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Frequency Analysis of Circular Cable Suspended Roofs

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تحليل الترددات للأسقف الدائرية المعلقة ذات الكابلات

الخلاصة:

الهدف الرئيسي لهذا البحث هو دراسة التحليل الديناميكي للأسقف الدائرية المعلقة ذات الكابلات حيث انها الشكل الأكثر شيوعا بين أشكال الأسقف المعلقة و ذلك باستخدام طريقة نطاق الترددات وعلي الرغم من أن هذه الطريقة تستخدم في الأساس لتحليل المنشات خطية التصرف الا أنه يمكن استخدامها في دراسة الأسقف المعلقة ذات الكابلات و ذلك لأن السرعة المتغيرة للرياح في بعض المناطق الجبلية هي المؤثر الديناميكي الرئيسي حيث يمكن إهمالها في المناطق غير الجبلية مقارنة بالسرعة المتوسطة للرياح وقد تمت دراسة و تحليل العديد من الأسقف الدائرية المعلقة ذات الكابلات باستخدام برنامج حاسب آلي تم انشائة بواسطة المؤلف الثاني قائم على طريقة نطاق الترددات وقد تم استخدام هذه النتائج في عمل رسومات بيانية لا بعدية حيث تم تحليل هذه الرسومات النتائج لاستنباط تأثير العوامل المختلفة على الترددات الطبيعية لمثل هذة المنشات.

Abstract

The main aim of the present work is to investigate the dynamic analysis of circular cable suspended roofs which are the most common forms of suspended roofs using the frequency domain method. Although frequency domain analysis is used basically for lincar systems, it may be used to analyze cable suspended roofs for which, wind is considered as the fundamental dynamic factor affecting in these structures. This is due to the fact that the fluctuating wind speed, except for sites in mountainous areas, may be neglected comparing with the mean wind speed.

Many circular cable roofs have been analyzed using a computer program constructed by the second author based upon the frequency domain analysis. The results werc used in making non-dimensional graphs. These graphs werc used to investigate the factors that affect the natural frequencies of circular cable suspended roofs.

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Introduction:

Cable structures are one of the two categories of tension structures, which include both cable structures and membrane structures. In tension structures, the main load-carrying members transmit the applied loads to the foundations or other supporting elements by direct tensile stresses without flexure or compression [1].

The development of high tensile steel cables has made it possible for the designers to transmit large axial forces in tension at a relatively low cost and so, cable suspended roof is the economic solution to cover large open areas without interior columns. They have been used to cover different types of buildings such as stadiums and sport halls, swimming pools and water reservoirs, concert hall, cooling towers, hangars, warehouses and factories.

Cable suspended roofs must be designed not only for static loads but also for dynamic loads. The failure, of the 853 m Tacoma Narrows bridge in 1940, because of aerodynamic oscillations, constant wind of 41 m.p.h[2],showed that it was not adequate to design structures for static stability only[3].

Circular roofs in plan may be radial, with tension ring, or of intersecting cables/cable beams. Generally, behavior of cable roofs is softening nonlinear. Also, they are lighter and more flexible than other forms of structures and, as a result, they are more resistant to earthquakes and more sensitive to wind than conventional structures.

A complete dynamic analysis includes:

1) A frequency analysis;

2) Establishment and formulation of the dynamic loading;

3) Estimation and formulation of the structural damping; and

4) A dynamic analysis.

The frequency domain method is limited to the analysis of linear-behavior structures. It is also, practically, applied to some nonlinear behavior, such as cable roofs taking only the nonlinear response due to the mean wind speed component into account and not for the case where there is no load on the structure. Apart from the assumptions with respect to the statistical characteristics of wind, the main assumption made in order to make the method possible is that the amplitudes of the fluctuating component of wind are sufficiently small compared to the mean wind speed and can be ignored. Generally, this assumption is justified except for sites in mountainous areas. This method is based on the spectral density function that enables the use of closed-form solutions, of the random loading. So, the accuracy of analysis will vary with the type of spectral density function used and it may be advisable to construct spectral density functions, for important structures, from recordings at site, concerned. Frequency domain method underestimates the response in the cable roofs as they are softening structures. As cable systems are nonlinear structures, both stiffness and frequency vary with the amplitude of response. As a result both natural frequencies and damping will also vary. For this reason, the closed-form solutions to obtain frequencies are no longer valid and an iterative process is needed. During the analysis, structural damping is assumed to be constant because of lack of information, and because the values given in the codes of practice tend to be conservative.

2- Frequency Domain Analysis

The eigenvalue equation may be written in general matrix notation as:

$$K\phi - \omega^2 M\phi = 0 \qquad (1\text{-a})$$

Or $$K\phi - \lambda M\phi = 0 \qquad (1\text{-b})$$

Where:

K = the tangent stiffness matrix at the static equilibrium position,

M = the mass matrix, which is diagonal since masses are lumped at nodes,

ϕ = the mode-shape matrix, and

$\omega^2 = \lambda = N \times N$ = corresponding natural frequencies.

The determination of eigenvalues, ω^2, is of fundamental importance to the frequency domain method of analysis, in which the distribution of energy of random forces such as wind are given as function of their frequency content in terms of power spectra.

Structural damping is usually not included when one is formulating the eigenvalue problem, as it increases the numerical effort considerably and has only a second effect on the calculated frequencies.

In the iterative method, the eigenvalues ω^2 and eigenvectors ϕ are determined by optimizing an assumed mode-shape vector through an iterative procedure on either:

$$\omega^2\phi = M^{-1}K\phi \qquad (2)$$

Or $$\frac{\phi}{\omega^2} = K^{-1}M\phi \qquad (3)$$

Iterations on Eqn (2) will cause the assumed eigenvector to converge towards the mode corresponding to the highest eigenvector and hence the highest frequency; iterations on Eqn (3) will cause the assumed eigenvector to converge towards the eigenvector corresponding to the lowest frequency. Eqn (2) involves the inversion of the mass matrix M, which when the matrix is diagonal is achieved by simply inverting each of the elements on the time to be inverted. This problem can be avoided by calculating the lowest eigenvalue and eigenvector as follows:

Let $$B\phi_i = \left[\alpha\ I - M^{-1}K\right]\ \phi_i \qquad (4)$$

Where: α = constant larger than the highest eigenvalue;

I = the unit matrix; and

B = square matrix having the same order of matrices M and K.

