THEORETICAL STUDY OF THE EFFECT OF INJECTION CONDITIONS ON THE LAMINAR BOUNDARY LAYER CHARACTERISTICS DEVELOPING ON A FLAT-PLATE.

PART(2): EFFECT OF SLOT HEIGHT AND INJECTION ANGLE.

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ABSTRACT:

This paper is devoted to the effect of slot height and injection angle on the physical and integral characteristics of the laminar boundary layer developing on a flat plate. The results showed that the effect of the slot height and injection angle has the same trend. The increase of injection angle and slot height leads to reduction of the friction coefficient.

1. NOMENCLATURE:

b : Slot height thickness (m).
$\bar{B}$: Slot height ratio, is the ratio of slot height(b), to the laminar boundary layer thickness(δ)
C_f : Skin friction coefficient,($\tau_w / \frac{1}{2} \rho u_e^2$).
C_p : Pressure coefficient, $\frac{(p-p_\infty)}{\frac{1}{2} \rho u_e^2}$
L : Flat-plate length
H_{12} : Shape factor parameter, $(\frac{\delta_1}{\delta_2})$
H_{23} : Energy parameter, $(\frac{\delta_2}{\delta_3})$
P : Static pressure at any x- direction $(\frac{N}{m^2})$
U : Velocity component in x-direction (m/s)
u_e : Free-stream velocity at the edge of the boundary layer = $0.99\ u_\infty$, (m/s).
u_{jet} : The jet velocity (m/s).
u_{max} : The first maximum velocity, on the profile, measured from the wall, (u_{max}/u_∞).
u : Outer edge velocity of the boundary layer, at the slot position, (m/s).
v : Velocity component in y-direction (m/s).
x : Distance along a flat-plate measured from the first iteration section (m).
y : Distance normal to a flat-plate measured from the plate (m).

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y_{min} : Location of the minimum velocity on the profile,
δ : Boundary layer thickness (m).
δ_1 : Boundary layer displacenent thickness, $\int_0^\delta (1-\frac{u}{u_c}).dy,(m)$
δ_2 : Boundary layer momentum thickness, $\int_0^\delta \frac{u}{u_e}(1-\frac{u}{u_e})dy,(m).$
δ_3 : Boundary layer energy thickness, $\int_0^\delta \frac{u}{u_e}(1-(\frac{u}{u_e})^2)dy$
ρ : Fluid density. (Kg/m^3)
μ : Dynamic viscosity. $(N.sec/m^2)$.
ν : Kinematic viscosity (m^2/sec).
α : Angled injection.
λ : Injection ratio, is the ratio of the jet velocity to the outer edge velocity, (u_{jet}/u_∞).
Re_L : Flow Reynolds number $=(u_e L/\nu)$

2. INTRODUCTION:

The formulated in a previous work [1] problem extension contributes study of the effect of (B) and (α). The full details of the model formulation and the solution procedure can be found in [2].

3. DISCUSSION:

The computer program designed for the calculation ristics on a flat-plate with injection was fed with the dimensionless slot height ($\bar{B}$) and the injection angle (α). The problem is solved here for:

$\bar{B}$: 0.5 , 0.6 , 0.7 and 0.8, and
α : 0.0 , 4.0 , 8.0 and 12°

In addition the physical properties of an incompressible air, the flow used in this study, are $\nu = 15.10^{-6}$ m^2/s. The (Re) was taken to be constant at $Re_L = 10^5$, which corresponds to free stream velocity = 34.4(m/s) on a plate with L=220mm. The vecocity is quit small to suit the condition of incompressible fluid flow. The problem was solved for isothermal and steady flow conditions. The slot height was chosen to be the same order of magnitude of δ_1.

3.1-1- Effect of $\bar{B}$-slot height ratio-on velocity frofile :

Figure (3-1) shows the effect of variation $\bar{B}$ on the velocity profile. The values of λ and α are kept constant at values of (0.4 and 0.5) respectively, while $\bar{B}$ was assumed to be 0.5 , 0.7 and 0.8. The same was repeated at different values of λ (0.8 , 1.2 and 1.4) as shown in figure (3-2) from these figures it can be noticed that:

1. An increase of slot height at const (λ) and ($\alpha°$) leads to increase of u_{max} and u_{min}. The distance y_{min} and y_{max} also increase . This can be explained by the following. Increase of ($\bar{B}$) at constant (λ) means increase of mass flow rate in the wall vicinity. This leads to increase of velocity in the vicinity of the wall . The shift of y_{max} and y_{min} off the wall is explained by the same reason.
2. The boundary layer thickness increases by increasing $\bar{B}$,

3.1-2: Effects of $\bar{B}$ on decay-maximum velocity profile and its location :

Figure (3-3) shows the effect of $\bar{B}$-variation on the first maximum velocity and the variation of its location. The values of $\lambda = 0.4$ and 1.2 are chosen for the comparison purpose.

From this figure it can be observed that:

1. Generally, the variation of $\bar{B}$, leads to the first maximum velocity change. The rate of decay is smaller as $\bar{B}$ increases. The rate of decay of u_{max} is nearly linear for $\lambda \leqslant 0.4$ as shown in figure (3-3). The rate of u_{max} decrease in case of low $\bar{B}$ is bigger than in case of hight $\bar{B}$. This can be explained as follows: At constant injection ratio (λ), the friction and mixing losses which are proportional to the velocity are constant . The kinetic energy of the flow in case of small $\bar{B}$ is relativaly small with respect to its value at higher $\bar{B}$. Reducing the friction losses of the kinetic energy in both cases gives a smaller decrease rate for case of high $\bar{B}$.

2. The variation of y_{max} is linear in the beginning of the mixing zone but downstream this linearity is not hold true. The location of(u_{max}) is increasing downstream,. At small injection ratios ($\lambda \leqslant 0.4$) this relation is linear all over the section, but at higher λ (1.2),this relation becomes non-linear at higher-x.

The increase of $\bar{B}$ leads to increase of (y_{max}) since the increase of y_{max} downstream direction can be explained by the stretching of the semi-bounded jet in the down stream direction. The higher the $\bar{B}$, the higher the y_{max} at any section-x.

3.1-3) Effect of-$\bar{B}$ on the minimum velocity on the profile and its location:

The effect of $\bar{B}$ variation on the minimum velocity and its location, is shown in figure (3-4) . This figure indicates that:

1. Generally, the minimum velocity on the profile is growing along the mixing region, but the rate of growth is relatively slow as-$\bar{B}$ increases. The increase of the minimum velocity is slightly slower at the smallest values of λ (0.4)while the effects are more appreciable for the highest values of λ (1.2).

2. An increase of $\bar{B}$, the location of the u_{min} , increases;the behaviour of u_{min} can be explained as follows:-

   The expansion of the jet in the x-direction induces an additional velocity component. This velocity is added to the local velocity of the boundary layer. The resultant is an increasing velocity in the x-direction. Increase of $\bar{B}$ means a decay of the jet at higher x, which leads to a smaller rate of u_{min} increase. The increase of y_{min} in x-direction is explained on the same principle.

