

## HIGH FREQUENCY TRANSFORMER MODELING BASED ON TIME DOMAIN SOLUTION

### نمذجة المحولات كعلاقة مع الزمن تحت تأثير الترددات العاليه

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#### ملخص البحث

نظرا لأهمية محولات القدرة في نظم القوى الكهربائية حيث أنها أحد الأجزاء الرئيسية في أي منظومة كهربية ونظرا لأرتفاع ثمنها فإن هذا البحث يهدف إلى الدراسة النظرية للمحولات تحت تأثير الظواهر العابرة الناتجة من الصواعق الكهربائية وإجراءات الفصل والتوصيل. لذلك فإن هذا البحث يقدم نموذج جديدا يمكن من دراسة الاستجابة العابرة لمحول القوى تحت تأثير الظواهر العابرة. تعتمد هذه الطريقة على اختبار دالة الترددات للمحول باستخدام مولد ترددات ذو جهد منخفض ولذلك فإن هذه الطريقة لا تحتاج إلى بيانات تصميم المحول ولا تتأثر بالاختلافات في الخواص الكهربائية الناشئة نتيجة التجاوز في التصميمات المبدئية. وأيضاً تأخذ في الاعتبار تقادم العزل الداخلى للمحول نتيجة للتشغيل مما يغير من خواص العزل. للحصول على الاستجابة العابرة لمحول القوى من اختبار دالة التردد تم تطبيق طريقة لابلاس وبمقارنته النتائج بطريقة أخرى تعتمد على ثوابت الحالة أثبتت الطريقة المقدمه دقتها وصلاحتها التامة.

#### Abstract

A new model for the high frequency transformer simulation is presented in this paper. The power transformer, as vital and expensive pieces of the power system, should be studied and investigated for all possible operational stresses. To this extend, a wide range of frequency covering all possible transient conditions is to be investigated. In this work a new model, which enables the calculation of transformer transient voltage response is presented. The model is based on measurements in high frequency domain using low voltage high frequency generator. The introduced new model is simple to be programmed using available programs like, "MATLAB and MAPLE". Comparison with other models shows high accuracy and excellent validity.

#### 1-Introduction.

Surge waves may be produced in power systems either by normal switching processes or by lightning. Surge waves have great fatal effects on the different components of any power system; especially on its transformers. A power transformer, being a vital and expensive piece of equipment in power system, needs critical attention from the stand point of its insulation design and performance under both steady-state and transient stresses [1].

The modeling of power transformer under transient conditions can be divided into, modeling based on internal parameters and modeling based on measurements.

*i- Modeling based on internal parameters:* Such modeling are based on the accurate knowledge of the transformer geometrical dimensions and internal parameters. With the help of these data, different ways can be used to get informations about equipment

inductances and capacitances. Self and mutual inductances have been calculated between sections of windings by Fergestad and Henriksen [2-3]. The Calculation of parameters of self and mutual matrix of this model involves the solution of complex field problems and requires information on the physical layout and construction details of the transformers.

By means of integral transform techniques a nearly accurate formulae for self and mutual inductances for winding sections, or turns has been calculated by Wilcox [4-8]. The model takes into consideration the iron core effects in the calculation of self and mutual inductances of the transformer.

Recently, De Leon and Semlyen [9-12] developed a model to represent the leakage inductance of the transformer including the iron core effect. In this model, the leakage inductance of the winding turns are calculated using image method, and the leakage inductance matrix is deduced.

***ii-Modeling based on measurements:*** Models belonging to this class are based on the simulation of the frequency characteristics at the terminals of the transformer by means of complex equivalent circuits or other closed representation forms. Woivre[13] calculated the transformer transient overvoltages from frequency response by using Fourier Transform.

