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COUETTE FLOW OF ELECTRON PLASMA IN WEAK INHOMOGENEOUS MAGNETIC FIELD

A.M. ABOURABIA
Department of Math., Faculty of Science, Monofia University, Shebin El-Koom, EGYPT.

ABSTRACT

We demonstrate the steady Couette flow of warm electron plasma of inhomogeneous density and subjected to a weakly variable magnetic field with the neglect of plasma waves under certain conditions. The plasma, obeying Debye approximation, behaves as an ideal gas in the transition region. It is shown, using the B.G.K. model of Boltzman equation and the half range polynomial expression technique for the electrons distribution function, that the shear momuntum is conserved. The flow, drift and slip velocities are compared. The bounadry conditions are built in the presence of partial reflections from the walls.

I. Introduction:

The consideration of problems with boundary surfaces is still of modest interest for the case of a plasma. In leterature |Shedlovskiy,1967| the case of Couette flow of fully ionized plasma in an external electric field is presented. The method is based on replacing the Boltzmann equation by moment equations with the simultaneous introduction of a two-stream Maxwellian distribution functions. |Abdel-Gaid,1972| treated the problem of shear flow of a rarefied gas consisting of charged particles moving in constant electric and magnetic fields, taking into account that the particles arriving to the plates are reflected diffusly with complete energy accommodation. In his thesis |Mahmoud,1985|studied the case of a rarefied charged gas in constant magnetic field with partial deffuse reflection at the walls. An approximate solution of the Boltzmann equation of the modefied Lui-Lees type is found to yield simple analytic expression for the flow and slip velocities.

The purpose of the present paper is twofold:

i) First, to present the formulation of the problem of Couette flow of a rarefied electron plasma of inhomogeneous density and subjected to a weak inhomogeneous external magnetic field on the basis of the Debye Screening theory.

ii) Second, to apply an alternative method to derive the transport properties of the plasma flow.

It is better in the begining to write a list of symbols that would be used in the text:

a – the interparticle distance $\sim n^{-\frac{1}{3}}$,

B_n – the coefficient of half range polynomial expansion,

C – nondimensional electron velocity,

c – speed of light,

D – Debye radius ,

d – plates separation,

e – electron charge ,

g – parameter of plasma nonideality = $(D^3 n)^{-1}$,

H – Magnetic field.

J – Current density,

K – Boltzmann's constant , Kn – Knudsen number,

k_D – Debye wave number, k – wave number ,

M – Shear Mach number,

M^* – Cyclotron Mach number, m – electron mass,

N_D – electron number in Debye sphere,

n – electron density,

$P_\perp$ – transverse plasma pressure.

P_{xy} – pressure deviator| Shear momentum |,

v – velocity component

T – temperature.

$v_\perp$ – transverse velocity = $v_x^2 + v_y^2$,

v_T – thermal velocity $= \sqrt{\frac{2KT}{m}}$

V_d – drift velocity , x, y – transverse coordinates,

z – longitudinal coordinate,

α – inverse of v_T^2,

β – effective addiabatic coifficient,

δ - degree of rarefaction

ε - reflection coefficient,

λ - wavelength,

ν - electron collission frequency

κ, κ' - relative gradients of magnetic field and electron density,

μ - thermal viscosity coefficient,

μ_c - viscosity coefficient due to coulomb interaction,

ω_p - plasma frequency $(= 4\pi ne^2/m)^{\frac{1}{2}}$,

Ω - cyclotron frequency $= \frac{eH}{mc}$.

II. The physical problem:

In our problem of steady Couette flow the two infinite parallel plates move in the xy plane apposite to each other in the x-direction with velocities $\pm W/2$. The plates are impermeable, uncharged and dielectric. The spase between the plates is filled with a inhomogeneous electron gas, under the influence of weak inhomogeneous external magnetic field. the whole system is supposed under constant temperature. Deviation from the concept of cold plasma in our work is essential, because completely cold electron stream will have only one value of x-momentum for a given energy, but owing to the proposed shear motion, the electrons would still have a distribution of momentum in the transverse x and y directions. This incorages one to introduce a model of warm or Maxwellian electron gas. In order that the space charge wave be desipated, the plasma oscillations become dispersive |Marshall,1985|

$$\omega^2 = \omega_p^2 + 3k^2 v_T^2 .$$

The phase velocity slows and approaches the thermal velocity v_T of the electrons as k becomes large. A bound on the collective behavior is set when k approaches k_D or $\lambda \lesssim D$, where $D = v_T/\omega_p$. In that case the wave motion is damped.