3-2- Effect of slot height on the boundary layer characteristics:

3.2.1- Effect of slot height ($\bar{B}$) on boundary layer thickness (δ):

Figure (3-6) shows the effect of $\bar{B}$-variation on the variation of (δ) at different values of λ (0.4, 0.8, 1.2 and 1.4), in case of tangential α=0.0. From this figure it can be seen that the increase of $\bar{B}$ leads to increase of δ . Since α =0.0, or the injection is tangent to the plate the increase in δ is mainly due to the increase of the flow area by the slot height.

3.2.1-2- Effect of the slot height on displacement thickness (δ_1):

Figure (3-7) shows the effect of - $\bar{B}$ on the variation of (δ_1) for different values of λ (0.4 , 0.8, 1.2 and 1.4) respectively in case of α = 0.0 (tangential injection). From this figure it can be observed that generally, an increase of $\bar{B}$ leads to increase of (δ_1), since the mass flow increases by increasing of $\bar{B}$ at the same injection ratio. So, by increasing $\bar{B}$, the mass flow rate increases , and hence (δ_1). In addition to that inspite of (δ) increase with $\bar{B}$, the ratio of (δ_1/δ) increases with the increases of $\bar{B}$.

3.2.1-3- Effect of $\bar{B}$ on the momentum thickness (δ_2):

Figure (3-8) shows the variation of δ_2 as a function of x for different $\bar{B}$ at λ= 0.4, 0.8, 1.2, and 1.4 respectively, in the case of tangential injection (α= 0.0). From this figure it can be noticed that , the same behaviour of (δ_1) variation is repeated for (δ_2). This increase is due to the increase of the kinetic energy in the wall region as a result for the injection.

3.2.1-4- Effect of $\bar{B}$ on energy thickness (δ_3):

Figure(3-9) shows , the energy thickness (δ_3) at different slot height ratios at constant λ (0.4 and 0.0). The same was repeated for different values of λ (0.8, 1.2 and 1.4). From this figure it can be seen that, the main trend of δ_3 variation, along the flat plate, it the same trend of δ_1 variation. The rate of increase of (δ_3) as a function x is slightly smaller than the case of (δ_1) and slightly higher than the case of (δ_2).

mixing region. It can be noticed also that at the smallest values of α, ($\alpha \leq 4^\circ$), the injection angle has no effect on (δ_1). At the higher values of α ($\alpha > 4^\circ$) , the effect begins to appear. The increase of (δ_1) at higher limits of α , is explained as the following: Since δ_1 is calculated by means of local velocity (u) in the tangential direction, and u is connected with the injection angle then (δ_1) varies with α-variation. The increase of , leads to decrease of u in the vicinity of the wall, then the area under the velocity curve increases, so the resultant effect of increasing α , is the decrease of u in the vicinity of the wall and then (δ_1) increases.

3.3-4: Effect of injection angle (α) on the momentum thickness (δ_2):

Figure (3-16) shows the effect of α -variation on (δ_2) at the same mentioned conditions in the previous section. From this figure it can be noticed that the same trend as (δ_1) is repeated. As α increases, (δ_2) increases within the mixing region. The change in (δ_2) is more pronouced in the case of small values of α . The increase of $\bar{B}$, leads to magnification of the effect of α. It is intersting to notice that the increase of the injection angle (α) for constant velocity ratio(λ) means decrease of tangential component of the injection flow. This means that, the calculation of the tangential velocity injection ratio is less than that calculated by the injected flow rate. One can find that the increase of α , in same way, is equivalent to decrease of λ. This is valid for case when $\lambda > 1$, when $\lambda < 1$, this notice is not hold true, especially for (δ_2). The reason for that may be explained by the velocity distribution in the wall region.

3.3-5: Effect of injection angle (α) on the energy thickness (δ_3):

As it can be seen from figure (3-17) where the (δ_3) values along the plat were computed for different values of α (0.0, 4.0 and 12°) for λ=0.4 and 1.2, $\bar{B}$= 0.5 and 0.8, that , the behaviour of (δ_3) is the same as (δ_2). The reasons for this behaviour are the same as for δ_2.

3.3-6: Effect of injection angle (α) on the coefficient of friction (C_f) and coefficient of pressure (C_p):

Figure (3-18) shows the variation of (C_f) along the plate. It can be noticed from this figure that, increase of α, leads to decrease of C_f. This may be explained in the analogy with λ , by the local velocity . The local velocity decreases with increasing (α) so with the increase of α, C_f decreases. One can also noticed that the effect of $\bar{B}$, on both (C_f) and (C_p) is analogous to the effect of $\bar{B}$ on C_p. This happens because the increase of either of α or $\bar{B}$ leads to decrease of the local velocity.

3.2.1-5- Effect of slot height on the shape factor H_{12}:

Figure (3-10) shows the variation of H_{12} along the plate . The effect of $\bar{B}$ on H_{12} is small,the change of H_{12} is within 5% , for the change of $\bar{B}$ from 0.5 to 0.8. The greater values of $\bar{B}$ was not considered, since the change of the surface will effect the accuracy of the mathematical model. So, H_{12} within 5% error can be considered for the slot height less than (0.8).

3.2.1-6- Effect of slot height on the energy parameter (H_{23}):

From figure (3-11), it can be seen that, H_{23} does not vary by the variation of $\bar{B}$ in the investigation limit. The change of $\bar{B}$ from (0.5) to (0.8) leads to a relative change of H_{23} estimated to be 0.31%. These figures show also that practivally H_{23} may be considered independent of $\bar{B}$ in case of ($\bar{B} \leq 0.8$).

3.2.1-7- Effect of $\bar{B}$ on friction coefficient (C_f):

The results of the computed (C_f) is shown on figure (3-12) for case of $\bar{B} = 0.5, 0.6 , 0.7$ and 0.8 at $\alpha = 0.0$ and (λ) was taken to be 0.4 , 0.8, 1.2 and 1.4. This results, show that along the plate the main characterstics of C_f are still valid. The increase of $\bar{B}$ leads to decrease of (C_f) . This is the result of decrease of the local velocity in case of increase of $\bar{B}$. It can be noticed also that the decrease rate of (C_f) in the case of small $\bar{B}$ is higher than the rate of high $\bar{B}$.

3.2.1-8- Effect of $\bar{B}$ on coefficient of pressure C_p:

The effect of $\bar{B}$ on the pressure coefficient C_p is analogus to be effect of λ . Increase of $\bar{B}$ leads to decrease of p_{min} as shown in figure (3-13).

3.3- Effect of injection angle (α) on the boundary layer charactristics:

3.3-1- Effect of injection angle on boundary layer thickness :

Figure (3-14) shows the effect of α variation on (δ). From this figure it can be seen that, (δ) is not affected by α-variation. This means that in all the examined cases; the injection flow is mixed with the main flow, the kinetic energy is rearranged. This accured in the vicinity of the wall . The outer edge of the boundary layer was not affected by the injection.