From Eqn (2) it follows that:

$$M^{-1}K\phi_i = \omega_i^2 I\phi_i \qquad (5)$$

Substituting of the expression for $M^{-1}K\phi_i$ given in Eqn (5) into Eqn (4) yields:

$$B\phi_i = \left[\alpha\ - \omega_i^2\right]\ I\phi_i = \left[\alpha\ - \omega_i^2\right]\ \phi_i \qquad (6)$$

Assuming an initial vector ϕ_i, iteration of Eqn (6) will yield the highest value of $\left[\alpha\ - \omega_i^2\right]$ and hence the lowest possible value of ω_i^2. Thus

$$\begin{aligned}\omega_i^2 &= \omega_l^2 \\ \phi_i &= \phi_l\end{aligned} \qquad (7)$$

Iteration algorithm based on Eqs (2) and (6) will yield the highest and lowest natural frequencies and the corresponding mode-shape for any structure.

The Raleigh quotient

Pre-multiplication of each term of Eqn (1-a) by ϕ^T yield:

$$\phi^T K\phi - \omega^2\phi^T M\phi = 0 \qquad (8)$$

Hence $$\omega^2 = \frac{\phi^T K\phi}{\phi^T M\phi} \qquad (9)$$

The expression for ω^2 given by Eqn (9) is called Rayleigh quotient. It has the property that for even approximately correct values of eigenvectors or mode-shape vectors the values for the frequencies are reasonably correct. This can be seen simply by pre-multiplying each term in Eqn (8) by ½. This yield:

$$\frac{1}{2}\phi^T K\phi = \frac{1}{2}\omega^2\phi^T M\phi \qquad (10)$$

Which states that the maximum strain energy ($\frac{1}{2}\phi^T K\phi$) is equal to the maximum kinetic energy ($\frac{1}{2}\omega^2\phi^T M\phi$) due to the mode shape vector ϕ.

2.1 Mass, damping and stiffness matrices

2.1.1 Mass properties

Mass matrix of cable element may be expressed in one of the following:

1) **Lumped-mass matrix**

Assuming that the entire mass of a structure is concentrated at the points at which the translational displacements are defined, the lumped mass matrix for a cable element is given by:

$$[M_L]=\frac{mL}{2}\begin{bmatrix}1&0&0&0&0&0\\0&1&0&0&0&0\\0&0&0&0&0&0\\0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&0\end{bmatrix} \quad (11)$$

Where m is the mass per unit length of the cable element.

2) **Consistent-mass matrix**

$$[M_C]=\frac{mL}{6}\begin{bmatrix}2&0&0&1&0&0\\0&2&0&0&1&0\\0&0&2&0&0&1\\1&0&0&2&0&0\\0&1&0&0&2&0\\0&0&1&0&0&2\end{bmatrix} \quad (12)$$

Lumped-mass system needs effort than consistent-mass system for two reasons:

i) Lumped-mass matrix is diagonal.

ii) Rotational degrees of freedom can be eliminated from a lumped-mass analysis, whereas all rotational and translational degrees of freedom must be included in the consistent-mass

2.1.2 Damping matrix

Damping matrix is expressed as:

$$[C]=2\xi\omega[M] \quad (13)$$

Thus, the damping matrix will only be diagonal if the mass matrix is also diagonal.

Eqn (13) implies that the damping forces at different points in a structure are proportional to the distribution of mass and that the damping ratios decrease and are very small in the higher modes of vibration.

2.1.3 Stiffness matrix

The stiffness matrix of pin-jointed pretensioned link is given by:

$$[K]=\frac{EA-T_0}{L_0}\begin{vmatrix}GG^T & -GG^T\\ -GG^T & GG^T\end{vmatrix}+\frac{T_0}{L_0}\begin{vmatrix}I & -I\\ -I & I\end{vmatrix}$$

. . . . (14)

Where I is a unit matrix of dimension (3×3), and

$G=\{l \quad m \quad n\}$ Where l, m and n are the direction cosines of the member.

2.2 Reduction of NDOF

When the mass of a structure is assumed to be concentrated at the nodes, it is usual to consider only the inertia due to transnational movements and to ignore that due to rotation. This assumes that the lumped masses are concentrated as point masses with radii of gyration equal to zero. Thus in the case of flexible structures, such as cable roofs, where the joints rotate, the elements on the leading diagonal of the mass matrix corresponding to the rotational degrees of freedom will be zero. In such case, the mass matrix can not be inverted. Therefore the elements related to the rotation need to be eliminated from the stiffness matrix. Condensation or reduction of the stiffness matrix may also be desirable to reduce the overall degree of freedom, of structures with a very large number of DOF in order to reduce the numerical problem.

Three condensation methods are mentioned below:

2.2.1 Static condensation method [4, 5, 6, and 7]

Stiffness equation of any structure may be written using partition of matrices as:

$$\begin{vmatrix} [K_{\theta\theta}] & [K_{\theta x}] \\ [K_{x\theta}] & [K_{xx}] \end{vmatrix} \begin{vmatrix} \{\theta\} \\ \{x\} \end{vmatrix} = \begin{vmatrix} \{0\} \\ \{P\} \end{vmatrix} \quad (15)$$

Where $\{\theta\}$ and $\{x\}$ are the displacement vectors corresponding to θ and x degrees of freedom, respectively, where θ are the secondary coordinates to be condensed and x are the primary coordinates (remaining coordinates). Carrying out Gauss-Jordan elimination, Eqn (15) may be written as follows:

$$\begin{vmatrix} [I] & -[\overline{T}] \\ [0] & [\overline{K}] \end{vmatrix} \begin{vmatrix} \{\theta\} \\ \{x\} \end{vmatrix} = \begin{vmatrix} \{0\} \\ \{P\} \end{vmatrix} \quad (16)$$

It should be noted that in Eqn (15), it is assumed that at the dependent degrees of freedom θ, the external forces are zero. Eqn (16) is equivalent to both:

$$\theta = \overline{T}x \quad (17)$$

And $$\overline{K}x = P \quad (18)$$

Where $\overline{T}$ is the transformation matrix given by?

$$[\overline{T}] = -[K_{\theta\theta}]^{-1}[K_{\theta x}] \quad (19)$$

In Eqn (18), that shows the relationship between the displacement vector x and the force vector P, $\overline{K}$ (the reduced stiffness matrix)and may be expressed by the following transformation of the system matrix:

$$\overline{K} = T^T KT \quad (20)$$

Where $$T = \begin{bmatrix} \overline{T} \\ I \end{bmatrix} \quad (21)$$

Similarly, the reduced mass and damping matrices will be expressed as:

$$\overline{M} = T^T MT \quad (22)$$

$$\overline{C} = T^T CT \quad (23)$$

Eqn (17), which expresses the relationship between displacement vectors x and θ, may also be rewritten, to calculate the modal shape matrix of the system, as:

$$\phi = \begin{vmatrix} \{\theta\} \\ \{x\} \end{vmatrix} = \begin{vmatrix} [\overline{T}] \\ [I] \end{vmatrix} \{x\} \quad (24)$$

