A UNIFIED INTERACTIVE APPROACH FOR SOLVING MULTIPLE CRITERION NONLINEAR INTEGER PROGRAMMING PROBLEM

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ABSTRACT

This paper introduces a unified interactive approach for solving multiple criterion nonlinear integer programming problems. The proposed algorithm collects the characteristics of four interactive approaches. These approaches are stem, trade off cutting plane, (GDF) procedure, and Tchebycheff procedure.This paper presents a combined interactive approach between stem and trade off cut ting plane methods, a combined interactive approach between GDF and Tchebycheff procedure will also be introduced. Finally, the paper also presents a proposed algorithm for a unified approach to solve the multiple - criterion nonlinear integer programming (MCNLIP) problems which combines the characteristics of both the combined approaches.

Key words Multiple criterion programming, Interactive procedures, Efficient solution, Tchebycheff metrics.

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1- Introduction

The purpose of multiple criterion mathematical programming is to optimize k different criterion functions subject to a set of constraints. This paper presents the collection of the characteristics of the four interactive approaches already mentional in the abstract. The stem method has been proposed by Benayoun Demontgolfier, Tergny and Laritchev (1971) [4] .This approach considers the reduced feasible region method for solving multiple criterion programming. The second trade off cutting plane method was developed by Musselman and Talavage. It isolates the best- compromise solution by iteratively reducing the objective space [7]. The third GDF method was developed by Geoffrion - Dyer-Feinberg. It a line search procedure for solving multiple criterion program [5]. Finally, the fourth method is the weighted Tchebycheff procedure, which is a weighting vector space reduction method for multiple criterion nonlinear integer programming problems. It has been proposed by Steuer and Choo (1983). [3]. This paper comprise: the first combined approach solves a point in the reduced feasible region $S = S^{(h)}$ whose criterion vector is closest to z^* (ideal solution) according to the weighted Tchebycheff metric defined by $\lambda \in R^k$, and isolates the best compromise solution by iteratively reducing the objective space (equivalent to the reduction of the feasible space) by cutting planes., The second combined approach deals with the Tchebycheff procedure by generating a useful line search direction which help the decision maker to generate a good weighing used in Tchebycheff program, and The unified proposed algorithm collects the characteristics of the two combined approaches.

2-Problem formulation

A mathematical formulation of multiple criterion nonlinear integer programming problem(MCNLIPP) can take the follwing form (P):-

Max $f(x) = \{ f_1(x) , f_2(x) , \ldots , f_k(x) \}$

Subject to $S^{(0)}$

, $S^{(0)}=\{ x / g_j(x) \leq 0 , x \geq 0 ,$ x is integer.$\}$, $j = 1,2,\ldots,m$

Where x is an n-dimensional vector of continuous decision variables , $S^{(0)}$ is a decision space ,and f(x) is a vector of k real valued functions ,considering that the ideal solution exists.

3- First combind approach

This approach combines the characteristics of stem method and trade off cutting plane method. Let us consider the follwing definitions.

Definition 1 (efficient solution) [6]

A solution $x^* \in S$ is said to be efficient if for any $x \in S$ satisfying $f_k(x) > f_k(x^*)$, $f_j(x) < f_j(x^*)$. for at least one other index , $j \neq k$.

3-1 The proposed first combined algorithm

The solution of the problem (P) can be summarized in the follwing steps using the first combined approach.

Step (1)

By individually optimizing each objective function to obtain the ideal criterion vector $z^* \in R^k$.

That is $z_i^* = \{ \text{Max} = f_i(x) / x \in S^{(0)} \}$ $\quad i = 1, 2, \ldots, k$.

and construct a pay off table as table (1) .

Step (2)

By normalizing the weights $\lambda \in R^k$, $\lambda_i = \dfrac{mi}{\sum_{j=1}^{k} mj}$,

$$mi = \begin{cases} \dfrac{zi^* - zi}{zi^*} \left[\sum_{j=1}^{k} (\nabla zi)^2 \right]^{-\frac{1}{2}} & z_i^* > 0 \\ \dfrac{zi - zi^*}{zi} \left[\sum_{j=1}^{k} (\nabla zj)^2 \right]^{-\frac{1}{2}} & z_i^* \le 0. \end{cases}$$

Where $z_i = \max z_i(x^j)$.

