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AN OPTIMIZED FRESNEL RHOMB FOR THE USE AT A LASER LINE

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ABSTRACT

It is shown that the traditional theory of constructing the Fresnel rhomb is inadequate if the rhomb is intended for use at a single wavelength. The theory was introduced for a rhomb working in a definite spectral region. Consequently, it considers in the first place high achromaticity of the rhomb (i. e., least variation of the retardance δ with the wavelength λ). Other characteristics were considered as of secondary importance. For use at a single wavelength, the retardance has a fixed value and the theory must be modified to minimize the sensitivity of the retardance δ to variations in the angle of incidence and to decrease the size of the rhomb. This implies shifting the value of the reflection angle θ from its usual position on the ascending side of the(δ-θ) relation to a central position at the bottom of the curve. An optimized rhomb for use at the He-Ne line 632.8 nm is described.

INTRODUCTION

The Fresnel rhomb and its different uses as a $\pi/2$ compensator in optical systems and polarization measurements were discussed by several authors [1-5]. There are three factors which characterize the accuracy of the rhomb. These are the achromaticity, sensitivity to variations in the angle of incidence and the size of the rhomb. As high achromaticity (i.e., least variations of the retardance δ with the wavelength λ) is always an essential requirement, the theory of constructing the rhomb as presented by Fresnel and is still adopted restricts the device parameters to values providing high achromaticity at the expense of the other two factors. To understand the importance of these two factors in the performance of the rhomb, Fig. (1) represents a typical rhomb for the use in the visible spectrum[2]. The retardance is exactly 270° at λ = 589 nm (n = 1.511) and between λ = 365 and 768 nm it varies about 2.5° due to the variation of n with λ (n = 1.506 -1.531) . The rhomb is highly achromatic as the values of n and θ are selected for this purpose . However , as the rhomb is sometimes used at a focal point to minimize its size, the angles of incidence for different rays of the beam will deviate from norml incidence. For the above typical values , the retardance will be seriously modified for small variations in the angle of incidence. In addiation the ratio of the long to short sides for the above values causes an undesired increase in the volume which calls for additional birefringent effects and consequently a change in the value of the retardance. For the use at a single spectral line, the retardance has a fixed value depending upen the parameters of the device. It is then essential to select these parameters taking into consideration other factors that affect the accuracy of the rhomb. In this work, an optimized Fresnel rhomb for use at the He-Ne laser line 632.8 nm is described.

CONSTRUCTION OF THE RHOMB

Linearly polarized light falling normal to the entrance face of the rhomb will be totally reflected twice before emerging . Due to each reflection , the parallel (p) and the perpendicular (s) components will suffer phase changes of [2]

$$\delta_p = \pi - 2 \tan^{-1} [n (n^2 \sin^2 \theta - 1)^{1/2}] / \cos \theta , \qquad (1)$$

$$\delta_s = -2 \tan^{-1} (n^2 \sin^2\theta - 1)^{1/2} / n \cos \theta \qquad (2)$$

The retardance is then $\delta = \delta_p - \delta_s$ and for the rhomb due to two reflections is given by

$$2\delta = 4 \tan^{-1} [-\cos \theta (n^2 \sin^2\theta - 1)^{1/2}] / n \sin^2 \theta \qquad (3)$$

The rhomb is actually $3\pi/2$ and not $\pi/2$ compensator as usually stated. Variations of δ_p , δ_s and δ with θ for a given value of n (1.50) are shown in Fig. 2 . For this value of n, the value of δ is very close to 135° at the bottom of the curve (δ is exactly 135° for n = 1.496). For higher values of n , the bottom of the δ curve shifts to lower θ values and becomes more sharp . The value of θ for which δ is minimum is

$$\theta_{min} = \sin^{-1} [2/ (n^2 + 1)]^{1/2} \qquad (4)$$

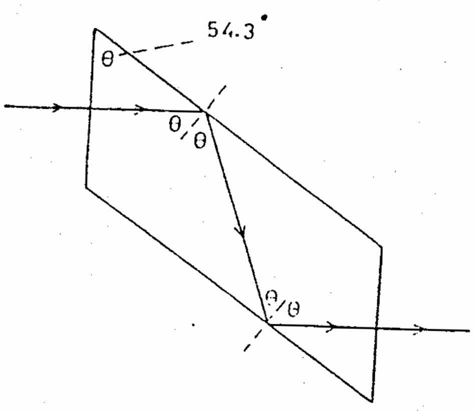

Fig. (1) Construction of a typical Fresnel Rhomb .