3.3-2- Effect of injection angle (α) on the boundary layer displacement thickness (δ_1):

The effect of α-variation on (δ_1) is shown in figure (3-15). The figure shows that, the increase of (α) leads to increase of (δ_1), till the end of the

4. CONCLUSION :

The mathematical model for calculation of the fluid flow with injection in case of laminar flow developing on a flat-plate enabled to study the effect of injection parameters , (α, $\bar{B}$ for constant Re_L) on the characteristics of the boundary layer . The following conclusion can be drawn from the previous discussion.

1. The mixing zone length increases by increasing the injection ratio (λ) as well as by increasing ($\bar{B}$) . The injection angle (α) partically has no significant effect on the length of the mixing zone.
2. The first maximum velocity (u_{max}) , decays along the mixing region, in the same manner, distance from the wall where the maximum velocity exists is continueosly departing the wall. The minimum velocity (u_{min}) increases also along the mixing region and its location (y_{min}) is departing the wall.
3. The effect of slot height ($\bar{B}$), on the boundary layer characterstics is opposite to the effect of λ for same Re_L, λ and injection angle (α).
4. The effect of injection angle (for $0 \leq \alpha \leq 12°$) has the same trend as the effect of slot height ($\bar{B}$) for the same Re_L and λ.
5. The friction coefficient (C_f), increases with increasing the slot height and injection angle (α) . This means that the pressure along the surface decreases in the main flow direction, which is suitable for elemination the flow separation.
6. The increase of injection angle (α), and the slot height ($\bar{\bar{B}}$), leads to decrease of C_f. This means that, the drag on the plate decreases which leads to increase of lifting force, when this system is used on an airofoil.

5. REFERENCES:

1. Weheba, S. M., Khalifa, B. A.,Hewedy,N.I.I.,Elmayet,M.M.
Theoretical study of the effedt of injection conditions on the laminar boundary layer characteristics developing on a flat-plate.
Part(1) Effect of Injection Ratio (λ) .
Eng. Res. Bull., Faculty of Eng. & Tech., Menoufia Univ.
Vol. V , Part 1, 1983, PP(255).
2. El-Mayet, M. M.
Boundary layer injection.
M.Sc. Thesis, Faculty of Eng. & Tech., Menoufia Univ.
(1983)..

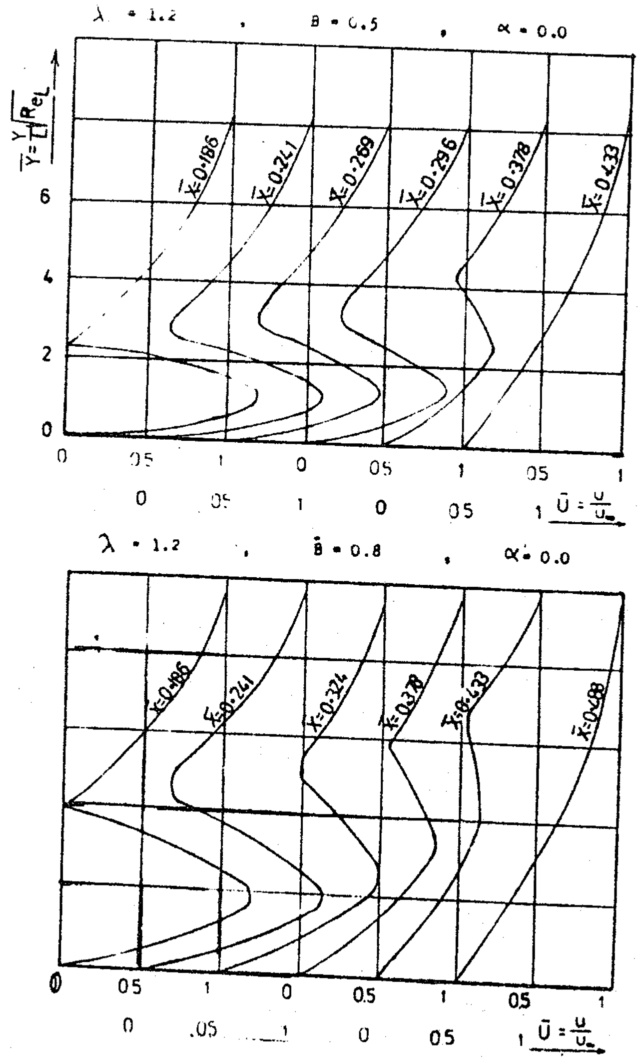


Fig.(3-2)effect of $\bar{B}$ on velocity profile .