Step (3)

By solving the weight min max program.

Min α

Subject to : $x \in S^{(1)}$,

$S^{(1)} = \{ x / x \in S^{(0)}, \ \alpha \ge \lambda_i (z_i^* - z_i) \ , x \ge 0, x \text{ is integer} , i = 1, 2, \ldots, k, \alpha \ge 0 \}$

For the solution $x^{(1)}$. In this step solves for the point in the reduced feasible region $S^{(1)}$ whose criterion vector closest to z^* according to the weighted Tchebycheff metric defined by $\lambda_i \in R^k$.

Step (4)

By letting $z^{(1)} = z(x^{(1)})$ and comparing $z^{(1)}$ with z^*.

If all components of $z^{(1)}$ are satisfactory , stop $(z^{(1)}, x^{(1)})$ as a final solution . Otherwise choose a concave decision maker utility *function U and go to Step (5).*

Step (5)

By interacting the decision maker to obtain the local trade off ratios at $x^{(h)}$.

A combined approach needs (k-1) local trade off ratios at each iteration .

Definition 2 (The weighted Tchebycheff metric) [4]

We recognize $\| z_i^* - z_i \|^{\lambda}_{\infty} = \max\limits_{1 \le i \le k} \lambda_i \, | z_i^* - z_i |$ as a member of the family of weighted-Tchebychff metrics for measuring the distance between z_i (current solution) , z_i^* (ideal solution) , $\lambda = \{ \lambda_i / \lambda_i \in R^k, \lambda_i \ge 0, \sum_{j=1}^{k} \lambda_j = 1 \}$.

Definition 3 (best compromise solution) [6]

The best compromise solution to multiple criterion nonlinear integer programming problem (MCNLIPP) is an efficient solution that maximizes the decision maker's preference function.

Definition 4 (local trade off ratio) [7]

The local trade off ratio (marginal rate of substitution) between the objectives $f_j(x)$ and $f_i(x)$ at solution $x^{(h)}$, U is the utility function.

Is

$$T_{ij}^{(h)} = \left(\frac{\partial u / \partial fj}{\partial u / \partial fi} \right)_{x^{(h)}}$$

Let us define the pay off tables, [4]

A pay off table of the form table (1) below, where the rows are criterion vectors resulting from individually maximizing each objectives , the z_i^* entries along the main diagonal from the vector of maximal criterion values(over the efficient set) .the *minimum value in the i^{th} column of the pay off table is an* estimate of a minmum criterion value of the i^{th} objective over S.

	z^1	z^2	.	.	.	z^k
z^1	z_1^*	.	.	.		z_{1k}
z^2	.	z_2^*	.	.		z_{2k}
.	.	.	z_3^*			.
.	.	.	.	.		.
z^k	z_{k1}	.	.	.		z_k^*

Table (1)

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	z^1	z^2			z^k
z^1	z_1^*	.	.	.	z_{1k}
z^2	.	z_2^*	.	.	z_{2k}
.	.	.	z_3^*		.
.	.	.	.	.	.
z^k	z_{k1}	.	.	.	z_k^*

Table (1)

Step (6)

By generating a trade off cut constraint, that is an additional constraint which is used to eliminate a certain portion of the reduction space $S^{(1)}$.

$$p(h) : \sum_{j=1}^{k} T_{ij}^{(h)} (f_j(x) - f_j(x^{(h)}) \geq 0$$,[7] is a trade off cut.

Step (7)

By solve the follwing program:-

Max Y^*

Subject to

$y \in S^{(h+1)}$

$y \geq 0$, is integer.

Where $S^{(h+1)} \quad S^{(1)} \cap P^{(h)}$

If $Y^* < \varepsilon$ or $Y^* = 0$, stop Otherwise determine a secondary point , $y^{(h+1)} = x^{(h)} + \rho\ (y^{(h)} - x^{(h)})$ and go to step 8.

Step (8)

Use the point $y^{(h+1)}$ and solve the program, [7].

$$Max\ \alpha + \varepsilon \sum_{j=1}^{k} T_{ij}^{(h)} (f_j(x) - f_i(y^{(h+1)})$$

Subject to

$f_j(x) - f_j(x^{(h+1)}) \geq \alpha$, $j = 1, 2, \ldots, k$

$x \in S^{(1)}$

Step (9)

If the solution is satisfactory, Then go to step (10) , otherwise let $h = h + 1$, and go to step (5).