On the other hand, to avoid quantum effects, the DeBrouglie wavelength associated with the electron thermal motion must be related to the Debye length by the strong inequality

$$\lambda << \frac{g}{4\pi} D.$$

In our study, the plasma is supposed to be not so dense such that very weak correlation between electrons occurs i.e. $g \lesssim 10^{-2}$ |Isahara 1971|.

The intensity vector of the external magnetic field lies in the z-direction and takes the form of weak inhomogenety

$$\underline{H} = (H_o + y \frac{dH}{dy}) \hat{\underline{z}} = H_o (1 + \kappa y) \hat{z} . \tag{1}$$

The density of the plasma decreases along y-direction. Our Caculations assume a Maxwellian distribution of zero shear

$$f_o = n(y) \quad (\alpha/\pi)^{3/2} e^{-\alpha v^2} . \tag{2}$$

Here $n(y) = n_1 e^{-\kappa'(1-y)}$ where n_1 is the number density at the wall. In eqs.(1) and (2) the relative gradients

$$\kappa = \frac{1}{H_o} \frac{dH}{dy}\Big|_{y=0} , \quad \kappa' = - \frac{1}{f_o} \frac{\partial f_o}{\partial y}\Big|_{y=0} = - \frac{1}{n} \frac{dn}{dy}\Big|_{y=0} \tag{3}$$

are considered small and constant, furthermore κ' is assumed independent of the integrals of motion $v_\perp^2 = v_x^2 + v_y^2$ and v_z |Krall et al 1973|.

For f_o and $\underline{H}$ to be selfconsistent, Maxwell's equation

$$\underline{\nabla} \times \underline{H} = \frac{4\pi}{c} \underline{J} , \qquad (4)$$

implies the relationship

$$\frac{\kappa'}{\kappa} = \frac{H_o^2}{4\pi n K T} \equiv \frac{2}{\beta} . \qquad (5)$$

The balance between the kinetic and the magnetic pressures i.e. $\beta=2$, provides the condition of warm plasma and equats κ, κ' numerically. This will play an important role in what follows.

Often one uses the Boltzmann equation to calculate the transport properties in plasma |Krall et al,1973-Akhiezer,1974-Chakraborty, 1978| .

As is well known the Boltzmann equation has a limited range of validity, namely, the scattering events have to be well separated in time, this means small scattering rates. This enables one to use the descrit model |Frank-Kamenetskiy,1968| in which the fluactuating interaction is looked for as a collection of closely approaching pairs of electrons. The Coulomb collision. Besides, this puts restrictions on the upper limit of plasma density as was discussed above.

In the case of plasma near equilibrium, the energy is expressed similar to that for an ideal gas - $E_T = \frac{3}{2} n KT$. If the Coulomb force is not so strong, hence the interaction energy may be described by Debye theory-$E_c = 2\sqrt{\frac{\pi n^3}{KT}} e^3$. In other words, the condition of ideal plasma should be applied only if the electrostatic energy is a small correction to the thermal energy- $E_c << E_T$, provided that the number of electrons inside the Debye sphere is large

$$N_D = \frac{4\pi}{3} g^{-1} >> 1 . \qquad (6)$$

III. The BGK Approximation:

The relevant Boltzmann equation governing the present problem is reduced to an equation proposed by |Bhatnagar et al, 1954| and is called the BGK model equation

$$v_y \frac{\partial f}{\partial y} + |\underline{v} \times \underline{\Omega}| \,\Sigma_v f = -\nu(f - f_{LM}) \,. \qquad (7)$$

The electrons collision frequency ν is proportional to the number density in a plasma. One may write

$$\nu = \nu_1 \exp - \kappa'(1-y), \qquad (8)$$

where ν_1 is the collision frequency at the wall. The local Maxwell distribution function is

$$f_{LM} = n(y)(\alpha/\pi)^{3/2} \exp\left[-\alpha(\underline{v} - \underline{U})^2\right], \qquad (9)$$

U is the flow velocity.

