Menofia University Faculty of Engineering Shebien El-kom Basic Engineering Sci. Department. Academic Year : 2017-2018 Date : 30 / 12 / 2017	Minoufia University	Subject: Linear algebra. Code: 502 Time Allowed: 3 hours. Year: Master. level (500) Total Marks: 100 Marks.

Answer all the following questions:

[100 Marks]

Q.1 **(A)** Let C be the curve $f(t) = \left[t^2, 3t-2, t^3, t^2+5\right]$ in R^4, where $0 \le t \le 4$ find:

1. The point P on C corresponding to t=2.
2. The initial point Q and terminal point Q' of C.
3. The unit tangent vector T to the curve C when t=2.

(B) Let $A = \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix}$ and let $f(x) = 2x^3 - 4x + 5$ and let $g(x) = x^2 + 2x - 11$

Find: i) A^T ii) A^3 iii) $f(A)$.

Q.2 **(A)** Show that $A = \begin{bmatrix} \frac{1}{3} - \frac{2}{3}i & \frac{2}{3}i \\ -\frac{2}{3}i & -\frac{1}{3}i - \frac{2}{3}i \end{bmatrix}$ is unitary.

(B) Compute AB using block multiplication where $A = \left[\begin{array}{cc:c} 1 & 2 & 1 \\ 3 & 4 & 0 \\ \hdashline 0 & 0 & 2 \end{array}\right]$ and,

$B = \left[\begin{array}{ccc:c} 1 & 2 & 3 & 1 \\ 4 & 5 & 6 & 1 \\ \hdashline 0 & 0 & 0 & 1 \end{array}\right]$

(C) Let $\mathbf{u}_1 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$, $\mathbf{u}_2 = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$, $\mathbf{v}_1 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

and let $L : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ be a linear operator whose matrix representation with respect to the ordered basis is $\{\mathbf{u}_1, \mathbf{u}_2\}$ is $A = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$.

1. Determine the transition matrix from $\{\mathbf{v}_1, \mathbf{v}_2\}$ to $\{\mathbf{u}_1, \mathbf{u}_2\}$.

2. Find the matrix representation of L with respect to $\{\mathbf{v}_1, \mathbf{v}_2\}$.

(D) Given $A = \begin{bmatrix} 2 & 1 & 1 \\ 6 & 4 & 5 \\ 4 & 1 & 3 \end{bmatrix}$

1. Find elementary matrices E_1, E_2, E_3 such that $E_3E_2E_1A = U$; where U is an upper triangle matrix.

2. Determine the inverses of E_1, E_2, E_3. What is the lower triangle matrix L such that $A = LU$.

Q.4 (A) Show that $U = W$, where U and W are the following subspaces of R^3.

$U = span\ (u_1,\ u_2,\ u_3) = span\ (1,\ 1,-1),(2,\ 3,-1),(3,\ 1,-5).$

$W = span\ (w_1,\ w_2,\ w_3) = span\ (1,-1,-3),(3,-2,-8),(2,\ 1,-3).$

(B) Let $u_1 = (1,2,4)$, $u_2 = (2,-3,1)$, $u_3 = (2,1,-1)$ in R^3.

Show that u_1, u_2, u_3 are orthogonal, and write v as a linear combination of u_1, u_2, u_3 when v= (7, 16, 6), and v= (3, 5, 2).

(C) Find the dimension and the basis of the solution space W of the following system:

$x - 2y + 2z - s + 3t = 0$

$x - 2y + 3z + s + t = 0$

$3x - 6y + 8z + s + 5t = 0$

(D) Determine the truth table for the following standard POS expression:

$$(A+B+C)(A+\overline{B}+C)(A+\overline{B}+\overline{C})(\overline{A}+B+\overline{C})(\overline{A}+\overline{B}+C)$$

(E) Map the following standard SOP expressions on a Karnaugh map:

i) $\overline{A}\overline{B}CD + \overline{A}BC\overline{D} + AB\overline{C}D + ABCD + AB\overline{C}\overline{D} + \overline{A}\overline{B}\overline{C}D + A\overline{B}C\overline{D}$

ii) $(\overline{A}+\overline{B}+C+D)+(\overline{A}+B+\overline{C}+\overline{D})+(A+B+\overline{C}+D)$
$+(\overline{A}+\overline{B}+\overline{C}+\overline{D})+(A+B+\overline{C}+\overline{D})$