Menofia University Faculty of Engineering, Shebin El-Kom Dep. Mechanical Power Engineering Second Semester Examination 2017-2018 Date of Exam: 3 / 06 / 2018	Minoufia University	Subject: Engineering Mathematics (2) Code: BES112 Year : 1st year Time Allowed : 3 hours Total Marks: 100 marks

Answer the following questions: عدد الاسئلة (ثلاثة أسئلة) (الأمتحان في صفحتين)

Question 1 **25 marks**

(A) **Find the general solution of the differential equations:-**

(i) $(e^x + 1)\, y\, dy = (y+1)\, e^x\, dx$ (ii) $\frac{dy}{dx} = \frac{x+3y-5}{2x-y-3}$

(iii) $x\frac{dy}{dx} + y = y^2 \ln x$ (iv) $\frac{dy}{dx} + y \sec x = \cos^2 x$

(v) $\frac{dy}{dx} - 5y = -\frac{5}{2} x y^3$

(B) **Find the general solution of the differential equations :-**

(i) $\left(\frac{dy}{dx}\right)^2 - 2x\frac{dy}{dx} + y = 0$

(ii) $(D^2 + 9)\, y = \cos 2x + \sin 2x$ (iii) $(x^2 D^2 + 3xD - 3)\, y = x$

(C) **Solve the following system of simultaneous linear D. Eqs.:**

(i) $D y_1 - y_2 = 0, \quad -y_1 + D y_2 = 0$ (ii) $\frac{dx}{dt} = y, \quad \frac{dy}{dt} = x$

(D) Prove that any separable differential equation is an exact differential equation.

Question 2 **25 marks**

(A) Find the orthogonal trajectories of the curves:

(i) $y = c\, x^2$ (ii) $x^2 + y^2 = c^2$

(B) Find the area of the region R that enclosed by the parabola $y = x^2$ and the line $y = x + 2$.

(C) Find the mass and the center of mass (gravity) of a thin plate bounded by the parabola $y = 6x - x^2$ and the straight line $y = x$, given that it has a mass density $\rho(x, y) = 1$.

(D) Evaluate the following integral: $\int_0^4 \int_{\sqrt{y}}^2 y \cos x^5\, dx\, dy$

Question 3 — 50 marks

(A) Test the following *sequences* are convergence and monotonic (and/or) bounded:

(i) $\left\{1+\frac{1}{n}\right\}$	(ii) $\left\{\frac{n}{n+1}\right\}_{n=1}^{\infty}$	(iii) $\{(-1)^{n+1}\}_{n=1}^{\infty}$

(B) Test the convergence of the following series:

(i) $\sum_{n=1}^{\infty}\frac{(-4)^{3n}}{5^{n-1}}$ and find its sum	(ii) $\sum\frac{1}{(n+1)\,ln\,(n+1)}$
(iii) $\sum_{n=1}^{\infty}\frac{3}{\sqrt{n^2+n+1}}$ (by limit comparison test)	(iv) $\sum_{n=1}^{\infty}\left(1-\frac{3}{n}\right)^{n}$

(C) Determine whether the following series is *absolutely convergent*, *conditionally convergent*, or *divergent*:

(i) $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{n^2+4}{2^n}$	(ii) $\sum_{n=1}^{\infty}(-1)^{n-1}(0.3)^n$	(iii) $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{n+1}{n^2}$

(D) Find the radius and the interval of convergence of the following power series:

(i) $\sum_{n=1}^{\infty}\frac{(x-3)^n}{n}$	(ii) $\sum_{n=1}^{\infty}\frac{x^n}{\sqrt{n}}$

(E) Find the Fourier series of the function: $f(x)=x, \quad -\pi<x\le\pi$

(i) In the form: $f(x)=2\left[\sin x-\frac{1}{2}\sin 2x+\frac{1}{3}\sin 3x-\cdots\right]$

(ii) By giving an appropriate value to (x), show that: $\frac{\pi}{4}=1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots$

(F) Find Laplace transform of the following functions:

(i) $e^{-3t}\sin(4t)$ (ii) $G(t)=\begin{cases}0 & t<2\\ (t-2)^2 & t>2\end{cases}$

(iii) $\int_0^t\frac{\sin(t)}{t}$ (iv) $e^{-2t}t^2$ (v) $\frac{1-cos(t)}{t}$

(G) Find Inverse Laplace transform of the following function:

(i) $F(S)=\frac{1}{S^2-3S+2}$ (ii) $F(S)=ln\frac{S-1}{S+2}$ (iii) $F(S)=\frac{S\,e^{-\frac{2\pi}{3}S}}{S^2+1}$

(iv) $F(S)=tan^{-1}(S+1)$ (v) $F(S)=\frac{1}{S^2+2S+5}$

(H) Solve the differential equation $y''-3y'+2y=6e^{2t}$ using the Laplace transform method with initial conditions $y(0)=-3, y'(0)=5$.

This exam measures the following ILOs

Question Number	Q4-a	Q2-a	Q4-b	Q1-b	Q3-b	Q4-b	Q3-c		Q1-b	Q3-a	Q4-a
Skills	Q2-b				Q2-a	Q3-b			Q3-a		
	Knowledge &understanding skills				Intellectual Skills				Professional Skills		

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