It was proved by |Johnson,1982| that the deviation of density and temperature away from their equilibrium value are independent of the presence of shear flow, this permits one to linearise equation (7) by taking |Kogan,1969|

$$f_{LM} = f_o(1 + 2\,C_x U). \qquad (10)$$

The distance y and the velocity v are nondemensionalised such that $y = \tilde{y}\, d/2$ and $\underline{C} = \alpha_o^{\frac{1}{2}}\, \underline{v}$. The shear Mach number $M = \alpha^{\frac{1}{2}} W$. By puting $f = f_o(1+\phi)$: $\phi \ll 1$, and equations (8),(10) in the BGK equation (7), one obtains, the nondimensional linearized BGK equation for the problem. Droping ~ over y we get:

$$C_y\left[\frac{\partial \phi}{\partial y} - \kappa'(1+\phi)\right] - M^*(1+\kappa y)\left[C_y \frac{\partial \phi}{\partial c_x} - C_x \frac{\partial \phi}{\partial c_y}\right] = -\delta\left[\phi - 2C_x \tilde{U}\right]:$$

$$0 \le y \le 1 \qquad (11)$$

where $M^* = \alpha^{\frac{1}{2}}\Omega\, d/2$ designating the cyclotron Mach number,

$$\delta = \frac{M^* \nu_1}{\Omega} \exp{-\kappa'(1-y)} = \frac{M^* \nu_1}{\Omega} F \; ; \; F = \exp - \kappa'(1-y) . \tag{12}$$

Here κ', κ are multiplied by d/2 to be nondimensionalized. Equation (11) may be simplified further by writting |Kogan 1969|

$$\phi(y, \underset{\sim}{C}) = C_x \, \psi(y, C_y) \; . \tag{13}$$

The moment U is determined by $\underset{\sim}{\psi}$, by taking the scalar product with respect to the factor $e^{-C^2}/\pi^{3/2}$

$$U = \langle C_x | \phi \rangle \equiv \int C_x \, d\, \underset{\sim}{C} \, \frac{e^{-C^2}}{\pi^{3/2}} \, \phi$$

$$= \frac{1}{2} \int_{-\infty}^{\infty} \psi \, \frac{e^{-C_y^2}}{\sqrt{\pi}} \, dC_y \; , \tag{14}$$

and equation (11) becomes

$$C_y \left[C_x \frac{\partial \psi}{\partial y} - \kappa'(1 + C_x \psi) - M^*(1+\kappa y)\psi \right] + M^*(1+\kappa y) C_x^2 \frac{\partial \psi}{\partial C_y} =$$

$$= -\delta \, C_x \left[\psi - 2\tilde{U} \right] . \tag{15}$$

Our aim is to follow the behavior of the shear flow in y direction, therefore integrating equation (15) with respect to the weight factor $C_x \, e^{-C_x^2}/\sqrt{\pi}$, thus yields

$$C_y \left[\frac{\partial \psi}{\partial y} - \kappa \psi \right] = -\delta \left[\psi - 2\tilde{U} \right] , \tag{16}$$

as the basic equation to be solved. It is important to note that no approximations have been made in obtaining eq. (16) other than the well justified one of using the linearized BGK equation.

IV. Half-range polynomial approximation:

One may seek a solution of eq. (16) by using a half-range polynomial expansion technique originally suggested by |Gross et al 1957|. ψ is divided into two parts

$$\psi = \psi^{+} + \psi^{-} \quad (17)$$

where $\psi^{+}(\psi^{-})$ is that part of ψ describing upward (dowerward) going electrons . ψ^{+} and ψ^{-} are then expanded separately in polynomials in C_y, and an approximate solution to some specified order in these polynomials is sought. It is convenient here to use the set of independent (but not mutually orthogormal) polynomials C_y^n; thus one may write

$$\psi^{\pm}(y,C_y) = \sum_{n=0}^{\infty} B_n^{\pm}(y) C_y^n \,\Theta(\pm C_y), \quad (18)$$

Θ is the Heaviside function, with the properties

$$\Theta(C_y) = \begin{cases} 1: & C_y > 0 \\ 0: & C_y < 0 \\ \frac{1}{2}: & C_y = 0 \end{cases} \quad (19)$$

V. The Boundary Conditions:

The half-range nature of the expanssion is a way of incorporating into a polynomial expansion the discontinuity of the distribution function in velocity space which ordinarly occurs at the boundaries $y = \pm 1$.

The boundary conditions for the single component plasma is different but physically simpler than those suggested |Shedlovskiy 1967| for ordinary two conponent fully ionised plasma in that a fraction ε of the electrons hitting a surface leave that surface with a Maxwellian velocity distribution characteristic of that surface's velocity and temperature. The remaining $(1-\varepsilon)$ are assumed to undergo specular reflection. In terms of ψ, the boundary conditions become at the upper wall (y=1)

$$\psi^-(1,-C_y) = (1-\varepsilon)\psi^+(1,C_y) + \varepsilon M\, \Theta(-C_y) \tag{20}$$

$$\psi^+(1,C_y) = (1-\varepsilon)\psi^-(-1,-C_y) + \varepsilon M\, \Theta(C_y). \tag{21}$$

at the lower wall (y= -1)

$$\psi^+(-1,C_y) = (1-\varepsilon)\psi^-(-1,-C_y) - \varepsilon M\, \Theta(C_y) \tag{22}$$

$$\psi^-(-1,-C_y) = (1-\varepsilon)\psi^+(1,C_y) - \varepsilon M\, \Theta(-C_y). \tag{23}$$

