

**EFFECT OF LOAD UNCERTAINTY ON  
ECONOMICAL LOAD SCHEDULING.**

BY

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**ABSTRACT:**

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Among the input data required for system economical load scheduling is the forecasted load. The uncertainty in forecasted load is of the main factors affecting economical load scheduling (E.L.S.) problem.

This paper presents the solution of E.L.S. problem for electrical power systems. The plants limitations, demand and transmission constraints are of the factors considered. Factors influencing forecasted load load uncertainty are illustrated. Mathematical formulation and comprehensive analysis for corrections must be made to E.L.S. due to incremental load changes are introduced.

**1. INTRODUCTION:**

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In electric power system operation, an effort is made to predispach the power generation for supplying economically the uncertain load demand satisfying the reliability and security constraints. These constraints are due to generating units and transmission network limitations as well as the uncertainty in forecasted loads.

To study the effect of load uncertainty on the economical load dispatching, it is important to have the total operating cost of the system (under economical load dispatching) as a function of load uncertainty. Various methods have been developed to predict the operating cost of a given utility. The most simplified procedure is based on the load duration curve (LDC)[4]. An improved approach[5] for calculating the operating cost of a power system is that based on the combined load duration curve.

**2. Economical Load Dispatching Problem:**

Economical load dispatching is a complex problem. It is determined by the total generation required is assigned to the different generating units so that the general operating cost is minimum. The reliability and security constraints constrain economical load dispatching determination level of the different units at any time. The incremental cost method has been widely used for economical load dispatching. In this method, the incremental cost of all available units is assumed equal for any demand level.

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2.1. Security constrained dispatch:

The power system scheduling process can be formulated as an optimization problem with linear inequality constraints [1]. The objective of the optimization problem is to minimize system generation cost subject to generator, transmission and reserve constraints. The objective function is:

$$F_T = \sum_i^{NG} F_i (P_i) \dots\dots\dots(1)$$

$F_T$  (the total system cost) is to be minimized w.r.t. generator output  $P_i$ .  $NG$  is the number of generating units. The resulting schedule should satisfy the following constraints:-

a) Demand constraint:

The total generation schedule should meet the system demand and transmission losses:

$$\sum_i^{NG} P_i = P_d + P_L \dots\dots\dots(2)$$

where  $P_d$  is the load demand.

b) Generator constraints:

The generation schedule produced should be within the physical limitations of generating units. The limits considered are the operating capability and the machine response limitations;

$$P_{MAX_i} \geq P_i \geq P_{MIN_i}, \quad i=1 \text{ to } NG \dots\dots\dots(3)$$

$$P_{i0} + P_{U_i} \geq P_i \geq P_{i0} - P_{D_i} \dots\dots\dots(4)$$

Where:

$P_{MAX_i}$ ,  $P_{MIN_i}$  are the dispatch limits of generator  $i$ .

$P_{i0}$  is the initial generation level of unit  $i$ .

$P_{U_i}$  is the maximum pickup capability of unit  $i$ .

$P_{D_i}$  is the maximum load drop capability of unit  $i$ .

c) Transmission constraints:

The resulting flow through any line, group of lines and/or transfer interface should not violate any imposed limits;

$$PTM X_k \geq PT_k \geq PTM N_k, \quad k=1 \text{ to } NL \quad \dots\dots\dots(5)$$

Where:

$PTM X_k$ ,  $PTM N_k$  are the flow limits imposed on transmission line k

$PT_k$  is the flow in transmission line k.

$NL$  is the number of transmission lines.

d) System reserve requirements:

The generation schedule produced should insure that sufficient pickup capability exists to meet load uncertainties and/or source contingencies;

$$RMAX_i \geq RS_i, \quad i=1 \text{ to } NG \quad \dots\dots\dots(6)$$

$$PMAX_i - P_i \geq RS_i \quad \dots\dots\dots(7)$$

$$\sum_i^{NG} RS_i \geq RT \quad \dots\dots\dots(8)$$

Where:

$RMAX_i$  is the maximum reserve pickup of unit i.

$RS_i$  is the reserve left on unit i at a given output.

$RT$  is the total system reserve required.

The forecasted load is appropriately represented by a probabilistic distribution, reflecting the fact that there is a certain probability that system load may exceed the forecasted value. The stochastic deviations from the forecasted loads may significantly influence the power system security function. The security function expresses the system risk as a function of time in the near future. The system risk identifies a breach of security as a breach of specific limits on node voltage, power flow in transmission lines...etc. Such limits have to be agreed upon, and the system risk is the probability of exceeding these limits.

2.2. Effect of forecasted load uncertainty on E.L.S.:

2.2.1. Sources of load uncertainty:

Although it is possible now to reduce the maximum error of forecasted load to 4.0% with 1.5% standard deviation [8,9], there are an amount of uncertainty which could not be avoided. This uncertainty is a result of the following sources:-

- 1) Random component in the load behavior.
- 2) Random component in the weather sensitive load.
- 3) Recording and measurement errors of load and/or weather data.
- 4) Extreme weather effects.
- 5) Assumptions in load forecasting models.
- 6) Faults in the power system.

