1 = m_2 = \left(\frac{13\rho Al}{35}\right)_1 + \left(\frac{13\rho Al}{35}\right)_3 + m$$

$$;m_3 = \left(\frac{\rho I_p l}{3}\right)_1 + \left(\frac{\rho I_p l}{3}\right)_3 + I, \quad \text{and}$$

$$;m_{34} = m_{43} = (\rho I_p l/3)_3$$

where m is the mass of cam disc.

The elastic coefficients of k_h given in equation (29) are:

$$k_1 = \left(\frac{12EI_x}{l^3}\right)_1 + \left(\frac{12EI_x}{l^3}\right)_3 ;\ k_2 = \left(\frac{12EI_y}{l^3}\right)_1 + \left(\frac{12EI_y}{l^3}\right)_3$$

$$,k_3 = \left(\frac{GI_p}{l}\right)_1 + \left(\frac{GI_p}{l}\right)_3 \quad \text{and} \quad k_4 = -k_{34} = (GI_p/l)_3$$

Appendix (2)

The Calculation of the Fundamental Frequency

I - Present Work:-

The kinetic energy of the follower set have 5 DF.

$$(T)= \frac{1}{2}\ (1.4\ 57)\ \dot{q}_1^2 + \frac{1}{2}\ \dot{q}_1\ \dot{q}_4 + \frac{1}{2}\ (1.55714)\ \dot{q}_4^2 + 0.069643\ \dot{q}_4\ \dot{q}_8$$

$$+ \frac{1}{2}\ (0.53214)\ \dot{q}_8^2 + 0.25\ \dot{q}_8\ \dot{q}_9 + \frac{1}{2}\ (0.53214)\ \dot{q}_9^2 + 0.06964\ \dot{q}_9\dot{q}_{10}$$

$$+ \frac{1}{2}\ (0.5572)\ \dot{q}_{10}^2$$

The potential energy:-

$$(V) = \frac{1}{2}\ (2.2819)\ q_1^2 - (2.0944)\ q_1q_4 + \frac{1}{2}\ (5.64995)\ q_4^2 - 2.666\ q_4\ q_8$$

$$+ \frac{1}{2}\ (3.333)\ q_8^2 - 0.666\ q_8q_9 + \frac{1}{2}\ (3.333)\ q_9^2 - 2.666\ q_9q_{10}$$

$$+ \frac{1}{2}\ (3.555)\ q_{10}^2$$

Koster Work:-

$$(T)= \frac{1}{2}\ (1.6499)\ \dot{q}_1^2 + \frac{1}{2}\ \dot{q}_1\dot{q}_4 + \frac{1}{2}\ (1.3535)\ \dot{q}_4^2 + (0.0589)\ \dot{q}_4\dot{q}_8 + 0.52857\ \dot{q}_8^2$$

$$+ 0.25\ \dot{q}_8\dot{q}_9 + \frac{1}{2}\ (0.5285)\ \dot{q}_9^2 + (0.05286)\ \dot{q}_9\dot{q}_{10} + (0.5572)\ \dot{q}_{10}^2.$$

$$(V) = \frac{1}{2}\ (2.20154)\ q_1^2 - (2.0944)\ q_1q_4 + \frac{1}{2}(2.9833)\ q_4^2 - (.8888)\ q_4q_8$$

$$+ \frac{1}{2}(1.1852)\ q_6^2 - (0.29629)\ q_8q_9 + \frac{1}{2}(1.1852)\ q_9^2 - 0.888\ q_9\ q_{10}$$

$$+ \frac{1}{2}(0.888)\ q_{10}^2$$

The dynamic matrix of the follower set is given by:-

$$[D]_{5x5} = [K]^{-1}\ [m]$$

The largest eigenvalue is λ_1 (fundamental frequency)

$$\frac{T_r\ D^{k+1}}{T_r\ D^k} \leqslant \lambda_1 \leqslant (T_r D^k)^{\frac{1}{k}} ;\quad T_r \text{ is trace of matrix} [D]$$

The dynamic matrix [D] in present work is given by:-

$$[D] = \begin{bmatrix} 1.90933 & 1.62213 & 0.44116 & 0.325 & 0.1462 \\ 1.37099 & 1.528665 & 0.48065 & 0.35416 & 0.15895 \\ 1.21807 & 1.3814 & 0.4206 & 0.0169 & 0.1592 \\ 0.6683 & 0.6263 & 0.0276 & 0.362 & -0.338 \\ 0.4565 & 0.5178 & 0.0169 & -0.170 & 1.00246 \end{bmatrix}_{5x5}$$

The dynamic matrix in Koster work is given by:-

$$|D|_{5x5} \begin{vmatrix} 1.43647 & 1.40517 & 0.53852 & 0.52943 & 0.18606 \\ 1.61383 & 1.3306 & 0.55713 & 0.53625 & 0.19557 \\ 1.378548 & 1.2621999 & 0.24897 & -0.161072 & 0.13092 \\ 1.373668 & 1.17008 & -0.1090875 & 0.386697 & 0.28679 \\ 1.373918 & 1.258012 & 0.673965 & 1.2010614 & 2.06269 \end{vmatrix}_{5x5}$$

The dimension of the mechanism is taken as:-

$l_1 = a + b = 3 + 4 = 7$

$l_2 = 3$ cm.