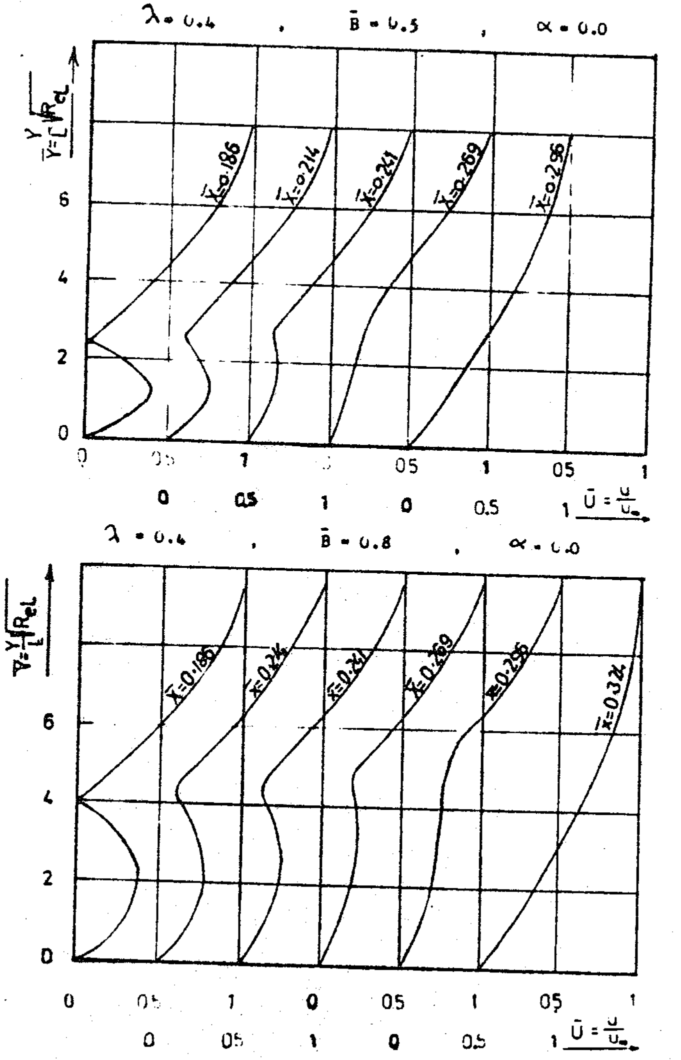
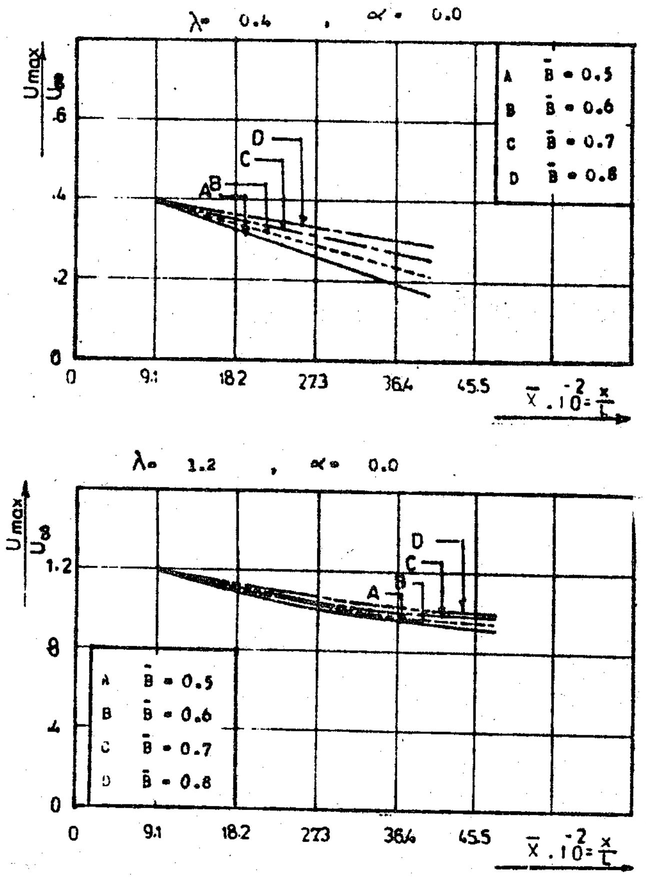
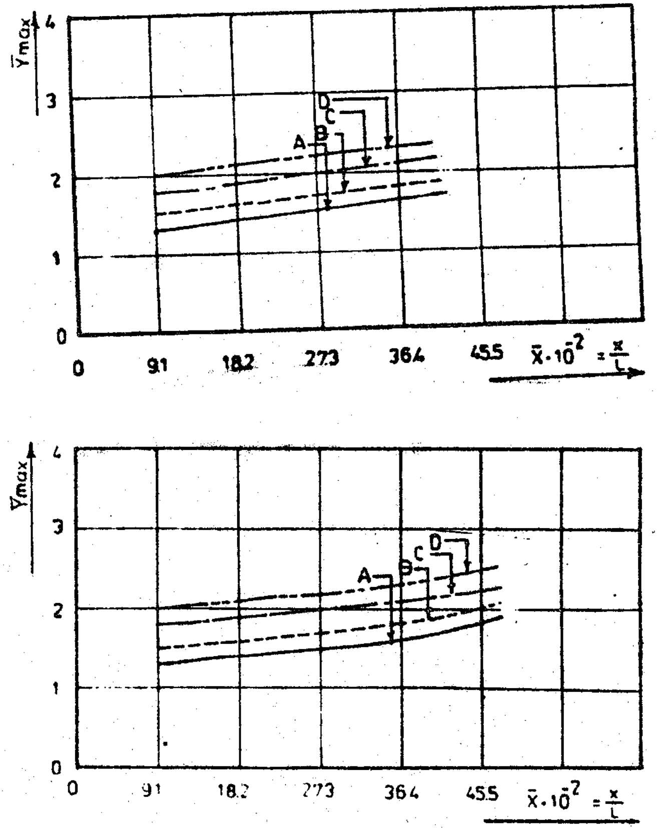


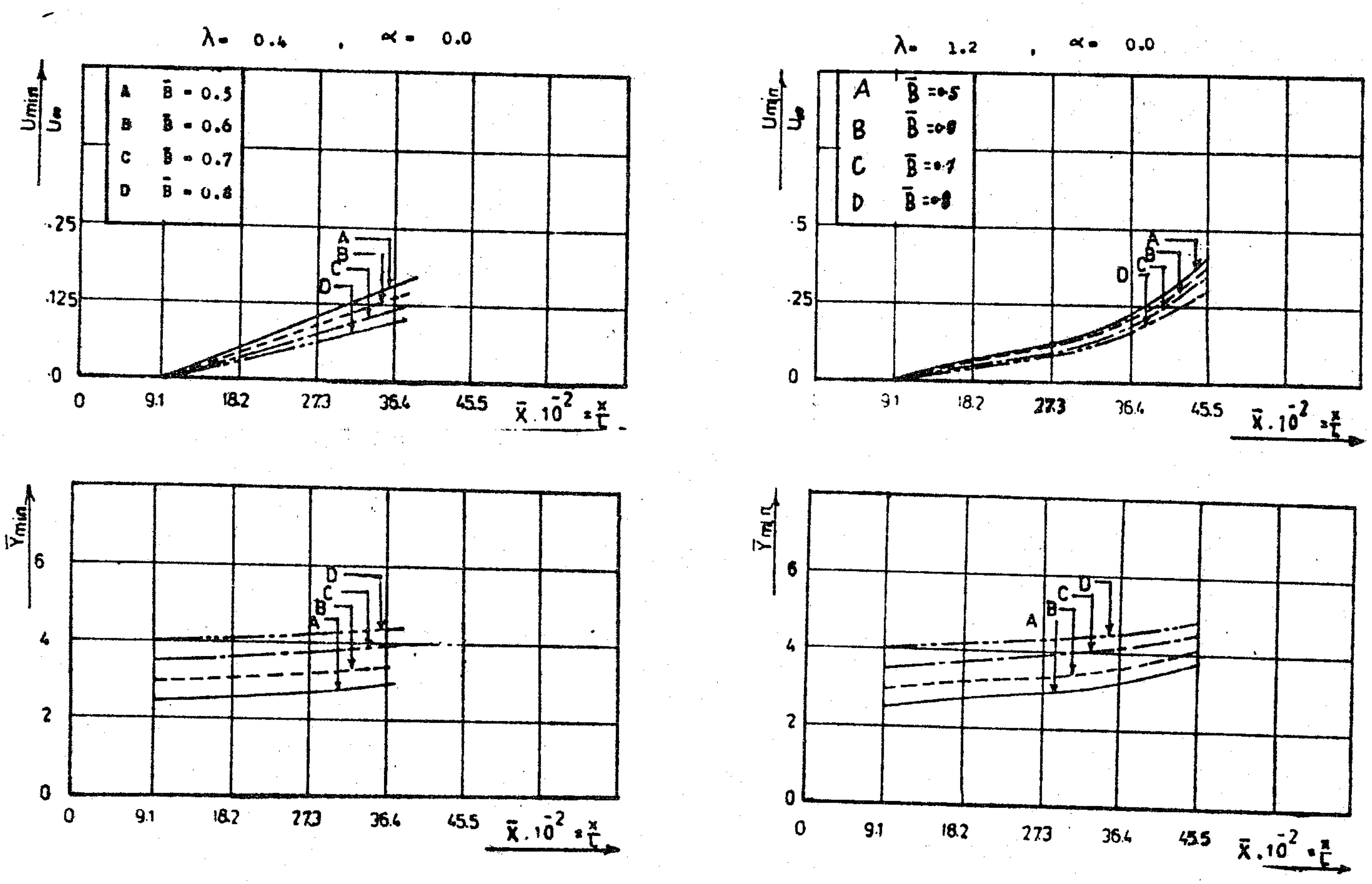
Fig.(3-1)effect of $\bar{B}$ on velocity profile.