Morched [14] uses model based on the frequency characteristics over the transformer admittance between its terminals over a given range of frequencies. The main advantage of this model is that it can simulate any type of multi-phase, multi-winding transformer. However, this model is more complicated and requires more effort for numerical calculations. R.T.M Vaessen provides a method based on the simple fact, that every transformer has a number of resonance frequencies, which can be shown up in theory in both transfer and admittance functions [15].

In developing countries, like Egypt for example, it is difficult to get such dimensions and construction informations, since most equipments are made abroad. Factories deliver these equipments usually by operation constructions without any design data. Moreover, the determination of the winding electrical parameters from design drawings can deviate mostly from the actual parameters, since actual physical dimensions often deviate from initial design assumptions. Even small differences can have a pronounced effect on the winding resonances, particularly in the higher frequency range. [16].

Due to these reasons, it is useful to find a simple transformer model depending on the actual measurements of the distribution transformer which ensure actual results. This model should not need the physical dimensions and electrical parameters of the transformer.

## 2-DESCRIPTION OF THE TRANSFORMER UNDER TEST

An actual distribution transformer drawn from service is used here as actual transformer model of the following data Table 1

Voltage ratio	3.3/38 kV
Connection	$\Delta/Y$
Rating	200 kVA

Table 1 Data of distribution Transformer

Before using this transformer in an experimental test, the standard test is carried out according to [IEC-1982] [17]. In this test, the non-tested phases of the side under test are shorted and earthed. The non-tested side is kept open-circuited as shown in Fig. 1.

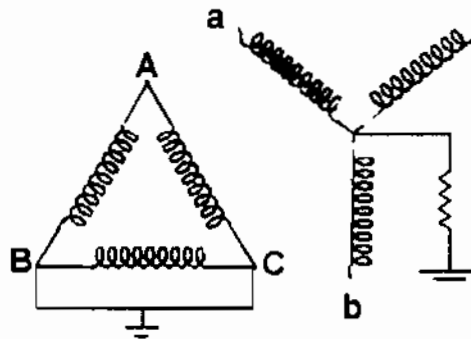


Fig.1 Standard test connection (IEC - 1982) [17]

### 3-THE NEW TRANSFORMER MODELING METHOD

#### 3.1 Two port network transformer

Any transformer can be considered as a block box simulated by passive two-port network [15]. As shown in Fig. 2. The terminal voltages and currents of the two-port can be related by two classes of network functions, namely, the driving point (dp) function and the transfer functions (tf) [18]

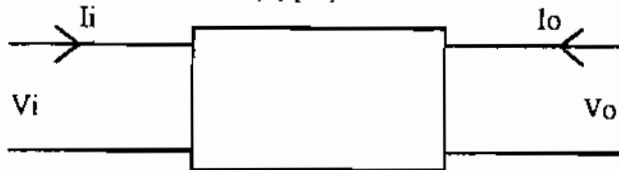


Fig. 2 Two-port network

The driving point (dp) function relates the voltage at a port to the current at the same port. For the input port with open-circuit output port the (dp) impedance function  $Z_i(s)$  is defined as [18] :-

$$Z_i(s) = \frac{V_i(s)}{I_i(s)} \tag{1}$$

where

$Z_i$  - the input impedance function

$V_i$  - the voltage at the input port

$I_i$  - the current at the input port

This function can be evaluated by measuring the values of the input voltage and the corresponding currents at the same port at different frequencies ranged from 10 Hz to 1.2 MHz using frequency generator.