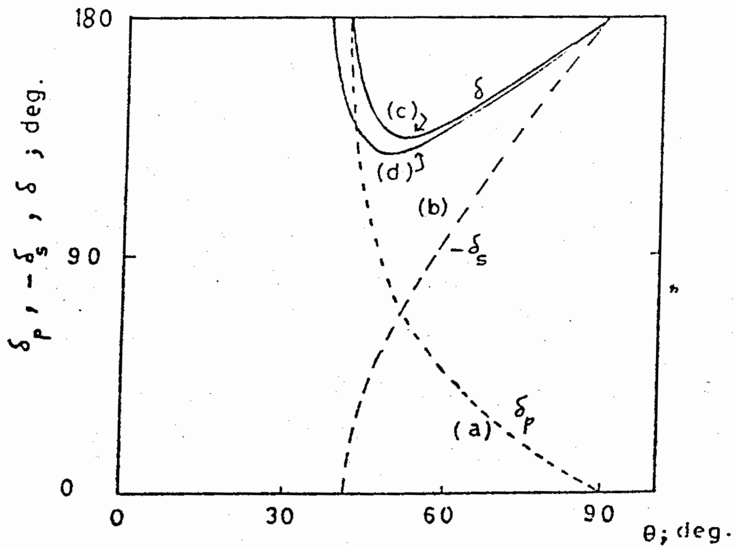

Fig. (2) Variation of δ_p (a), δ_s (b), and δ (c), with reflection angle θ for n = 1.50. Curve (d) represents δ for n = 1.60 .

The corresponding value of δ is

$$\delta_{min} = 2 \tan^{-1} [- (n^2 - 1) / 2 n] ; \qquad \pi /2 < \delta < \pi \qquad (5)$$

To understand the reasons for which the rhomb is almost always cut with values of θ (the acute angle which is equal to the reflection angle) between 54° and 55° to provide a retardance in the vicinity of 270° for refractive index values n between 1.50 and 1.54 , we refer to Fig . 2, curve (c) . The descending side of the (δ–θ) curve is too steep so that a small variation in the reflection angle θ due to possible error in the alinement of the optical system or when using the rhomb at a focal point, will cause a serious error in δ. Also the variation of δ with n in the area of the bottom is maximum , which contradicts achromaticity requirementes . Finally , too large values of θ will require high refeactive index glass and the volume will be exceedingly large (curve d) . For all these reasons , the rhomb is restricted to the above mentioned values .

As far as the rhomb is to be used at different wavelengths (n is variable), the above restrictions which provide a high degree of achromaticity are reasonable . For the use at a single wavelength , the parameters n and θ must be selected such as to minimize the sensitivity to angles of incidence and to decrease the size of the rhomb as mentioned before . Clearly , the retardance at the center of the bottom of the (δ - θ) relations for values of n close to 1.50 presents ideal conditions . Fig . 3 represents an optimized Fresnel rhomb for the use at the He - Ne line 632.8 nm. For $n = 1.497$ and $\theta = 52^\circ$ the retardance is 269.96°. For variations up to $\pm 3^\circ$ in the angle of incidene in air (or $\pm 2^\circ$ in the reflection angle θ), the retardance varies between 270.86° and 269.94°. In comparison with the rhomb of Fig . 1 , where the retardance is 270° for $n = 1.511$, then for similar variations in θ , the retardance assumes values between 272.56° and 268.34°. This means that the sensititity to variaitions in the angle of incidence is reduced almost five times in our suggested rhomb. Also, as the ratio of the long (L_1) to short (L_2) sides is [1]

$$L_1 / L_2 = 2 \tan \theta \sin \theta , \qquad (6)$$

then the volume of the rhomb is also reduced by about 11 % for the same entrance face due to the decrease in the value of θ .

Obviously , this optimized rhomb requires a glass of exactly a refractive index value $n = 1.497$ at $\lambda = 632.8$ nm . Since this may not be available , it is then possible to use a glass with a refractive index close to this value at the given λ . With a suitably deposited dielectric layer on one or both of the reflecting surfaces, it is possible to adjust the retardance to an exact value of 270° , Fig . 4 . In this cass , the reflection angle θ' at the film/air interface will be different from the acute angle of the rhomb . In effect , this is equivalent to changing the refractive index of glass to the required value of 1.497 . Coating processes in total internal reflection compensators

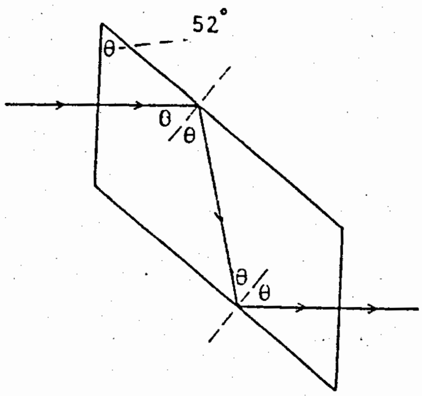

Fig. (3) Optimized Fresnel Rhomb for use at 632.8 nm.