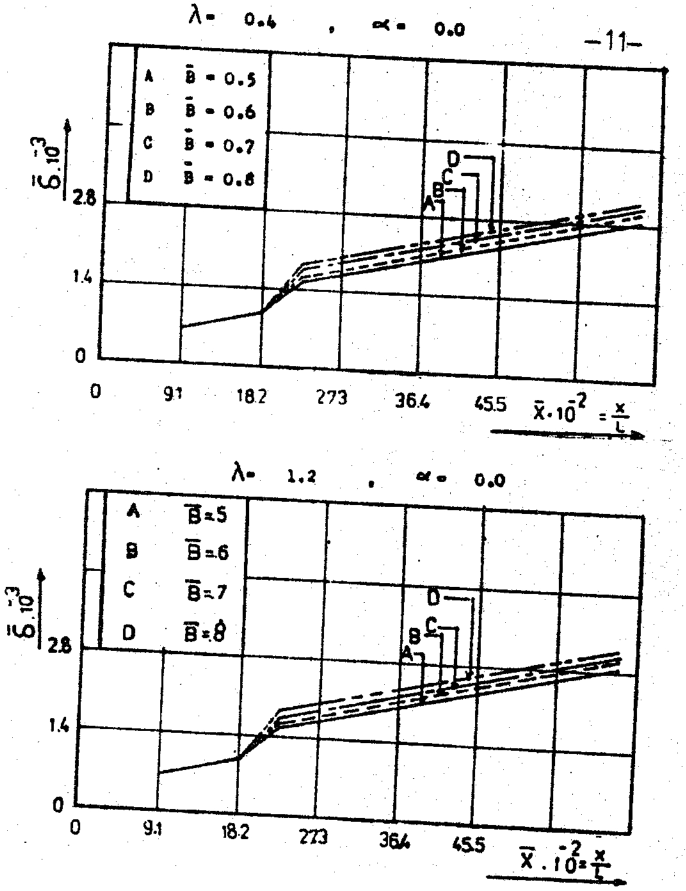
FIG(3- 3): EFFECT OF ($\bar{B}$) ON DECAY MAXIMUM VELOCITY PROFILE.



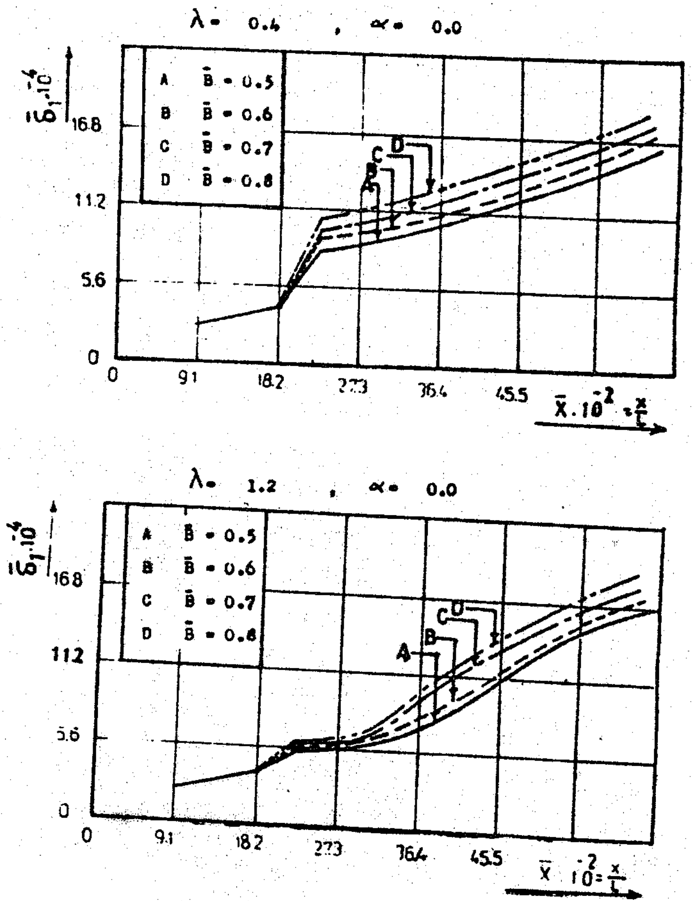
FIG(3- 3): EFFECT OF ($\bar{B}$) ON-LOCATION OF MAXIMUM VELOCITY PROFILE.



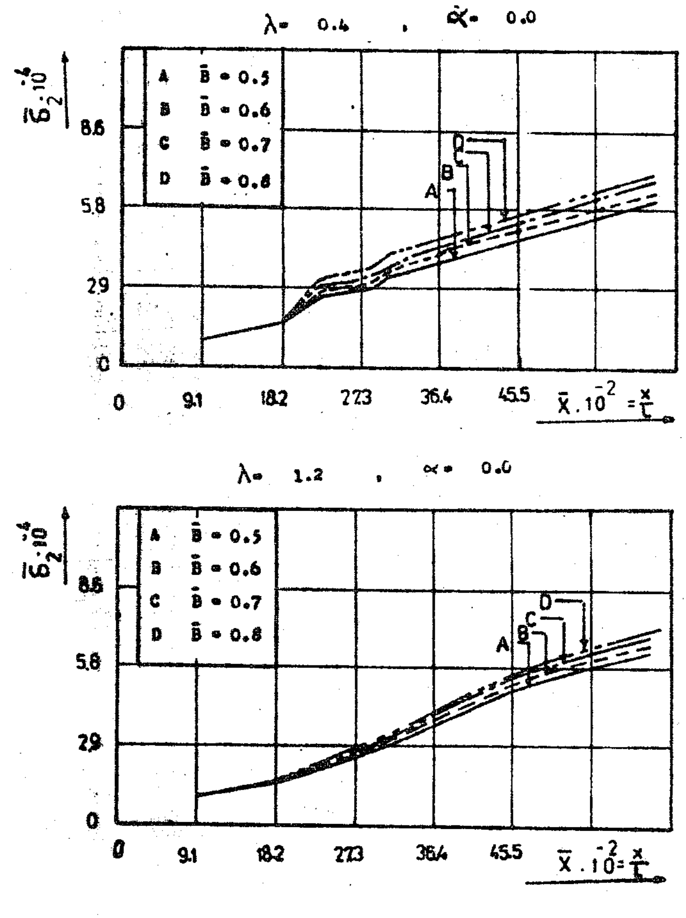
FIG(3-4): EFFECT OF (B̄) ON LOCATION OF MINIMUM VELOCITY PROFILE.