Similarly, the voltage transfer function (vtf), which is defined as the ratio between the voltage at the output port and the corresponding voltage at the input port. [18]. This can be expressed as follows

$$\text{Voltage transfer function (gain)} = \frac{V_o(s)}{V_i(s)} \tag{2}$$

where

$V_o$  - the output voltage

### 3.2 Proposed procedure of system identification technique

#### 3.2.1 Evaluation of the network function $H(s)$

The plotted magnitude and phase characteristics of network functions of power transformers (impedance and voltage gain) show clearly that all of the poles and zeros of the impedance function are simple as shown in Fig. 3. As the magnitude of the network function is practically bounded for all  $\omega$ , it can be assumed that all the poles are distinct and complex conjugate [19]. The mathematical expression for these network functions can be modified and presented in the following form :

$$H(s) = \frac{k(s - Z_1)(s - Z_2) \dots (s - Z_m)}{\prod_{p=1}^n (s^2 + Bs + \omega_p^2)} \tag{3}$$

Where

$H(s)$  Any network function

$\omega_{ip}$  is the pole angular frequency =  $2\pi f_{ip}$

$f_{ip}$  the frequency corresponding to the magnitude peaks

$B$  is the bandwidth which depend on the quality factor

$$B = \omega_{ip} / Q \tag{4}$$

The magnitude of  $Q$  depends on the flatness or sharpness of the curve at frequencies of poles.

The factor  $k$  of these expression can be estimated from the (DC) region of the network functions. Since at this region the input impedance and the voltage transfer function are nearly constant and each equal  $20 \log |k|$  [20].

The factors  $(s - Z_j)$  represent simple zeros. At low frequency (i.e.,  $\omega \ll Z_j$ ), these factors can be approximated by  $Z_j$  while at high frequency (i.e.,  $\omega \gg Z_j$ ), these factors can be approximated by  $s$ , and the slope of the high frequency asymptote is 20 dB/decade [20]

The denominator is defined taking into account its conditions namely all the poles must be distinct and complex conjugate. Therefore the denominator can be approximated to a second order algebraic expression in the form  $(s^2 + Bs + \omega_{ip}^2)$ .

### 3.2.2 Executive example to system identification

A good indication for magnitude and phase angle has been achieved by applying the proposed technique of system identification as can clearly seen.

Figure 3 shows the input impedance of the three phase distribution transformer in the case of standard test connection transformer, which its data is given in Table 1.

Figures 4,5,6 shows the voltage gain in phase (a) in low-voltage side (LVS). It can clearly seen that, the proposed system identification technique is gives good system identification.

### 3.3-TIME DOMAIN SOLUTION BASED ON LAPLACE TRANSFORM METHOD

It can clearly be seen that, the proposed technique is more easy and gives good results. An accurate formula is obtained which represents the two network functions for three phase distribution transformer under test. This formula represents the first step in our proposed technique. The second step depends on two simple equations, which can be written as follows:-