2.2.2 Dynamic condensation method [8, 9 , and 10]

Equation (15) will be dynamically extended and rewritten as:

$$\begin{vmatrix} [K_{\theta\theta}] - \omega_i^2[M_{\theta\theta}] & [K_{\theta x}] - \omega_i^2[M_{\theta x}] \\ [K_{x\theta}] - \omega_i^2[M_{x\theta}] & [K_{xx}] - \omega_i^2[M_{xx}] \end{vmatrix} \begin{vmatrix} \{\theta\} \\ \{x\} \end{vmatrix} = \begin{vmatrix} \{0\} \\ \{0\} \end{vmatrix} \quad (25)$$

Where ω_i^2 is the approximation of the i th eigenvalue which was calculated in the preceding step of the process. To start the process an approximate or zero value is taken for the first eigenvalue ω_1^2. After carrying out a Gauss-Jordan elimination of the unknown rotations θ, Eqn (25) will be rewritten as:

$$\begin{bmatrix} I & -\overline{T} \\ 0 & \overline{D}_i \end{bmatrix} \begin{bmatrix} \theta \\ x \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad (26)$$

Where: $\overline{D}_i$ = the reduced dynamic equation and expressed as:

$$\overline{D}_i = \overline{K}_{xx} - \omega_i^2 \ \overline{M}_{xx} \qquad (27)$$

The reduced mass and damping matrices are expressed respectively as:

$$\overline{M}_i = T_i^T M \ T_i \qquad (28)$$

$$\overline{C}_i = T_i^T C T_i \qquad (29)$$

Where, transformation matrix $\overline{T}_i$ is calculated using Eq. (17).

Using Eqn (27), the reduced stiffness matrix will be expressed as:

$$\overline{K}_i = \overline{D}_i + \omega_i^2 \overline{M}_i \qquad (30)$$

Finally, according to Eqs (26) and (30), the reduced eigen-problem is:

$$[\overline{K}_i - \omega_i^2 \ \overline{M}_i]x = 0 \qquad (31)$$

This equation is solved to obtain an improved eigenvalue ω_i^2, and also an approximation for the next order eigenvalue ω_{i+1}^2. The i th modal shape ϕ_i is given, using the eigenvector x for the reduced system, by Eqn (24).

2.2.3 Modified dynamic condensation method [11]

Firstly, setting $\omega_i^2 = 0$ in Eqn (30), it yields an unchangeable reduced stiffness matrix $\overline{K}$. Secondly, an approximated value is taken for the first eigenvalue ω_1^2 to calculate the corresponding reduced dynamic matrix $\overline{D}_i$ using Eqn (27).

The corresponding reduced mass matrix for the i th mode is given by:

$$[\overline{M}_i] = \frac{1}{w_i^2}[[\overline{K}] - [\overline{D}_i]] \qquad (32)$$

The reduced eigen-problem will be:

$$[\overline{K} - \omega_i^2 \ \overline{M}_i]x = 0 \qquad (33)$$

As mentioned before, this equation is solved to obtain an improved eigenvalue ω_i^2, and also an approximation for the next order eigenvalue ω_{i+1}^2. Also, the i th modal shape ϕ_i is given using Eqn (24).

3- Factors affect natural frequencies of circular cable roofs [12]

Using a computer program based on the frequency domain method mentioned before [13], the natural frequencies and mode-shapes of many circular cable roofs (nets, concave grids and convex grids) are calculated; Fig. (1). These natural frequencies are recalculated for different values of sag/span ratio, rise/span ratio, spacing between the nodes, the steel area of the sagging cable, the steel area of the hogging cable, the steel area of the hangers, the pretension of the sagging cable, the pretension of the hogging cable and the pretension of the hangars. To study the influence of any factor, the other factors are kept constant. The results are shown in non-dimensional graphs, Figs. (2) to (17). The analyzed roofs (nets, concave grids and convex grids) have the following properties: Diameter = 40.0m, Spacing = 4.0m, Sag = 1.60m (4%), Rise = 1.20m (3%), w = 0.1 t/m^2, initial tensions for all cables = 6.25 ton with modulus E= 1663t/cm^2 , steel area of all members = 13.92 cm^2 and Height = 10.00m.

3.1 Natural frequencies of circular cable nets

The results presented in Figs. (2) to (6) showed that:

1) Increasing the applying uniformly distributed loads decreases the natural frequencies.

2) Increasing the pretension increases the natural frequencies.

3) Increasing the cable steel area does not change the first natural frequency, whereas higher natural frequencies increase in a very slight rate that can be considered unchanged also.

4) Increasing the sag/span ratio decreases the natural frequencies.

5) Increasing the spacing between nodes decreases the natural frequencies.

Figures (7) and (8) represent the first two mode-shapes of the circular cable net with the properties mentioned before.

3.2 Natural frequencies of circular cable grids

The results presented in Figs. (9) to (17) show that:

1) Increasing the sag/span ratio increases the natural frequencies of both concave and convex grids in a small rate.

2) Increasing the rise/span ratio has approximately no effect on convex grids, whereas it increases the natural frequencies of the concave grids in a small rate.

3) Increasing the spacing between hangers decreases the natural frequencies.

4) Increasing the steel area of one of the two cables increases the natural frequencies in a very small rate so that it has practically no effect. Also, hangers, either ties or struts, have no effect on the natural frequencies.

5) Increasing the pretension of either the suspension or the pretensioned cables increases the natural frequencies. The same thing will happen when increasing the pretension of the hangers, but in a smaller rate.

4- Conclusions:

A frequency analysis of cable roofs is an essential step to complete their dynamic analysis. Frequency domain analysis of cable roofs is obtained using an iterative eigen-problem and a spectral density function of the dynamic load, which is most probably due to wind. The following conclusions are drawn:

1) Increasing the steel areas of cables or hangers has approximately no effect upon the dynamic stability of the structure. This is due to their very small own weight compared with the applied loading.

2) Increasing sag/span ratio of a cable net makes the structure more dynamically excitable, whereas increasing sag or rise to span ratios in cable grids slightly increases the dynamic stability of the structure.

3) The most efficient and economic solution to make the circular cable roof more dynamically stable, is to increase the pretension of the cables and the hangers.

4) Convex grids are dynamically better than concave grids and both are better than cable nets.