**2.2.2. Generalized load duration curve (GLDC):**

The need for better analysis that accounts for the load uncertainty has resulted in development of a modified LDC known as the equivalent load duration curve [3]. In its most general form the equivalent load contains four elements of demand:-

- 1) The deterministic component of load demand ( $P_d$ ).
- 2) The random component of load demand ( $P_r$ ).
- 3) Requirements for forced outages ( $P_f$ ).
- 4) Requirements for maintenance ( $P_m$ ).

Therefore, the combined load ( $P_c$ ) is:

$$P_c = P_d + P_r + P_f + P_m \quad \dots\dots\dots(9)$$

The deterministic component of load demand can be viewed as the expected load (forecasted) with a random component  $P_r$  distributed around it in a normal distribution [8,9] represented by its standard deviation. The demand for forced outage and the demand for maintenance are conceived of as the self demand of the power system with zero consumption of energy.

Maintenance requirements can be incorporated in the calculations of the GLDC by partitioning the total period involved to subperiods of constant maintenance and carry out the calculations for each subperiod separately. The results of various subperiods are then combined to yield the corresponding results for the whole period.

The random load component is incorporated in the calculations of the GLDC by scattering its probability distribution (normal distribution) to a sufficient number of step-sizes ( $k$ 's) as a fraction of its standard deviation. Thereby, the load demand is given by different levels each with certain probability. Hence, the partial generalized load duration curve  $t_o(P_c)$  containing  $P_d$ ,  $P_r$  and  $P_m$  which is fixed for each subperiod of constant maintenance (excluding  $P_f$ ) is:

$$\bar{t}_o(P_c) = \sum_0^{\infty} t(P_d) \cdot f(P_r) dP_d \quad \dots\dots\dots(10)$$

Where:

$t(P_d)$  is the LDC containing  $P_d$  and  $P_m$ .

$f(P_r)$  is the probability density function of load demand.

Equation (10) can be transformed to:

$$\bar{t}_0(P_c) = \sum_{k=0}^{\infty} t(P_d) \cdot P(P_k) \dots\dots\dots(11)$$

Where  $P(P_k)$  is the probability of having the uncertain load in the level  $k$ .

The forced outages of generating units can be incorporated in the GLDC using the recursive relation[6]:

$$\bar{t}_n(P_c) = (1-R_n) \cdot \bar{t}_{n-1}(P_c) + R_n \cdot \bar{t}_{n-1}(P_c(c_n)) \dots\dots(12)$$

Where:

- $c_n$  is the generating capacity of the  $n^{th}$  unit in MW.
- $R_n$  is the forced outage state probability of the  $n^{th}$  unit.
- $\bar{t}_n(P_c)$  is the partial GLDC including  $\bar{t}_{n-1}(P_c)$  and the demand for outages of the  $(n-1)^{th}$  unit.

2.2.3. Cost of total system generation:

The total operating cost of the power system can be obtained from the system incremental cost function and the load duration curve. For a deterministic load demand  $P_d$ , the incremental energy required  $dE$  is:

$$dE = t(P_d) \cdot dP_d \dots\dots\dots(13)$$

The total incremental cost  $dF_T$  of delivering  $dE$  is:

$$\begin{aligned} dF_T &= \lambda(P_d) \cdot dE \\ &= \lambda(P_d) \cdot t(P_d) \cdot dP_d \dots\dots\dots(14) \end{aligned}$$

Where:  $\lambda(P_d)$  is the system incremental fuel cost for demand  $P_d$ .

Therefore, the total operating cost of the system is:

$$F_T = \int_0^{P_{dmax}} \lambda(P_d) \cdot t(P_d) \cdot dP_d \dots\dots\dots(15)$$

Where  $P_{dmax}$  is the peak load demand.

To incorporate the load uncertainty in the total cost, equation (15) is transformed to:

$$F_T = \int_0^{P_{dmax}} \lambda(P_d) \cdot \bar{t}_0(P_d) \cdot dP_d \dots\dots\dots(16)$$

2.2.4. Cost of generation for each unit:

For the  $i^{th}$  generator, the energy delivered is:

$$E_i = \int_0^{P_{imax}} t(P_i) dP_i \quad \dots\dots(17)$$

Where:  $t(P_i)$  is the LDC for the  $i^{th}$  generator and  $P_i$  is its output power.

$t(P_i)$  is not known since it depends on the individual loading pattern of generator  $i$ .