VI. Explicit solution (zero order):

Following |Pomraning, 1963| and |Johnson,1983|, one may determine the coefficients of the solutions (18), by requiring that the solutions obey the moments

$$\int_{-\infty}^{\infty} dC_y \frac{e^{-C_y^2}}{\sqrt{\pi}} C_y^n\, \Theta(\pm C_y), \quad (n=0,1,\ldots,m) \ , \tag{24}$$

of the governing equation (16) and by truncating the sum in eq.(18) at the m th term. When m=0

$$\psi = B_o^+\, \Theta(C_y) + B_o^-\, \Theta(-C_y). \tag{25}$$

The nondimensional flow velocity is |cf eq.(14)|

$$U = \tfrac{1}{2} \int dC_y \frac{e^{-C_y^2}}{\sqrt{\pi}} \left[B_o^+\, \Theta(C_y) + B_o^-\, \Theta(-C_y) \right] = \frac{(B_o^+ + B_o^-)}{4}. \tag{26}$$

The pressure deviator is obtained from

$$P_{xy} = \frac{mn_1 F}{\alpha} \int d\underline{C}\, \frac{e^{-C^2}}{\pi^{3/2}}\, \phi\, C_x C_y$$

$$= \frac{mn_1 F}{\alpha} \frac{(B_o^+ - B_o^-)}{4\sqrt{\pi}}, \tag{27}$$

where from eq.(12): $F = \exp{-\kappa'(1-y)}$. (28)

The governing equation for this lowest order solution follows from eq. (16). By making use of the properties of the Θ-function given in eq. (19), one obtains

$$C_y\{\frac{dB_o^+}{dy}\,\Theta(C_y) + \frac{dB_o^-}{dy}\,\Theta(-C_y) - \kappa'[\,B_o^+\,\Theta(C_y) + B_o^-\,\Theta(-C_y)\,]\}=$$

$$= -\delta\{B_o^+[\,\Theta(C_y)-\tfrac{1}{2}\,] + B_o^-[\,\Theta(-C_y)-\tfrac{1}{2}\,]\,\}. \qquad (29)$$

The m=0 moments (24) of the governing eq.(29) give equations for $B_o^{\pm}$:

$$\frac{1}{2\sqrt{\pi}}[\,\frac{dB_o^+}{dy} - \kappa' B_o^+\,] = -\delta\,\frac{(B_o^+-B_o^-)}{4}$$

$$-\frac{1}{2\sqrt{\pi}}[\,\frac{dB_o^-}{dy} - \kappa' B_o^-\,] = \delta\,\frac{(B_o^+-B_o^-)}{4} \qquad (30)$$

The sum of eqs (30) gives

$$\frac{dh}{dy} - \kappa' h = 0 \quad ; \quad h \equiv B_o^+ - B_o^- \qquad (31)$$

so that

$$h = h_1 F^{-1} . \qquad (32)$$

The difference of eqs (30) gives

$$\frac{1}{2\sqrt{\pi}}[\,\frac{dg}{dy} - \kappa' g\,] = -\,\delta(\frac{B_o^+-B_o^-}{2}) \; : \; g \equiv B_o^+ + B_o^- . \qquad (33)$$

From which if follows that

$$g = g_1 \exp - \kappa'(1-y) - \frac{\sqrt{\pi}\delta_1 h_1}{\kappa'}[\,e^{-\kappa'(1-y)} - 1\,]. \qquad (34)$$

The constants of integration h_1 and g_1 may be evaluated from the boundary conditions (20-23)one obtains for the lowest order solutions that

$$B_o^-(1) = (1-\varepsilon)\,B_o^+(1) + \varepsilon\,M \qquad (20')$$

$$B_o^+(1) = (1-\varepsilon)\, B_o^-(-1) + \varepsilon\, M \tag{21'}$$

$$B_o^+(-1) = (1-\varepsilon)\, B_o^-(-1) - \varepsilon\, M \tag{22'}$$

$$B_o^-(-1) = (1-\varepsilon) B_o^+(1) - \varepsilon\, M \tag{23'}$$

solving one gets at the upper wall

$$h_1 = B_o^+(1) - B_o^-(1) = -\frac{2\,\varepsilon\,(1-\varepsilon)M}{(2-\varepsilon)} \tag{36}$$

$$g_1 = B_o^+(1) + B_o^-(1) = 2\,\varepsilon\, M \tag{37}$$

Comparison of (26) and (27) shows that

$$P_{xy} = \left(\frac{mn_1}{4\sqrt{\pi}\alpha}\right) Fh = \frac{mn_1}{4\sqrt{\pi}\alpha}\, h_1 = P_{xy1} = \text{ constant.} \tag{38}$$