Step (10)

Stop ($x^{(h+2)}$, $z^{(h+2)}$) as the final solution.

4 –The second combined approach

This combines GDF and Tchebycheff procedure. Now let us make these definitions.

The direction finding program [5]

With the known function $U : z^k \rightarrow R$ to be optimized over $S^{(0)}$. The direction finding program is defined as follows.

Max $\{[\nabla_x U (x^{h-1}]^T . x / x \in S \}$ with $y^h \in S$ is the optimal solution the direction is defined $d^h = y^h - x^{h-1}$.

A steepest ascent program [4]

With direction $d^h = \nabla_x^j U(x^{h-1})$ where $\nabla_x U(x^{(h-1)})$ is a gradient of U at x^{h-1} we define the step size program

Max { $U(x^{h-1} + t\, d^h) / x^{h-1} + t\, d^h \in S^{(0)}, 0 \le t \le 1$} with step size t* optimal. Let $x^h = x^{h-1} + t^* d^h$.

The proposed second combined algorithm

The solution of the problem (P) can be summarized in the follwing steps using the second combined approach.

Step (1)

Compute the ideal solution z* (the reference criterion vector) where

$z_i^* = \max \{ f_i(x), x \in S \}$ $i = 1, 2, \ldots, k$.

Step (2)

Generate an initial efficient solution by solving the follwing program:

$$P_k: \; Max \sum_{i=1}^{k} \frac{1}{k} fi(x),$$

Subject to : $x \in S^{(0)}$, x may be non-integer.

Step (3)

Set $h = h + 1$, $x^{(0)} \in S^{(0)}$ is an initial efficient solution.

Step (4)

Ask the decision maker to determine the utility function which is concave and differentiable .

Step (5)

Estimate the direction of the gradient of U at x^h [5].

$$\nabla U = \left[\sum_{j=1}^{k} Tij \frac{\partial fj}{\partial x1} + , \ldots , + \sum_{j=1}^{k} Tij \frac{\partial fj}{\partial xn} \right]$$

Step(6)

Solve the direction finding program:-

Max = { $\nabla U . y / y \in S^{(0)}$ }, y is integer.

With $y^h \in S$ define the direction $d^h = y^h - x^h$ this should suffice for generating a useful line search directions. i.e. is a good direction in which seeks to improve the objective function U[z(x)],

Step (7)

Specify the number of step wise points, display the stepwise criterion vectors $z[x^h + (j-1/p-1) d^h]$, $j = 1, 2, \ldots, p-1$

Step (8)

If the solution is satisfactory then stop. Otherwise go to step (9)

Step (9)

For all $z[x^h + (j-1/p-1)\, d^h]$ determine λ for each z.

Where $\lambda = \{ \lambda \in R^k \,/\, \lambda_i \in [0, 1],\ \sum_{j=1}^{k} \lambda j = 1 \}$, [2]

$$\lambda i = \begin{cases} \dfrac{1}{zi^* - zi}\left[\sum_{j=1}^{k} \dfrac{1}{zj^* - zj}\right]^{-1} & z_i^* \neq z_i \\ 1 & z_i^* = z_i \\ 0 & z_i^* \neq z_i \text{ but there exists index j such that } z_j^* = z_j \end{cases}$$

Step (10)

For each λ_i solve the weighting Tchebycheff program [3] for $i = 1, \dots, k$.

Min = α

Subject to : $S^{(1)}$,

$S^{(1)} = \{ x \in S^{(0)},\ \alpha > \lambda_i (z_i^* - z_i),\ f_i(x) = z_i ,\ \alpha \geq 0 \}$

Step (11)

Display the p criterion vector to the decision maker to choose which preferred. Let z^h designate the criterion vector selected by the decision maker as the most preferred from the sample of step (10) then go to step (12) otherwise go to step (5)

Step (12)

Compute the inverse image of the decision – maker's final criterion vector selection , he will terminate the algorithm if

(1) $(j-1/p-1) = t^* = 0$

(2) the solution is satisfactory to the decision maker.