5- References:

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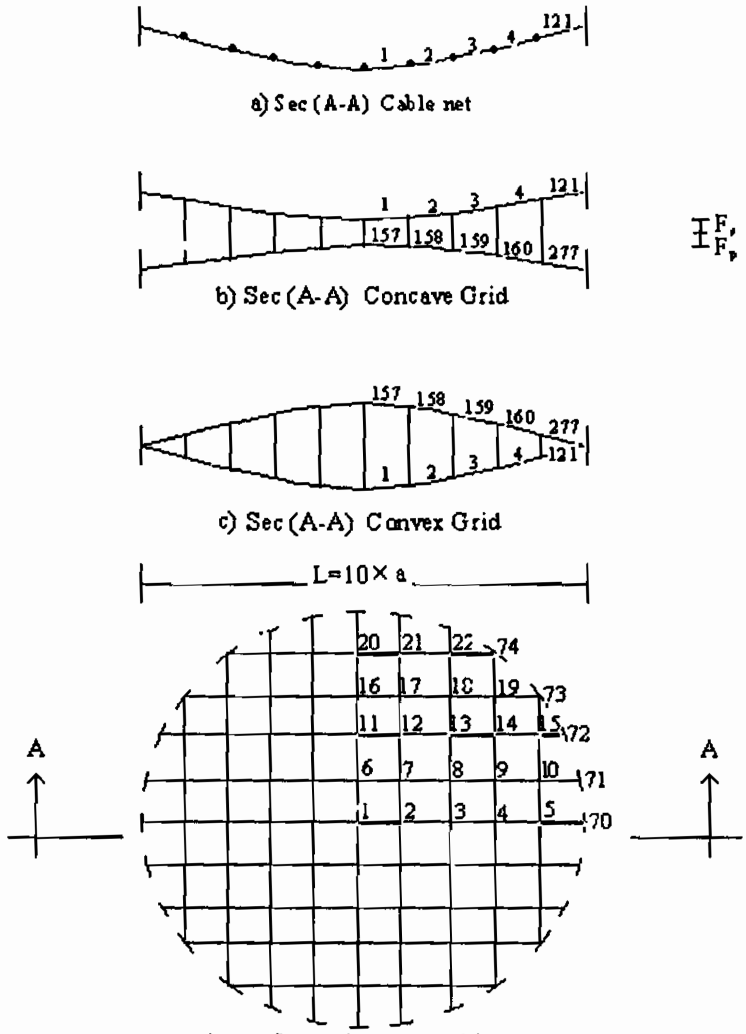

Plan of the circular cable Roof.

Fig. (1): (10×10) **Circular cable roof.**

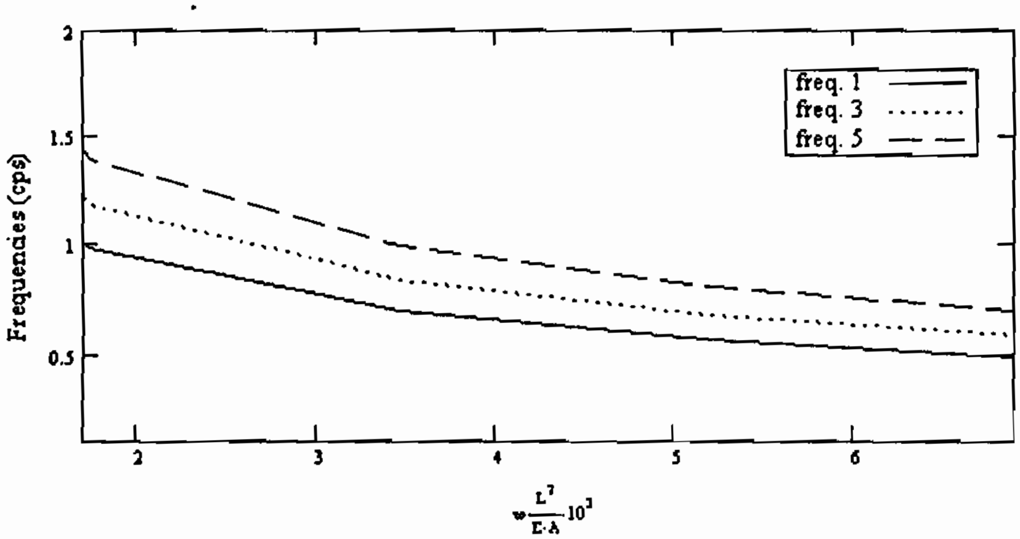

Fig. (2): Effect of increasing the applying load upon frequencies.

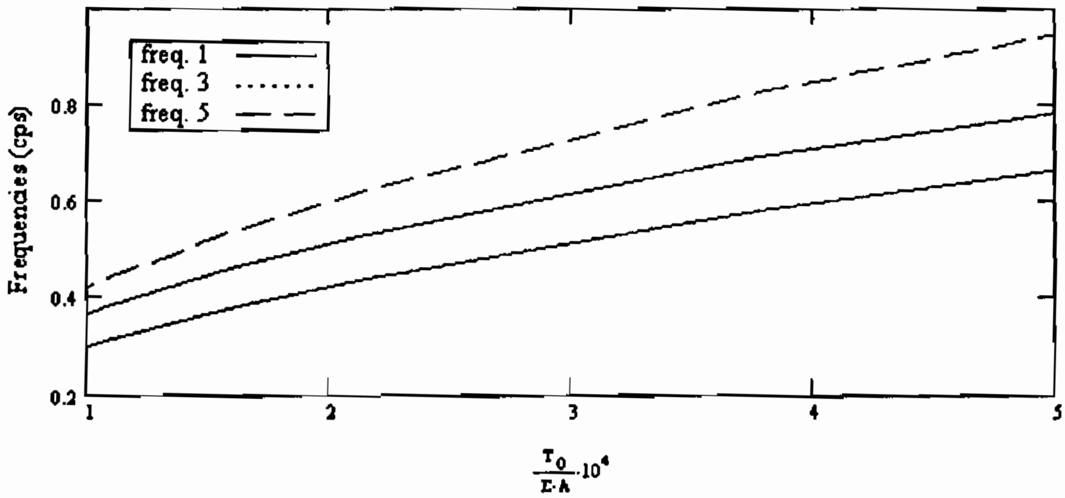

Fig. (3): Effect of increasing the pretension on frequencies in the circular cable net.