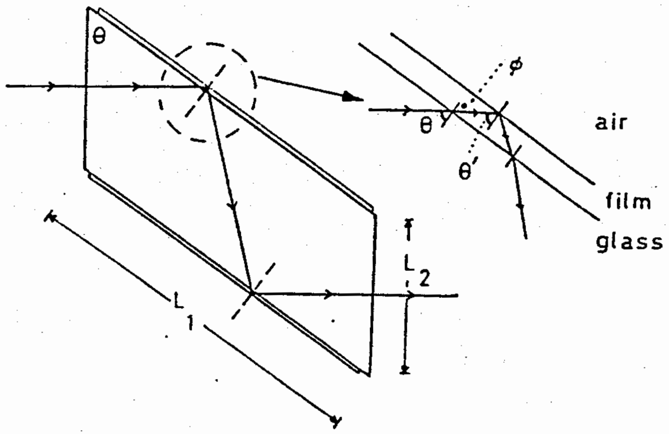

Fig. (4) Coated Fresnel Rhomb .

were extensively studied in several works . Of great importance are the articles of King and his co - workers [5.6] , Spiller [7] , and Filinski - Skettrup [8] . An important advantage of coating the reflecting surfaces is that it is possible by adjusting the layer thickness to eliminate the deviation in the retardance value produced by residual birefringence in glass . The exact analysis for the change in phase retardation due to the presence of a thin transparent film is given by vasicek [9] . In case of total internal reflection occuring at the film / air interface ,

$$r \exp(i\,\delta) = [r_1 - \exp(-i\,\delta_2)] / [1 + r_1 \exp(-i\delta_2)] \qquad (7)$$

where r is the resultant reflected amplitude , δ_2 is the overall phase difference due to the retardance introduced by the film and that caused by total internal reflection and r_1 is the Fresnel coefficient for the glass / film interface . This equation can be applied to calculate δp and δs to find the overall retardance $\delta = \delta_p - \delta_s$. It must be stated that if the refractive index of the film is lower than that of glass, the phase retardance will be increased as the reflection angle will increase and vice versa. In equation 7, δ_2 is equal to

$$\delta_2 = (2\pi/\lambda)(2n_1\, d \cos\phi) \qquad (8)$$

where λ is the wavelength of light , n_1 is the refractive index of the film , d is the thickness of the film and ϕ is the angle of refraction in the film [9].

SUMMARY

The rhomb described in this work was analyesd where it was found to be much less sensitive to variations in the angle of incidence and of smaller volume . These conditions provide the optimal characteristics for use at a single wavelength . In addition , the coating process , if necessary , is useful in correcting the value of the retardance by adjusting the film thickness to eliminate the effects of residual birefringence .

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معين فرسنل عالي الكفاءة للاستخدام
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يظهر هذا البحث قصور نظرية تصميم معين فرسنل عند استخدامة بالنسبة للضوء الاحادى الطول ، حيث تهتم النظرية فى المقام الاول بتقليل فرق الطور بين مركبتى الضوء المتعامدتين فى مدى طيفى معين وذلك على حساب باقى الخواص .

ونظرا لكثرة استعمال المعين فى قياسات ضوئية عند خط احادى الطيف فان فرق الطور بين مركبتى الضوء يكون فى هذه الحالة ثابتا اساسا وعلى ذلك فإن من المهم ان يعدل التصميم بحيث يقلل من حساسية فرق الطور بالنسبة للتغيرات فى زاوية السقوط الخارجية وكذلك لتصغير حجم المعين . والتعديل المقترح هو ازاحة زاوية الانعكاس الكلى الداخلى الى قيمة تكون حساسية فرق الطور عندها بالنسبة لتغير زاوية السقوط اقل ما يمكن ، وفى نفس الوقت تؤدى هذه الازاحة الى تصغير حجم المعين .

يقدم هذا البحث نموذجا مثاليا لمعين فرسنل للاستخدام عند خط الليزر شائع الاستعمال ٦٣٢.٨ نانومتر وذلك طبقا للتعديل المقترح .