5-The unified approach

The two approaches which are discussed in the previous sections are combined together in a unified approach. The proposed unified approach is more convergent than the other approaches . This approach is based upon the direction finding program (Frank Wolf algorithm [5]) to determine the good direction d^h and steepest ascent program [4] to determines the step size t*.The unified algorithm determine the best weighting vector λ^k and applied Tchebycheff program to the point closest to the ideal solution and constructs a promising space (trade off cut) [7] to eliminate a certain portion.

5-1 A proposed unified algorithm

The solution of problem (P) can be summarized in the follwing steps using the unified approach.

Step (1)

Compute an ideal solution z* [3].

Step (2)

Compute an initial efficient solution.

Step (3)

Apply a direction finding program to find $d^{(h)}$

Step (4)

Apply a steepest ascent program to find t*.

Step (5)

Compute $z_i(x^{(h)} + t^* d^{(h)})$ and find λ_i at z_i .

Where,

$$\lambda i = \begin{cases} \dfrac{1}{zi^* - zi}\left[\dfrac{1}{zi^* - zi}\right]^{-1} & z_i^* \neq z_i \\ 1 & z_i^* = z_i \\ 0 & z_i^* \neq z_i \text{ but there exists index j such that } z_j^* = z_j. \end{cases}$$

Step (6)

Solve the follwing program.

Min α

Subject to : $x \in S^{(h)}$,

$S^{(h)} = \{x / x \in S^{(0)},\ \alpha \geq \lambda_i (z_i^* - z_i) \ ,\ i = 1, 2, \ldots, k \}$

Step (7)

If the solution is satisfactory, go to step (12), otherwise go to step (11), and interact the DM to choose the best solution. If it does not satisfy the best solution go to step (8).

Step (8)

Construct a trade off cut (promising space)

$P^{(h)}: \sum_{i=1}^{k} Tij^{(h)} (f_i(x) - f_i(x^{(h)}) \geq 0$ [2].

Step (9)

Solve the following program

Max $Y = \nabla_x^i U . (y - x^{(h)})$

Subject to

$y \in S^{(h)} \cap P^{(h)}$

Step (10)

If $Y^* = 0$ or $Y^* \le \varepsilon$ go to step (11), otherwise let $h = h + 1$ and determine the secondary point

$y^{(h+1)} = x^{(h)} + \rho\ (y^{(h)} - x^{(h)})$. , $\rho = .5$, go to step (6).

Step (11)

Select the variable with the greatest fractional part, say x_j, and solve the program with additional constraint $x_j \le [x_j]$. *If the integer solution is satisfactory* stop .If not solve the same program with additional constraint $x_j \ge [x_j] + 1$. If the integer solution is satisfactory, go to step (12).

Step (12)

Stop with ($x^{(h)}$, $z^{(h)}$) as a final solution.

5-2 Illustrative example:-

To illustrate the unified algorithm, consider the following MCNLIP problem

$\text{Max} = \{ f_1(x) = x_1\ ,\ f_2(x) = x_2\ ,\ f_3(x) = x_3 \}$

Subject to

$x_1^2 + x_2^2 + x_3^2 < 225.$

$xi \ge 0$, x_i is integer. and $\varepsilon = .001$

Solution

To obtain the integer solution use the branch and bound technique, [1].

Step (1) :-

Compute the ideal solution by solving the follwing program:-

$\text{Max}\ \ f_i(x) = x_i \qquad i = 1,2,3$

Subject to

$x_1^2 + x_2^2 + x_3^2 \le 225 \qquad S^{(0)}$

$x_i \ge 0$ & is integer

then the ideal solution $Z^* = (15, 15, 15)$

Step (2) :-

Find the initial efficient solution

$$P_k : \sum_{i=1}^{3} 1/3\, f_i(x) = 1/3\,(x_1 + x_2 + x_3)$$

Subject to

$x_1^2 + x_2^2 + x_3^2 \le 225 \qquad S^{(0)}$

$xi \ge 0,$

$x^{(0)} = (8.65\ ,\ 8.66,\ 8.65\ )$

Step 3:-

Apply Frank - Wolf algorithm (direction finding program)

let $T_{11}^{(1)}$ 1 , $T_{12}^{(1)}$ .99 , $T_{13}^{(1)}$ 1

$$\& \quad \nabla_x i\, U = \left[\sum_{i=1}^{3} T_{1i}^{(1)} (\partial fi / \partial x_1), \sum_{i=1}^{3} T_{1i}^{(1)} \partial fi / \partial x_2, \sum_{i=1}^{3} T_{1i}^{(1)} \partial fi / \partial x_3 \right]$$