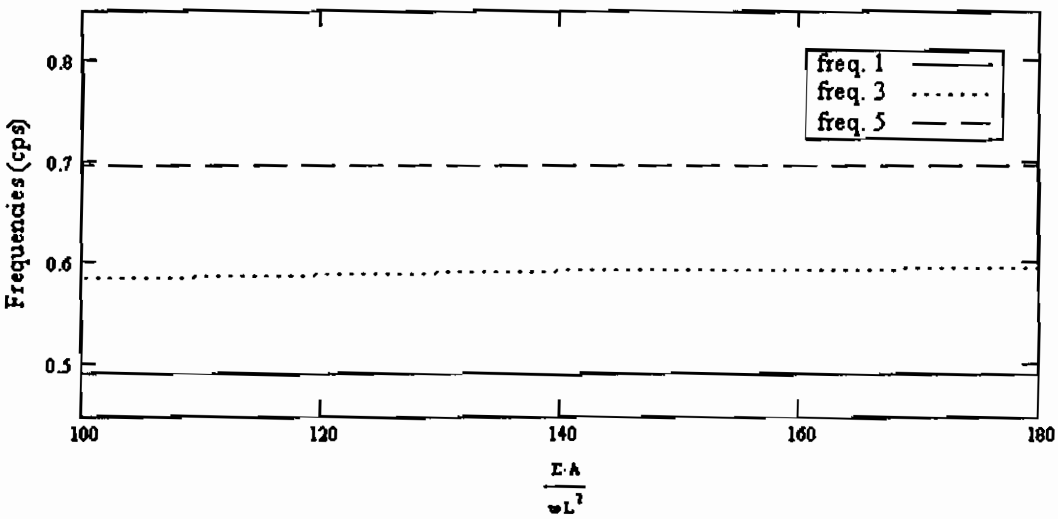

Fig. (4): Effect of increasing cable steel area on frequencies in the circular cable net.

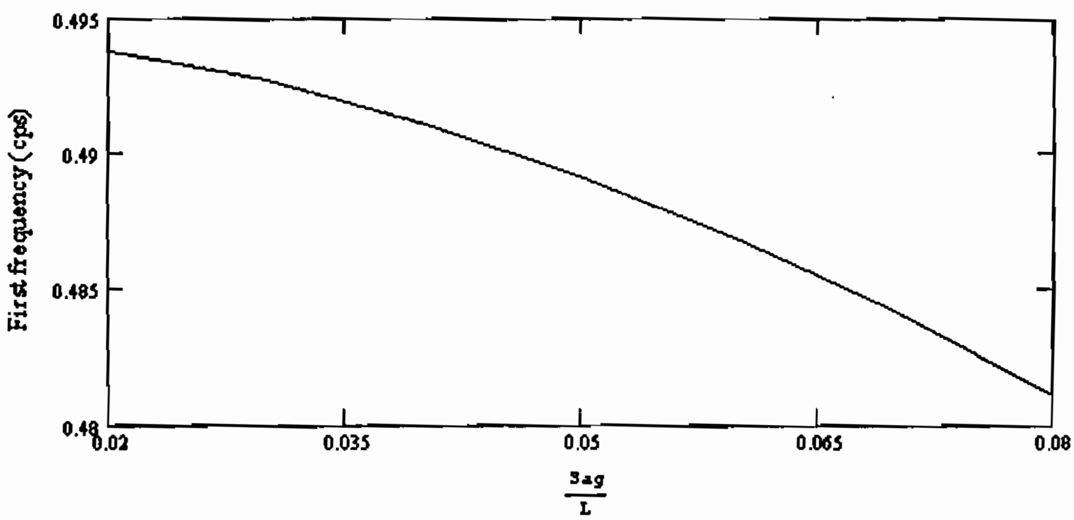

Fig. (5): Effect of increasing sag/span ratio on frequencies in the circular cable net.

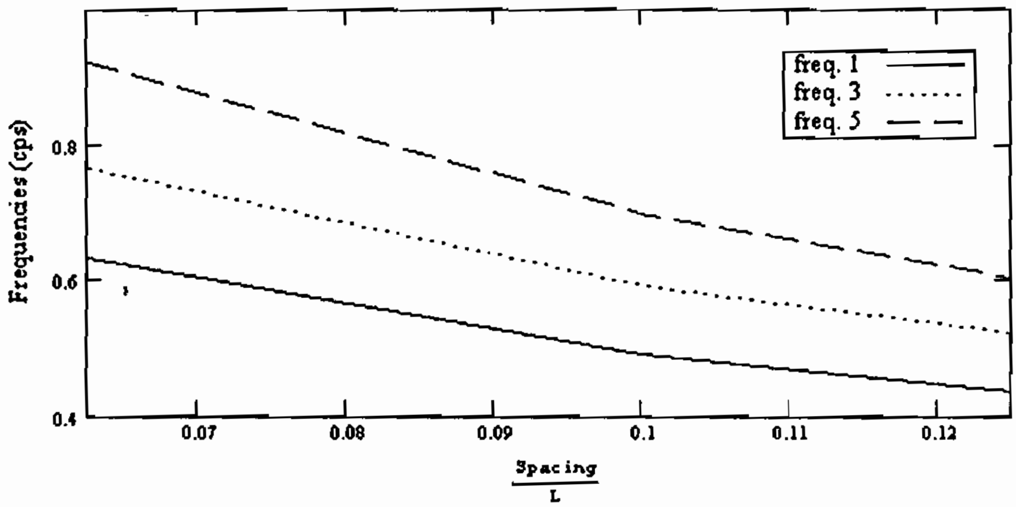

Fig. (6): Effect of increasing the spacing between nodes on frequencies in the circular cable net.

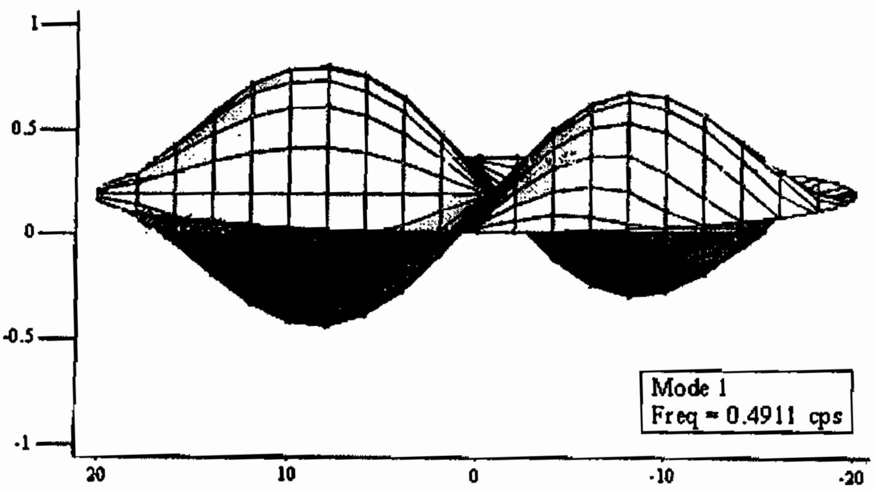

Fig. (7): The first mode-shape of the circular cable net.