$$V_o = I_i \cdot Z \tag{5}$$

$$V_o = I_o \cdot I_o \cdot *I_i \tag{6}$$

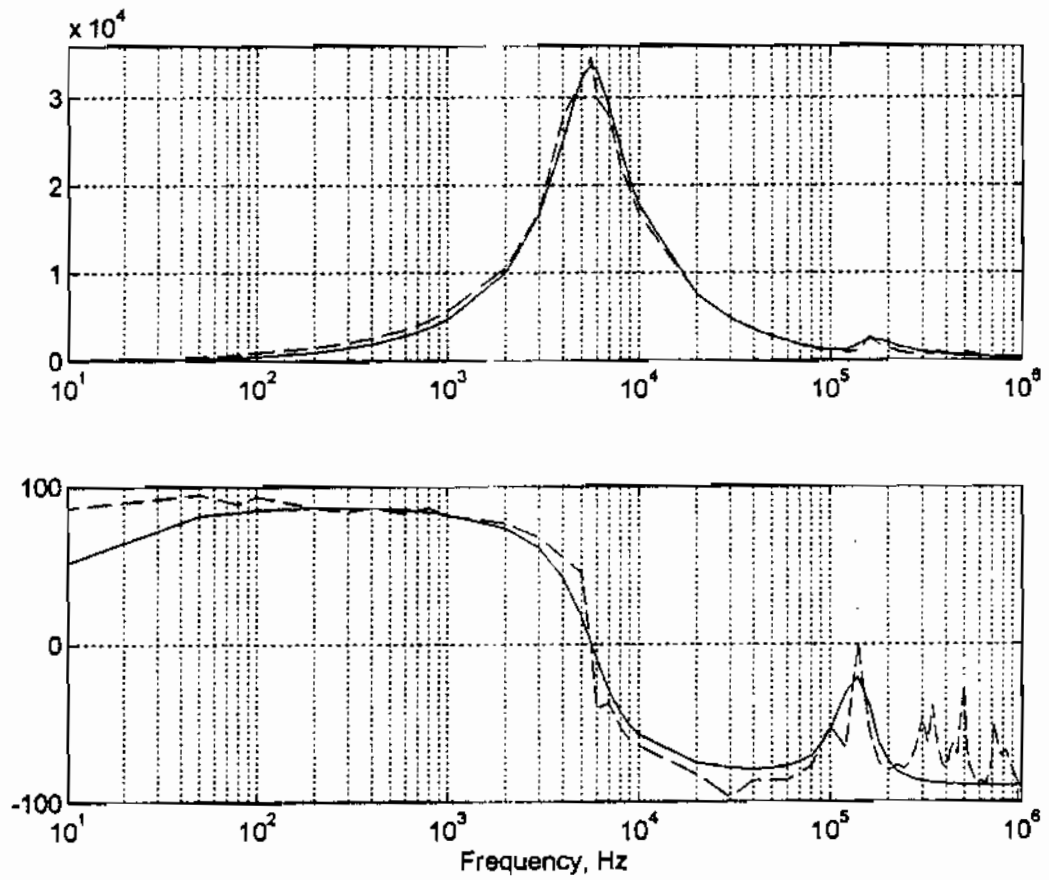


Fig. 3 Transformer input impedance (a) Magnitude ; (b) Phase angle ----- measurement, \_\_\_\_\_ curve fitting

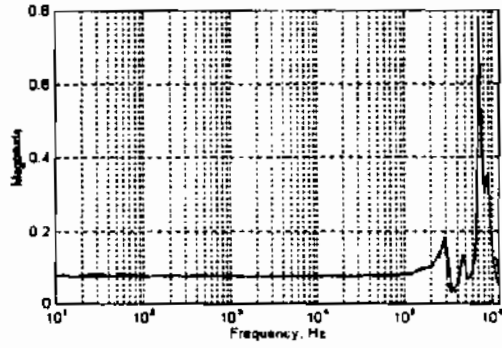


Fig. 4 Voltage gain at phase (a) for standard test connection ; — measured , - - - curve fitting

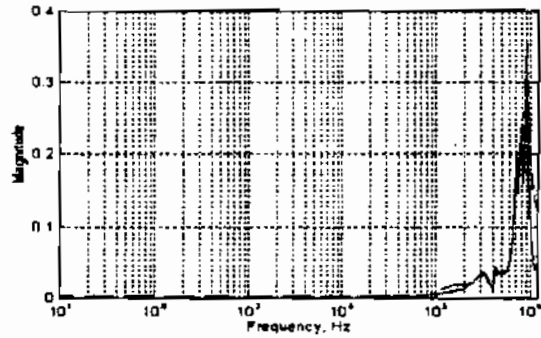


Fig. 5 Voltage gain at phase (b) for standard test connection ; — measured , - - - curve fitting

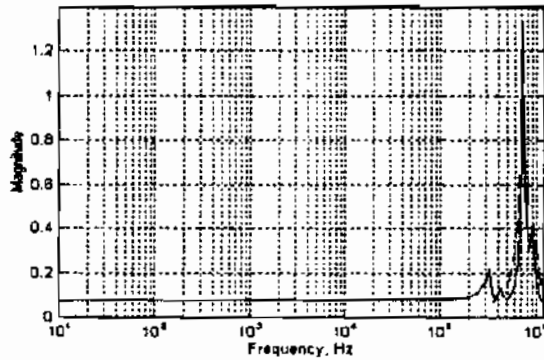


Fig. 6 Voltage gain at phase (c) for standard test connection ; — measured , - - - curve fitting