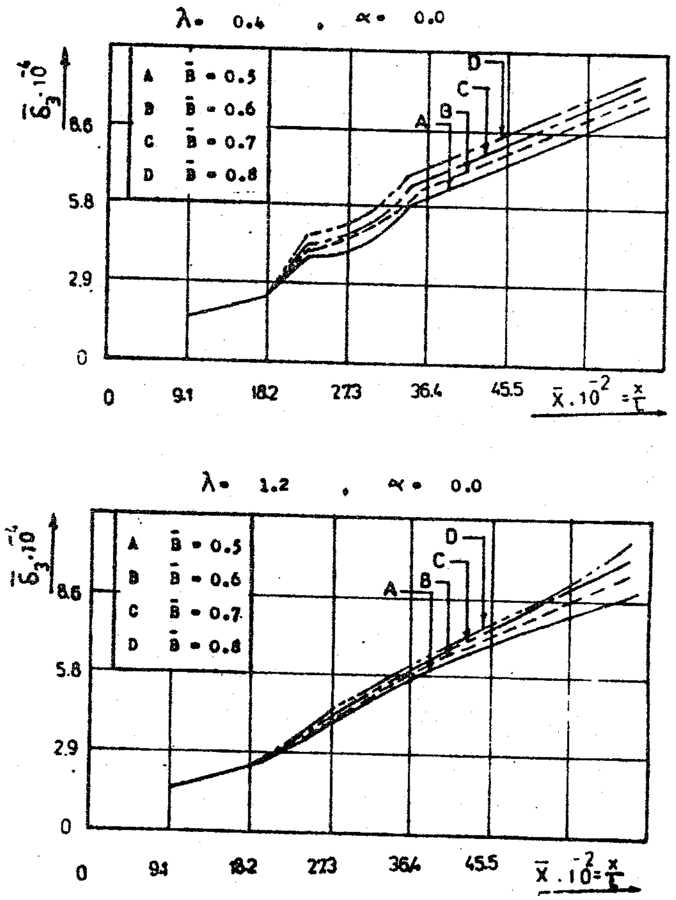
FIG(3- 6):EFFECT OF SLOT HEIGHT RATIO ($\bar{B}$) ON GROWTH-BOUNDARY LAYER THICKNESS(δ).



FIG(3- 7): EFFECT OF SLOT HEIGHT RATIO ($\bar{B}$)ON GROWTH-BOUNDARY LAYER DISPLACEMENT THICKNESS(δ_1).



FIG(3-8): EFFECT OF SLOT HEIGHT RATIO ($\bar{B}$)ON GROWTH-BOUNDARY LAYER MOMENTUM THICKNESS (δ_2).



FIG(3-9): EFFECT OF SLOT HEIGHT RATIO ($\bar{B}$)ON GROWTH-BOUNDARY LAYER ENERGY THICKNESS.

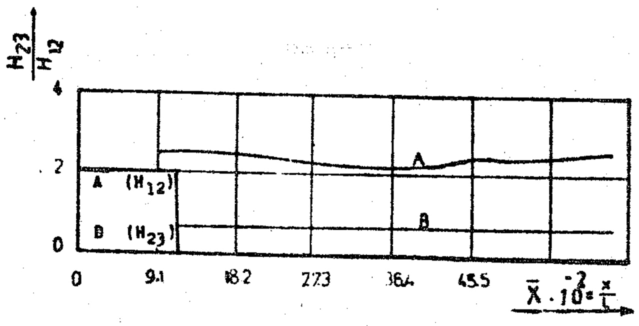
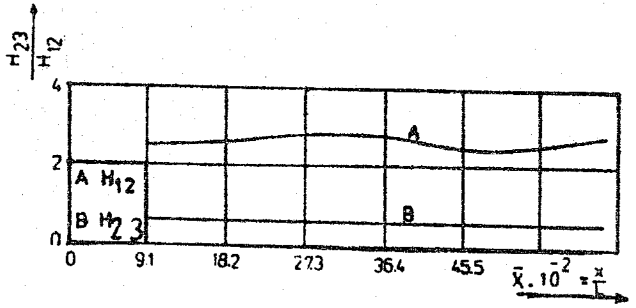
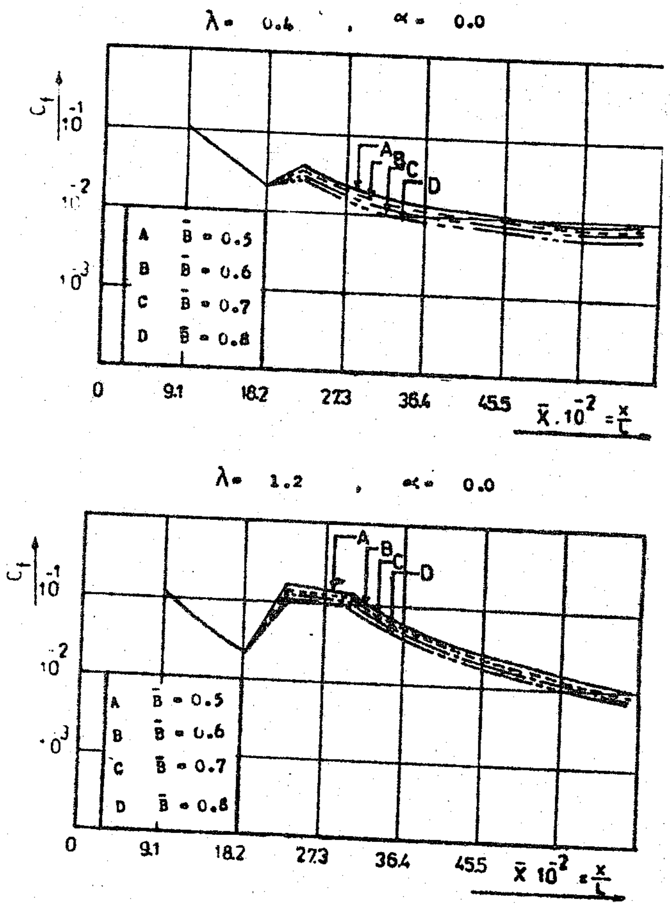