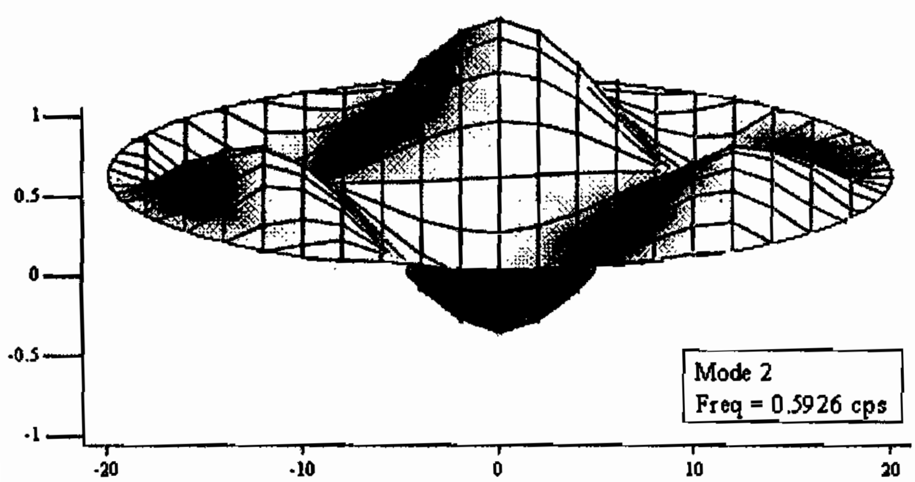

Fig. (8): The second mode-shape of the circular cable net.

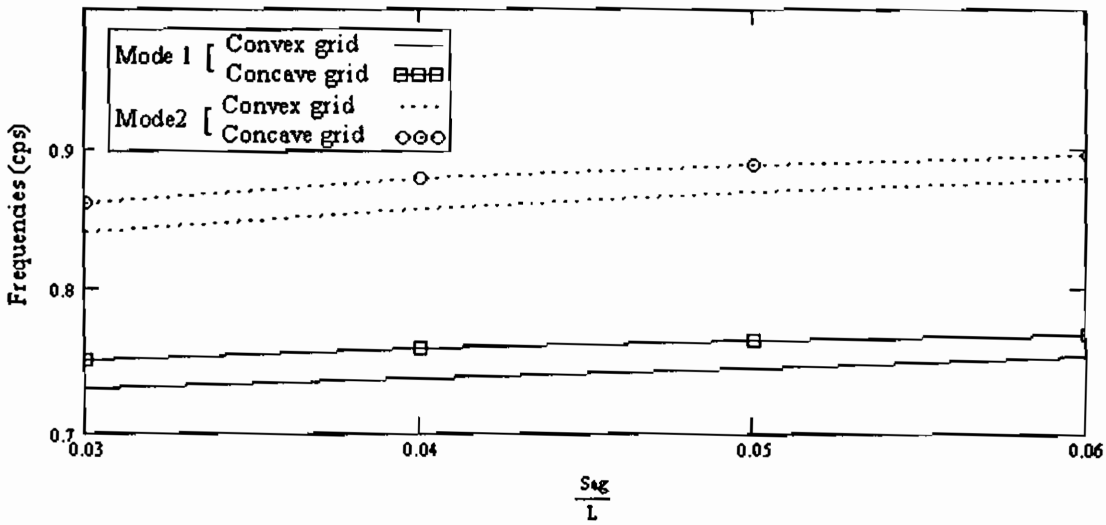

Fig. (9): Effect of the sag/span ratio on natural frequencies of the circular cable grid.

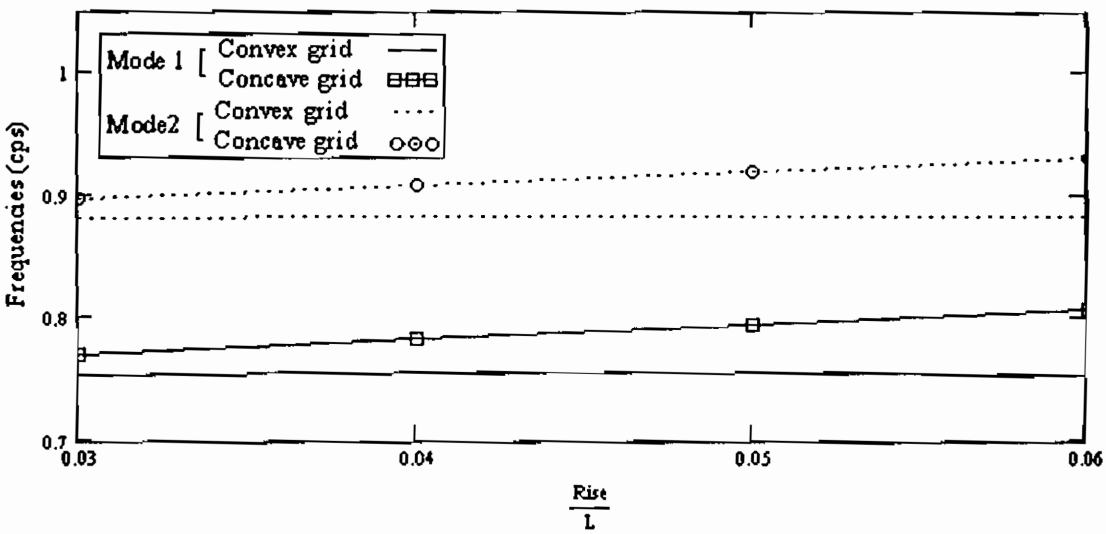

Fig. (10): Effect of the rise/span ratio on natural frequencies of the circular cable grid.

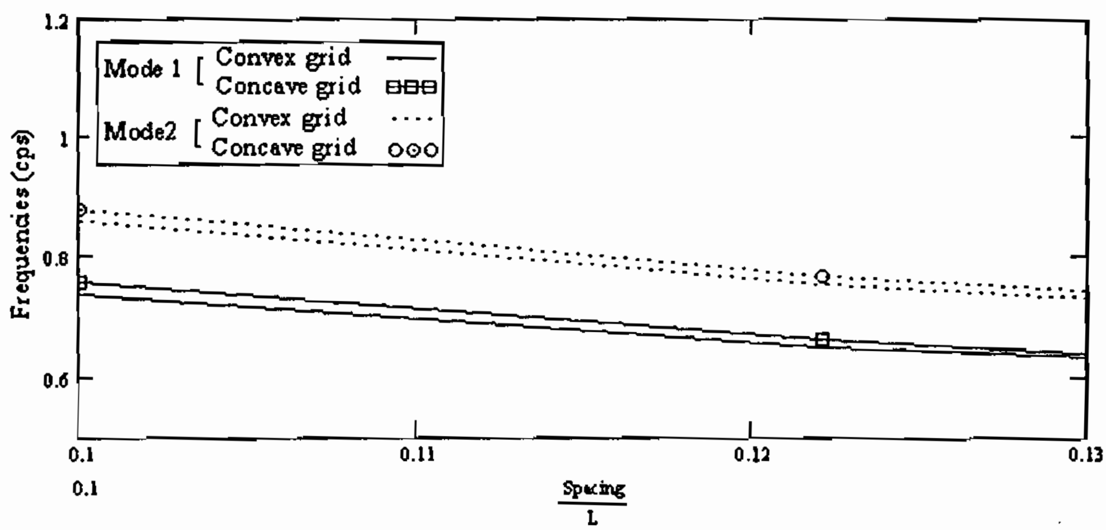

Fig. (11): Effect of the spacing between hangers and natural frequencies of the circular cable grid.