$l_3 = 1.5$ cm. $\qquad A = I \cdot 2\pi =$ constant

$l_4 = 1.5$ cm.

$l_5 = 1.5$ cm.

In present Work

$T_r\ D^{16} = 1640228853$

$T_r\ D^{15} = 441501314.5$

Hence largest eigenvalue is λ_1 (foundamental frequency)

$$\frac{1640228853}{441501314.5} \quad \lambda_1 \quad (441501314.5)^{\frac{1}{15}}$$

$$3.73253 \leqslant \lambda_1 \leqslant 3.7662$$

$$\therefore \lambda_1 = \frac{3.73253 + 3.7662}{2} = 3.749365 \quad S^2$$

∴ The foundamental frequency given by:

$$\omega_1 = 0.5184414 \quad S^{-1}$$

<u>In Koster Work:-</u>

$$T_r D^{16} = 6696135689$$

$$T_r D^{15} = 1601692192$$

The largest eigenvalue λ_1 is given by:

$$\frac{6696135689}{1601692192} \leqslant \lambda_1 \leqslant (1601692192)^{\frac{1}{15}}$$

$$\therefore 4.21823 \leqslant \lambda_1 \leqslant 4.118077$$

$$\therefore \lambda_1 = \frac{4.21823 + 4.118077}{2} = 4.1681535 \quad S^2$$

∴ The foundamental frequency is:

$$\omega_1 = 0.4896104 \quad S^{-1}$$

The relative deviation is given by:-

$$\ell = \frac{0.5184414 - 0.4896104}{0.5184414} \times 100 = 5.56\ \%$$

If the system represented as a single degree of freedom:-

<u>In present work:-</u>

$$K_f = 2.2819$$

$$m_f = 1.4857$$

$$\therefore \quad f = \sqrt{\frac{2.2819}{1.4857}} = 1.24173$$

<u>In Koster Work:-</u>

$$K_f = 2.20154$$

$$m_f = 1.6499$$

$$\therefore \quad w_f = \sqrt{\frac{2.20154}{1.6499}} = 1.1551$$

$$\eta = \frac{1.24173 - 1.1551}{1.24173} = 6.83\%$$

" صياغة نموذج لآليات الكامات باستخدام العناصر المحدودة "

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يهدف هذا البحث الى تطبيق طريقة العناصر المحدودة " Finite element approach "
للحصول على النموذج الرياضى لآليات الكامات أخرى ما يمكن للحقيقة ولتحقيق هذا البحث اختيرت
كامة تحرك تابع ذو حركة تذبذبية ويحمل هذا التابع مجموعة من الأجزاء لها جميع خواص الاهتزاز
الطولى والملتوى والمستعرض ٠ ولتطبيق هذه الطريقة تم تقسيم آلية الكامة الى ثلاث مجموعات
وهى مجموعة التابع " Follower set " ممثلة بمنظومة عديدة درجات الحرية ولها ١٣ درجة
حرية ومجموعة الكامة " Cam set " ممثلة بمنظومة لها ثلاث درجات من الحرية والمجموعة
التى تربط بين مجموعة التابع ومجموعة الكامة " Coupling set " وهى علاقة كيناتيكية تحتوى
على جميع الخواص الكيناتيكية لآلية الكامة ٠ أى أن فى هذا البحث تم تمثيل آلية الكامة
بمنظومة لها ١٦ درجة حرية ولتخفيض هذا العدد من درجات الحرية الى أى عدد من المحاور
المرغوب فيها من درجات الحرية تم ذلك بطريقتين أحدهما طريقة الاختزال "Elemination technique"
والطريقة الأخرى طريقة التكثيف "Condensation techinque" وقد وجد أن طريقة
التكثيف أفضل من طريقة الاختزال ٠ وقد وجد أن طريقة العناصر المحدودة مناسبة لتمثيل
آليات الكامات عن بعض الطرق المستخدمة من قبل وبالأخص فى تمثيل آليات المقعدة وتم
ذلك بمقارنتها ببعض الطرق التى استخدمها بعض الباحثين السابقيين كما تمتاز أيضا
بسهولة حساباتها باستخدام الحاسب العلمى " Computor " الذى يؤدى الى توفير
الوقت والجهد فى تصميم أجزاء الماكينات ٠