where

$I_i$  = the input impulse current

$V_i$  = the input impulse voltage

$Z$  = the input impedance

$V_o$  = the transient output voltage

Voltage gain =  $(V_o/V_i)$

Voltage transfer function (vtf) = Voltage gain

The above equations gives the transient voltage response of the distribution transformer in the time domain, but the formula of the input impedance and voltage transfer function is expressed in frequency domain by the proposed expression. Therefore, the transient voltage  $V_o$  must be expressed in terms of the time instead of the frequency. This has been achieved by using Laplace transform method:

### 3.3.1 Laplace Transform Method

The time-domain solution using Laplace transform is realized in two ways. The first, is done by using Laplace transform inversion while the second uses a scientific software called ,MAPLE, which depends mainly on Laplace transform.

### 3.3.2 Numerical Laplace Transform Inversion

The development of this method can be presented as follows [21,22] :

Consider the Laplace inversion formula

$$V(t) = \frac{1}{2\pi j} \int_{-\infty}^{+\infty} V(s) e^{st} ds \quad (7)$$

An alternate is to first remove the variable  $s$  from  $e^{st}$  by the transformation

$$z = st \quad (8)$$

and then approximate  $e^z$ . This is done numerically only once in the development of the method, and all further in versions use the results thus obtained.

Substituting Eqn. (8) into Eqn.(7) then,

$$V(t) = \frac{1}{2jt} \int_{-\infty}^{+\infty} V(z/t) e^z dz \quad (9)$$

Approximation the function  $e^z$  by a rational function (Pade approximation), gives



$$R_{N,M}(z) = \frac{P_N(z)}{Q_M(z)} \quad (10)$$

Where:  $P_N(z)$  and  $Q_M(z)$  are polynomials of order  $N, M$ , respectively. The classed form can be written as:

$$R_{N,M}(z) = \frac{P_N(z)}{Q_M(z)} = \frac{\sum_{i=0}^N (M+N-i)! (i^N) z^i}{\sum_{i=0}^M (-)^i (M+N-i)! (i^M) z^i} \quad (11)$$

From Eqns. (10) and (9) the approximation to  $V(t)$  to  $\hat{V}(t)$  becomes:

$$\hat{V}(t) = \frac{1}{2\pi j} \int_{\gamma} V(z/t) R_{N,M}(z) dz \quad (12)$$

Now the integral (12) can be evaluated by residue calculus by closing the path of integration along an infinite arc. The path along the infinite arc not contribute to the integral, therefore  $M$  and  $N$  are chosen such that the function has at least two more finite poles than zeros [22].

$$F(z) = V(z/t) R_{N,M}(z) \quad (13)$$

$$\int_C F(z) dz = \pm 2\pi j \sum (\text{residues at poles inside the closed path}) \quad (14)$$

where the positive sign applied when the path  $C$  is closed in the left half plane, whereas the negative one applied for the other case. For  $N < M$  :-

$$R_{N,M}(z) = \sum_{i=1}^n \frac{K_i}{z - Z_i} \quad (15)$$

Where  $Z_i$  are the poles of  $R_{N,M}(z)$  and  $K_i$  are the corresponding residues. Closing the path of integration around the poles of  $R_{N,M}(z)$  in the right half plane. The summation can be written as [22] :-

$$\hat{V}(t) = -\frac{1}{t} \sum_{i=1}^n (K_i V(z, t)) \quad (16)$$

This is the basic inversion formula. Real-time function can be evaluated using only the poles  $Z_i$  in the upper half plane [22] :-

$$\hat{V}(t) = -\frac{1}{t} \sum_{i=1}^M 2 \operatorname{Re}(k_i V(z_i, t)) \quad (17)$$

Where  $M' = M/2$ .