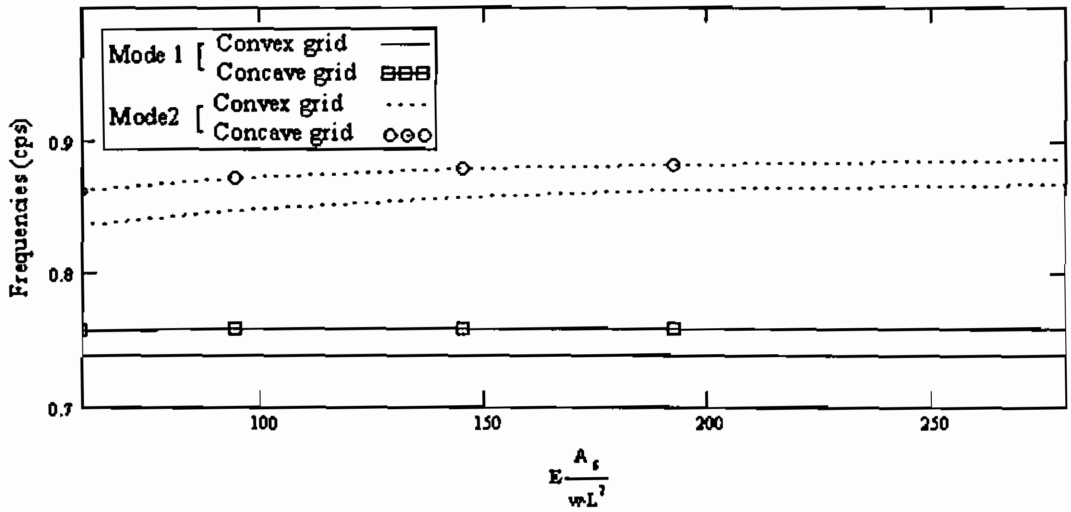

Fig. (12): Effect of the steel area of the suspension cables on natural frequencies of the circular cable grid.

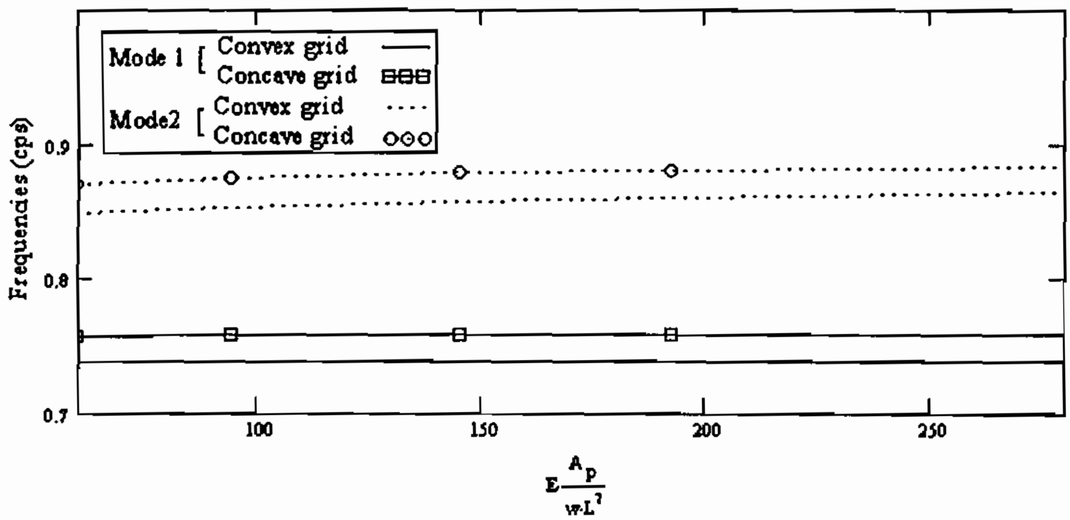

Fig. (13): Effect of the steel area of the pretensioned cables on natural frequencies of the circular cable grid.

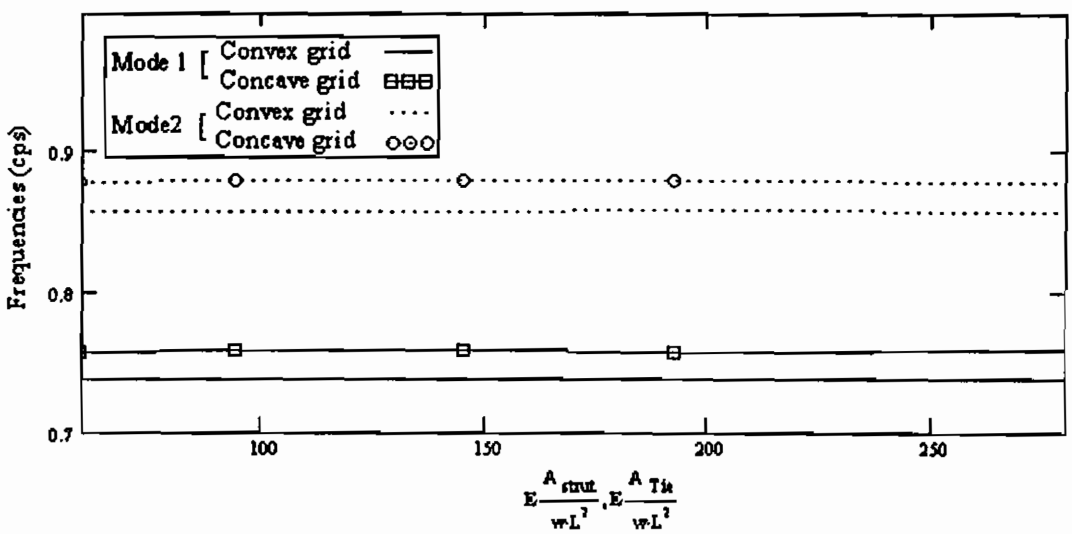

Fig. (14): Effect of the steel area of the hangers on natural frequencies of the circular cable grid.