$= (1, .99, 1)$

and solve

$\text{Max} = (1, .99, 1) . (y_1, y_2, y_3)$

Subject to

$y_1^2 + y_2^2 + y_3^2 \leq 225$

$yi \geq 0$, $i = 1, 2, 3$

where $U = -[(Z_1 - 15)^2 + (Z_2 - 15)^2 + (Z_3 - 15)^2]$

is the implicit DM's utility function which is concave.

and let the program:-

$\text{Max} \ 1.y_1 + .99.y_2 + 1.y_3$

Subject to

$y_1^2 + y_2^2 + y_3^2 \leq 225.$

$yi \ 0.$

The solution of that system $y^{(1)} = (8.68, 8.6, 8.68)$

let $d^{(1)} = y^{(1)} - x^{(0)} = (.03, .06, .03)$

Step 4 :-

Apply a steepest ascent program

That solves the program.

$\text{Max} = U(Z(x^{(0)} + t\, d^{(1)})$

Subject to

$(x^{(0)} + td^{(0)}) \in S^{(0)}$

$0 \leq t \leq 1$

i.e. $\text{Max} = -[(-6.35 + .03\, t)2 + (-8.66 + .06\, t)2 + (-8.65 + .03\, t)2]$

Subject to

$(8.65 + .03t)^2 + (8.66 + .06\, t)^2 + (8.65 + .03)^2 \leq 225.$

$0 \leq t \leq 1$

then the solution $t^* = .$

Step 5 :-

Compute $Z(x^{(0)} + .17\, d^{(1)})$

$Z \ (8.665, 8.67, 8.66)$

& compute $\lambda i \in R^3$ $i=1,2,3$

$$\lambda 1 = \frac{1}{15 - 8.66} \left[\frac{1}{15 - 8.66} + \frac{1}{15 - 8.67} + \frac{1}{15 - 8.66} \right] - 1 = .33$$

and also λ_2 , λ_3 .33

Step 6 :-

Solve the program

Min α

Subject to : $x \in S^{(1)}$,

$S^{(1)} = \{ x / \alpha \geq .33\,(15 - x_1), \alpha \geq .33(15 - x_2)\ , \alpha \geq .33\,(15 - x_3), x_1^2 + x_2^2 + x_3^2 \leq 225,$

$Z_1 = x_1\ , Z_2 = x_2, Z_3 = x_3\ , xi \geq 0$ & integer , i= 1,2,3 }

$Z^{(1)} = (8.66\ , 8.66\ , 8.66)$

$x^{(1)}\ \ (8.66\ , 8.66, 8.66)$,

Step 7:-

To find the integer solution select the variable with the greatest fractional part say x_1 , and add the constraint $x_1 \leq 8$ to $S^{(1)}$ and solve the program , then the solution is (8,8,8), also by the same way add the constraint $x_1 \geq 9$, then the solution is (9,8.485,8.485) Interact the DM to select the preferred solution .

If the decision maker would improve the solution go to step 7.

Step 8 :-

Compute T_{ij} at $x^{(1)}$ $\quad T_{11} = 1\ \ , T_{12} = 1\ \ , T_{13} = 1$

& construct a trade off cut

$p^{(1)}\ :\ \ 1(y_1 - 8.66) + 1\,(y_2\ -\ 8.66) + 1\,(y_3 - 8.66)$

Step 9 :-

Solve the program.

Max $Y^* = (y_1 -\ 8.66) +\ (y_2\ -\ 8.66) + (y_3\ - 8.66)$

$y \in S^{(1)} \cap P^{(1)}$

$y \geq 0$ & is integer

The solution is $(7.93\ , 7.99\ , 8.005)$,$Y^* = .0007 < \varepsilon$,

go step(11)

Step 10 :-

If $Y^* < \varepsilon$, go step(11)

Step 11:-

Select the variable with the greatest fractional part ,say y_2 and add the constraint $Y_2 \leq\ 7$ to $S^{(1)}\ \cap P^{(1)}$, then the solution is (9.368, 7, 9.3931), also by the same way ,add the constraint $y_3 \geq 10$, then the solution is (8.7 , 7 , 10) ,take $y_1 \leq 8$, Then the solution is (8 ,7, 9) .