### 3.3.3 Application of Software (MAPLE)

A software, called MAPLE is used to give an inverse-Laplace transform. This is realized in the following procedure

- (1) Finding the appropriate network function in the s-domain.
- (2) Defining the input signal in s-domain or in time domain and get the Laplace transform for it.
- (3) Inverting their product.

## 4- APPLICATION AND COMPARISON EXAMPLE.

The described new approach has been applied on the three phase distribution transformer under test, which data shown in Table I.

The response due to 1.2/50  $\mu$ s impulse current wave with 1 pu crest value is determined by using the two methods based on Laplace transform as shown in Fig. 7.

The results using the two solution techniques of using Laplace transform method have been compared with each other as well as with a known method based on the State space method published by Soysol [19]. Soysol solves a group of nonlinear equations having real and imaginary parts to achieve the driving point function in frequency domain. That after he uses the State space method to get the solution in time domain. It should be mentioned that the proposed method, based on the Laplace transform method, has not the limitations associated with the State space method, namely, that the degrees of numerator and denominator may not be differing by more than one as constrained by Soysol [23]

## 4-CONCLUSION

This paper presents a developed model to simulate the behavior of distribution transformers. The present model is simple and applicable for any distribution transformer. A simple method of the system identification with good accuracy of the transformer gain at different phases in the case of standard test connections is developed. Two methods of time domain solution based on Laplace

transform method are presented, appreciated and compared with other solution based on the State space method. The new high frequency transformer modeling method is not only able to simulate any kind of transformer for all high frequency range accurately, but it also overcomes the solution limitation associated with other time domain solution methods.

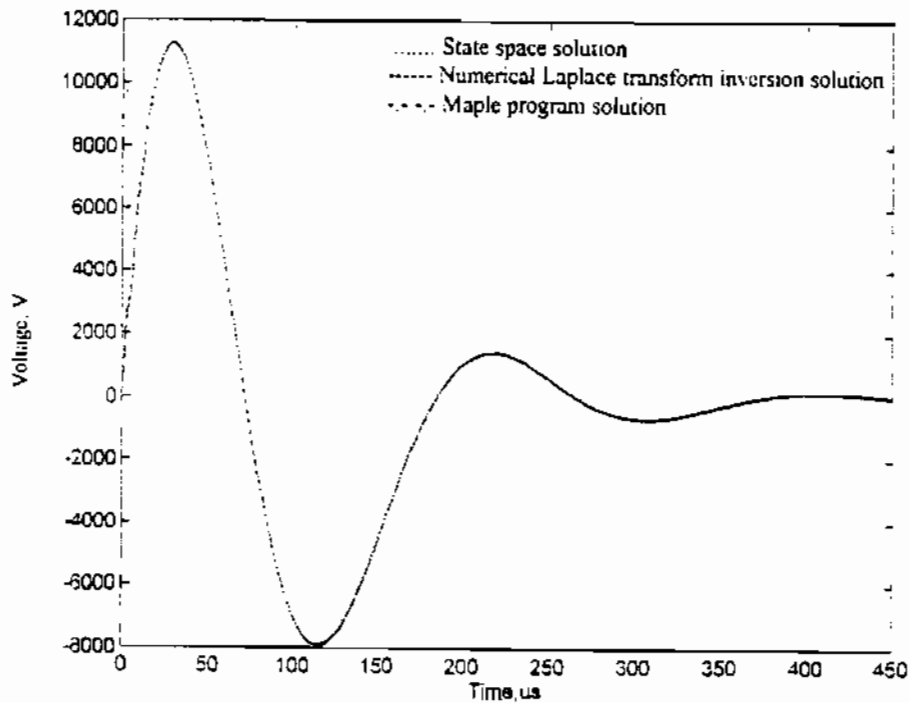


Fig 7 Transient voltage response under the action of 1.2/50 μs impulse current wave using the parameter of the power transformer under test.

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