FIG.(3-10)EFFECT OF $\bar{B}$ ON H_{12} AND H_{23}

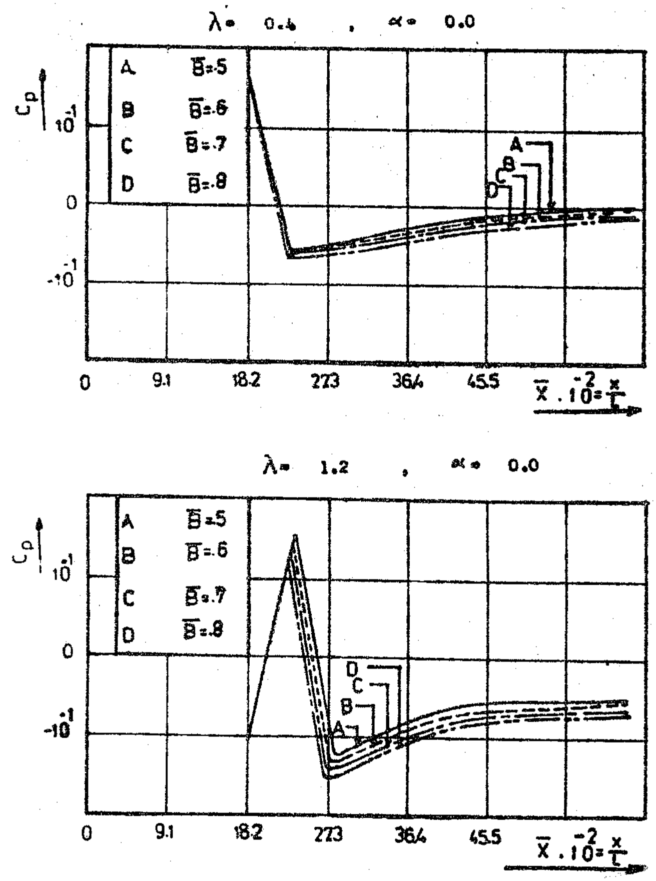


FIG(3-11)EFFECT OF $\bar{B}$ ON H_{12} AND H_{23}

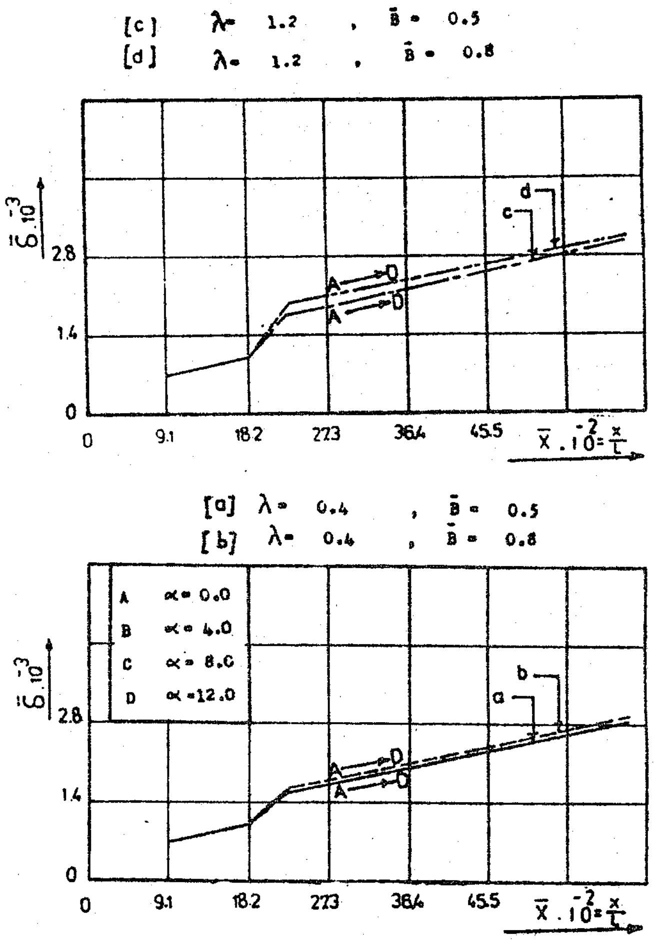

$\lambda = 0.6$, $\alpha = 0.0$

$\lambda = 1.2$, $\alpha = 0.0$

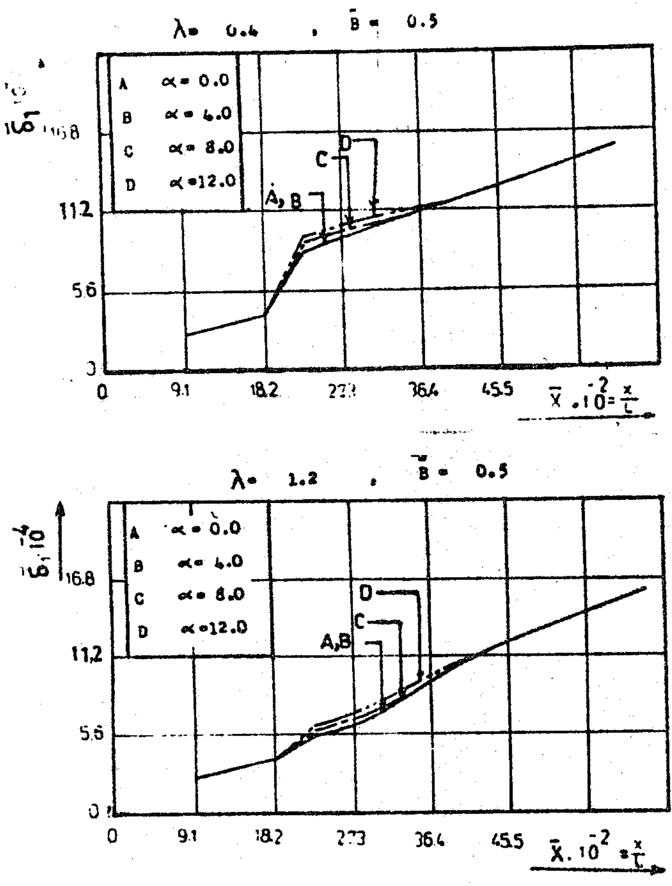
FIG(3-12): EFFECT OF SLOT HEIGHT RATIO ($\bar{B}$) ON (c_f).



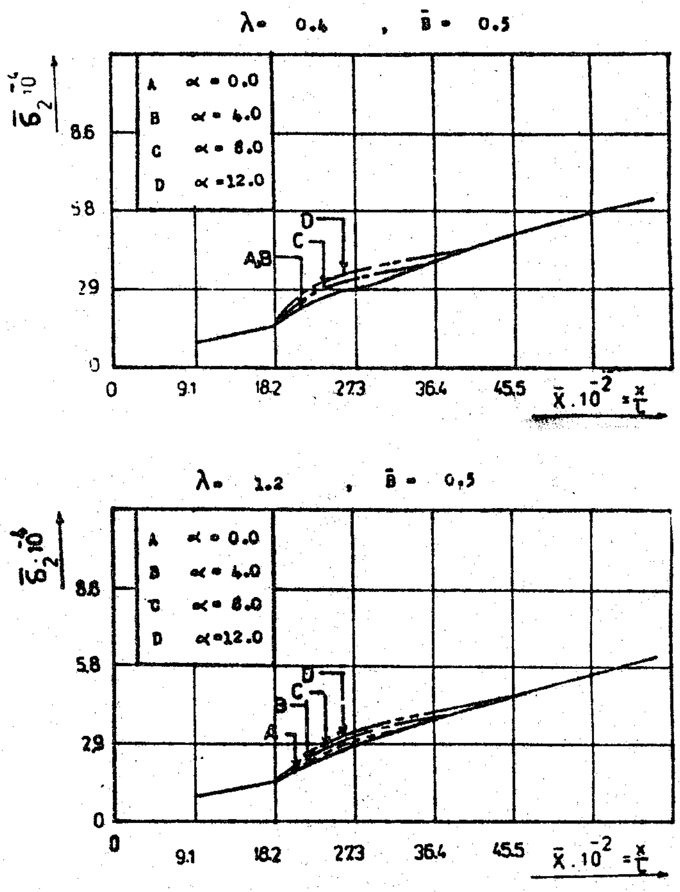
FIG(3-13): EFFECT OF SLOT HEIGHT RATIO ($\bar{B}$) ON (cp).