So, equation (17) can be rewritten as:

$$E_i = \int_0^{P_{dmax}} t(P_i) \cdot \frac{dP_i}{dP_d} dP_d \quad \dots\dots(18)$$

At the point of equal incremental cost there exists:

$$t(P_i) = t(P_d) \quad \dots\dots(19)$$

Denoting the ratio of the  $i^{th}$  generator load  $P_i$  to the total system load  $P_d$  by  $m_i (P_d)$ , hence,

$$P_i = m_i (P_d) \cdot P_d \quad \dots\dots(20)$$

Which results by differentiation to:

$$\frac{dP_i}{dP_d} = m_i (P_d) \quad \dots\dots(21)$$

Therefore, equation (18) becomes,

$$E_i = \int_0^{P_{dmax}} t(P_d) \cdot m_i(P_d) dP_d \quad \dots\dots(22)$$

The cost of the  $i^{th}$  unit  $F_i$  is:

$$F_i = \int_0^{P_{dmax}} \lambda (P_d) \cdot t(P_d) \cdot m_i(P_d) dP_d \quad \dots\dots(23)$$

If there is uncertainty in the load demand, equation (23) is transformed to:

$$F_i = \int_0^{P_{dmax}} \lambda(P_d) \cdot t_o(P_d) \cdot m_i(P_d) dP_d \quad \dots\dots(24)$$

3. Sensitivity of Generation Scheduling to Load Uncertainty:

Since the forecasted load is always subjected to errors, it is necessary to consider frequent updating of the system load dispatching obtained from the optimization based on this forecast. Because the accuracy of the optimum generation schedule is influenced by load uncertainty, the optimum generation will no longer be the optimum one, and after the forecast errors are known correction to the original schedule is necessary.

For incremental change in the forecasted parameters, change in the incremental cost of delivered power  $\Delta\lambda(t)$  and incremental change in the optimum generation schedule  $\Delta P(t)$  are given [10] by:

$$(t) = a_{11}(t) \Delta H_P(t) + a_{12}(t) \Delta V - a(t) \Delta H_\lambda(t) \quad \dots\dots(25)$$

and

$$\Delta P(t) = H_{PP}^{-1} (\Delta H_P - H_{P\lambda}(t) - H_{PV} \Delta V) \quad \dots\dots(26)$$

Where H is a Hamiltonian (scalar) defined as:

$$H(\underline{P}, \lambda(t), V(t), t) = \sum_{i=1}^m F_i(P_i) + \lambda(t) \cdot (P_d(t) + P_L(P) - \sum_{i=1}^{m+n} P_i + \sum_{j=1}^n V_j(t) \cdot q_j(P_{m+j}) \quad \dots\dots(27)$$

Where:

- V is the water conversion factor for hydro units (£/cuft).
- m is the number of thermal units.
- n is the number of hydro units.
- $q_j$  is the water discharge for the  $j^{th}$  hydro unit.
- $V_j$  is the water conversion factor for the  $j^{th}$  hydro unit.

$$a(t) = (H_{\lambda P}^t H^{-1} H_{P\lambda})^{-1}$$

$$a_{11}(t) = a(t) H_{\lambda P} H_{PP}^{-1}$$

$$a_{12}(t) = -a_{11}(t) H_{PV}$$

An error in the load forecast  $\Delta P_d$  results in the following special cases:-

$$H \neq 0 \quad \dots\dots\dots(28)$$

and  $H_P = 0 \quad \dots\dots\dots(29)$

Therefore,  $\Delta H_\lambda = -\Delta P_d \quad \dots\dots\dots(30)$

and  $\Delta H_P = 0 \quad \dots\dots\dots(31)$

Assuming no change in the water conversion factor i.e.,

$$\Delta V = 0 \quad \dots\dots\dots(32)$$

Substituting equations (30, 31, 32) in equation (25), we have;

$$\Delta \lambda(t) = a(t) \Delta P_d \quad \dots\dots\dots(33)$$

The above equation shows that the uncertainty in load forecast results in loss of system economy.

The change in the optimum generation schedule  $\Delta P(t)$  due to changes in the load forecast is give by substituting equations (31, 32, 33) in equation (26) as:

$$\Delta \underline{P}(t) = -H_{PP}^{-1} H_{P\lambda} a(t) \Delta P_d \quad \dots\dots\dots(34)$$

Putting  $b(t) = -H_{PP}^{-1} H_{P\lambda} a(t)$ , equation (34) becoms:

$$\Delta \underline{P}(t) = b(t) \Delta P_d \quad \dots\dots\dots(35)$$

The above equation shows that for uncertain load forecast the optimum generation schedule requires frequent corrections as soon as forecast errors are known.

#### 4. CONCLUSIONS:

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It could be concluded that in the presenance of forecasted load uncertainty, it is important for power system operation to consider the effect of this uncertainty on economical load dispatching problem. The E.L.S. problem among the system different generating plants is analyzed. The maximum plant generation, transmission limits are of the constraints considered. To include the effect of load uncertainty G.L.D.C. is developed.



The cost of generation of each plant is also illustrated. It is shown that, in the presenance of load uncertainty, the power system operator judgement is demanded to modify and update the system generation schedule frequently.

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