Another example

Let us consider the following example

Max $\{ f_1(x) = x_1\ , f_2(x) = x_2 \}$

Subject to $x \in S^{(0)}$,

$S^{(0)} = \{ x_1 + x_2\ \leq 10, 2\,x_1 + x_2 \leq 18, 5\,x_1 + 9\,x_2 \geq 45, x_1^2 + x_2^2 \leq 100,$

$x_1\ , x_2$ are integer }, $\varepsilon = .01$

Solution

Step (1)

The ideal solution is (9, 10)

Step (2)

The initial efficient solution is (4.1, 5.89).

Step (3)

Apply the Frank- Wolf program to find the direction $d^{(1)}$

Let $U = (Z_1 + 1)^2 (Z_2 + 1)^2$, solve the direction finding program, then the solution $y^{(1)} = (9, 0)$ then $d^{(1)} = (4.9, -5.89)$.

Step (4)

Apply the steepest ascent program to find t*

$t^* = .06$

Step (5)

Compute $Z_i(x^{(0)} + t^* d^{(1)})$, and find λ_1, λ_2

$\lambda_1 = 1/9 - 4.394 [1/9 - 4.394 + 1/10 - 5.536]^{-1} = .429.$

then $\lambda_2 = 1 - \lambda_1 = .57$.

Step (6)

Solve the following program

Min α

Subject to : $x \in S^{(1)}$,

$S^{(1)} = \{\alpha \geq .429(9 - x_1), \alpha \geq .57(10 - x_2), x \in S^{(0)}\}$.

Then the solution is $x^{(1)} = (3.864, 6.135)$.

Use branch and bound technique to give the integer solution as follow

Select the variable with the greatest fractional part ,say x_1, add the constraint $x_1 \leq 3$ to $S^{(1)}$, solve the program ,then the solution is (3 , 5.48), also add the constraint $x_1 \geq 4$, the solution is (4 , 6),

Step (7)

Interact the DM to select the preferred solution.

If the decision maker desire to improve that solution go to step 8.

Step (8)

Construct the trade off cut $P^{(1)} = (y_1 - 3.864) + .86(y_2 - 6.135)$

Step (9)

Solve the program

Max $(y_1 - 3.864) + .86(y_2 - 6.135)$

Subject to : $y \in S^{(1)} \cap P^{(1)}$.

Then the solution is (3.87 , 6.128), $Y^* = .003$.

Step(10)

If $Y^* < \varepsilon$ go to step (11)

Step(11)

Use the branch and bound technique for solving nonlinear integer programming, i.e Select the variable with the greatest fractional part say y_1, and add the constraint

$y_1 \le 3$ to $S^{(1)} \cap P^{(1)}$, then the solution is (3 , 7) ,$U = 1024$,and also add the constraint $y_1 \ge 4$ then the solution is (4 , 6), U = 1225.

Step(12)

Finally the decision maker choose the best compromise solution which is (4, 6).

Conclusion

We introduce an interactive unified approach for solving multiple criterion nonlinear integer programming problems which is converges to the best compromise solution more than the other approaches , at any iteration the decision maker is shown the best step size t* , and a good direction d^h to help the decision maker to arriving the best compromise solution . An illustrative example is solved in details to illustrate the validity of the proposed unified algorithm.

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يقوم هذا البحث أسلوب موحد تفاعلى لعلاج مشاكل البرمجة الصحية الغــــير خطيــة متعددة الأهداف. ويتميز هذا الأسلوب بجمع خصائص ومميزات أربعة طرق تفاعلية لعـــلاج مشاكل البرمجة المتعددة الأهداف هى طريقة Stem، طريقة Trade of cutting plane، طريقة GDF، طريقة Tchebycheff.

تم عمل خوارزمين لدمج كل طريقتين من السابق ذكرهم وأخيراً تم دمج الأربعة فـــى أسلوب واحد موحد وتم وضع خوارزم مرقم لهذا الأسلوب مع أمثلة عددية توضـــح ممـــيزات هذا الأسلوب الموحد.