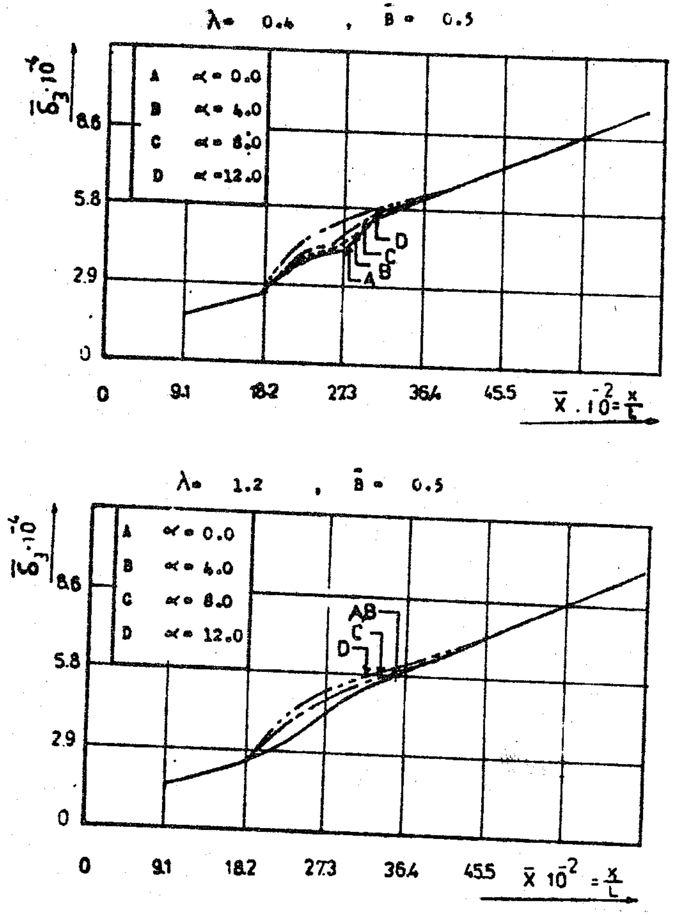
FIG(3-14) EFFECT OF INJECTION ANGLE (α) ON GR TH- BOUNDARY LAYER THICKNESS (δ).



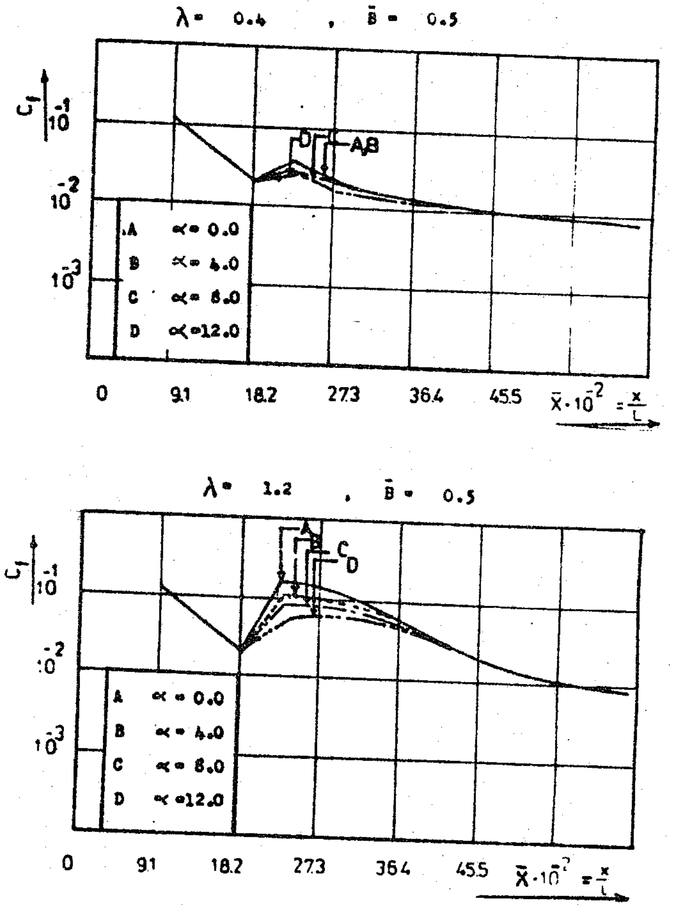
FIG(3-15): EFFECT OF INJECTION ANGLE (α) ON GROWTH-BOUNDARY LAYER DISPLACEMENT THICKNESS(δ_1).



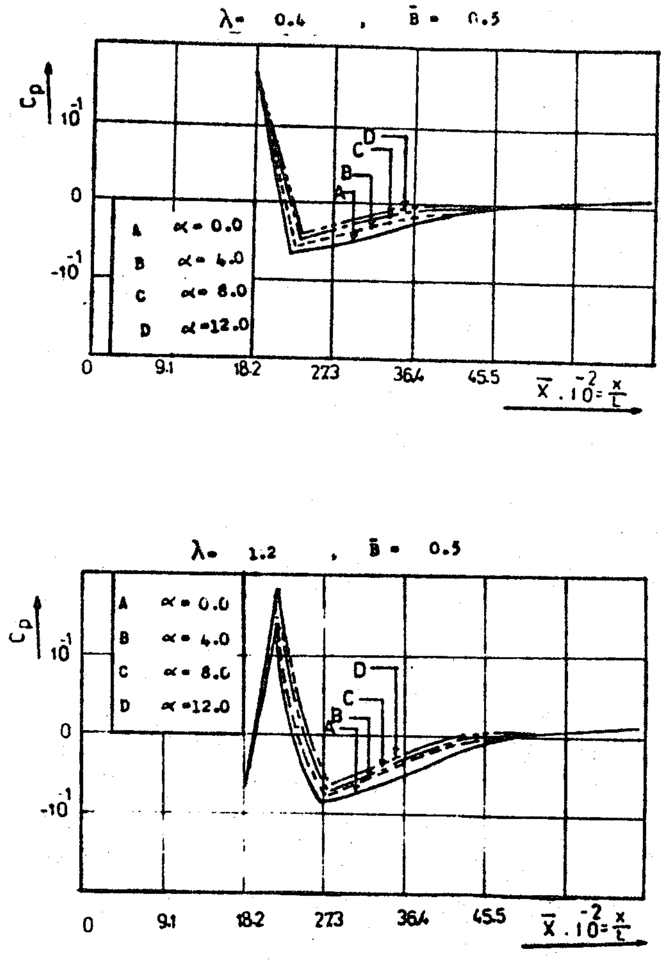
FIG(3-16): EFFECT OF INJECTION ANGLE (α) ON GROWTH-BOUNDARY LAYER MOMENTUM THICKNESS (δ_2).



FIG(3-17): EFFECT OF INJECTION ANGLE (α) ON GROWTH-BOUNDARY LAYER ENERGY THICKNESS (δ_3).



FIG(3-18): EFFECT OF INJECTION ANGLE (α) ON (cf).



FIG(3-19): EFFECT OF INJECTION ANGLE (α) ON (cp).