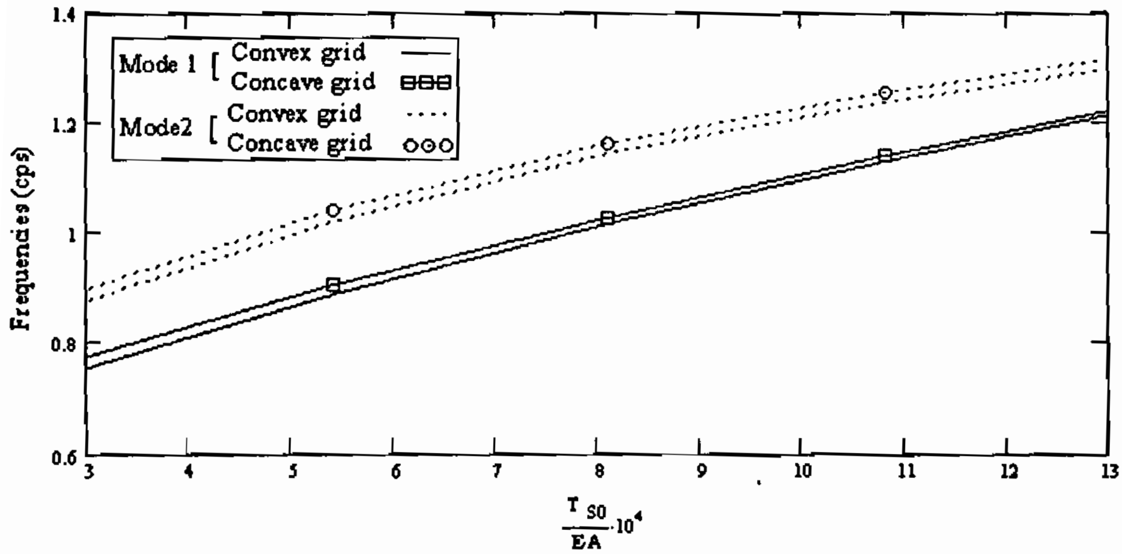

Fig. (15): Effect of the pretension of the suspension cables on natural frequencies of the circular cable grid.

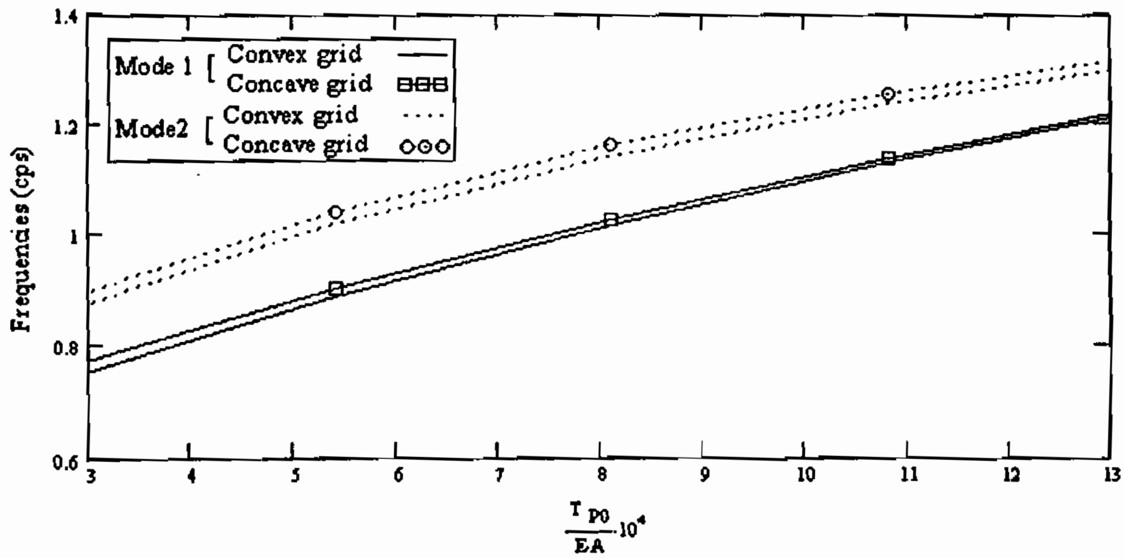

Fig. (16): Effect of the pretension in he pretensioned cables on the natural frequencies of the circular cable grid.

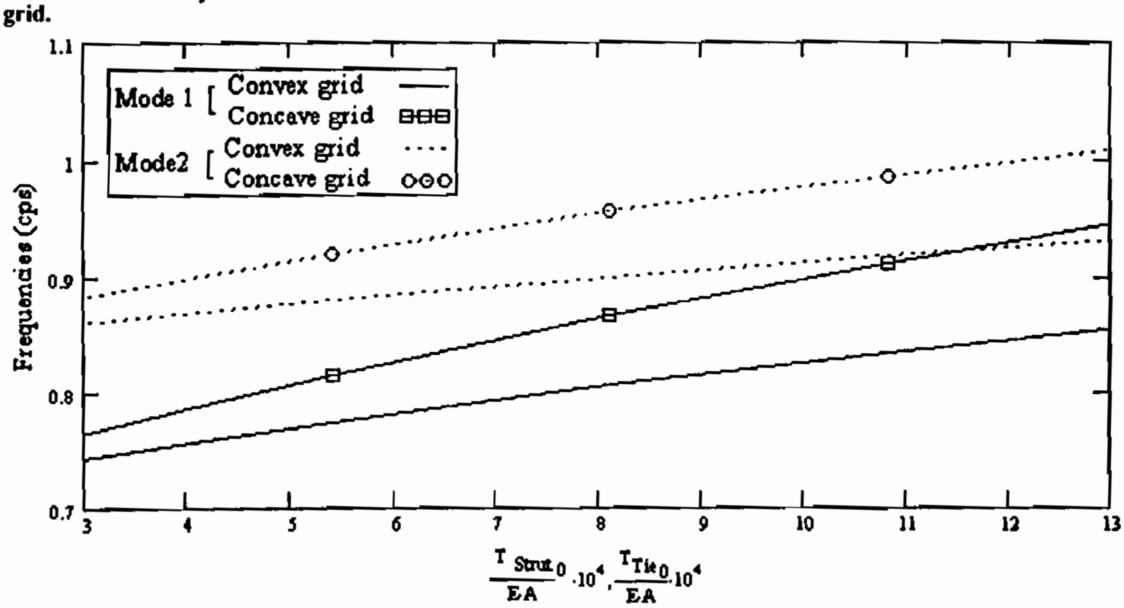

Fig. (17): Effect of the pretension in the hangers on the natural frequencies of the circular cable grid.