The direct integration of eq. (16) with respect to the weight factor $C_x^2 \frac{e^{-C^2} dC}{\pi^{3/2}}$, after some labor, leads to the solution

$$P_{xy} = P_{xy1} e^{\kappa'(1-y)} \quad : 0 \le y \le 1. \tag{39}$$

It is seen from eq.(38) that the law of conservation of shear monetum is obtained, this is an advantage of using the half range polynomial expansion. The same result could be determined from eq.(39) by rigorously demanding that

$$-\kappa' << 1 , \tag{40}$$

i.e. now, the condition of small inhomogenity for both megnetic field and density numbers is Justified. This allows one to linearize eq. (34),

$$g = g_1 - [\, \kappa'\, g_1 - \sqrt{\pi}\, \delta_1 h_1 \,](1-y), \tag{41}$$

and the dimensional flow velocity $U_x(y) = \frac{1}{\sqrt{\alpha}}\, U$ is

$$U_x(y) = \frac{1}{4\sqrt{\alpha}} g = \frac{\varepsilon M}{d\sqrt{\alpha}} \{d/2 - [\kappa' + \sqrt{\pi}\, \delta_1 (\frac{1-\varepsilon}{2-\varepsilon})] (d/2-y)\}, \qquad (42)$$

(y here is dimensional). Eqs. (38) and (42) display the familier relation.

$$P_{xy} = - \mu \frac{dU_x}{dy} , \qquad (43)$$

by which the coefficient of visocity amounts to

$$\mu = \mu_1 \left[1 + \frac{\kappa'}{\sqrt{\pi}} (\frac{2-\varepsilon}{1-\varepsilon}) Kn_1 \right]^{-1} . \qquad (44)$$

Here Kn_1 is the Knudsen number at the wall,

$$\mu_1 = \frac{mn_1}{\pi\alpha\nu_1} .$$

VII. Discussions and comparisons:

1- It is helpfull to bear in mind orders of magnetudes for varios quantities of interest in electron plasma.

Typical values may be taken to be

$\varepsilon = 0{,}9$, $T = 577\ K^o$, $n \sim 10^{10} cm^{-3}$ corresponds to the degree of rarefaction in the transition region $\delta \sim 1$.

$d/2 = 1$ cm , $H_o = 0{,}1$ Gauss, $\Omega = 0{,}178 \times 10^7$ rad/sec,

$c = 3 \times 10^{10}$ cm/sec.

$m = 9 \times 10^{-28}$ gm, $e = 4{,}8 \times 10^{-10}$ cgs, $K = 1{,}38 \times 10^{-16}$ cal/deg,

$\nu_1 = 10^5 sec^{-1}$. Shear spead W will be considered in the order of 10^5 cm/sec, which can be reached in plasma laboratories or observed in astrophysical investigations.

These values correspond to

$\alpha^{-\frac{1}{2}} \sim 1{,}33 \times 10^7$ cm/sec $= v_T$, $g = 2 \times 10^{-2}$, $M^* = 0{,}134$,

$M = 0{,}75 \times 10^{-2}$, $D = 6{,}9 (\frac{T}{n})^{\frac{1}{2}} = 1{,}657 \times 10^{-3}$ cm,

$a \sim (\frac{1}{n})^{1/3} = 4{,}6 \times 10^{-4}$ cm , $N_D \sim 200$.

2- The evaluation of the relative gradients κ, κ' are controlled by several factors, let us prove it!

At y=0, i.e. along x-axis , the rate $\frac{dH}{dy}$ may be deduced using Maxwell's equations:

$$-\left.\frac{dH}{dy}\right|_{y=0} = \frac{4\pi J_o}{C} = -\frac{4\pi e n_o U_x(0)}{C} ,$$

substituting from eq. (44) at y=0 and dividing both sides by H_o,

$$\frac{1}{H_o}\frac{dH}{dy} = \frac{4\pi\ e n_o}{CH_o}\frac{\varepsilon M}{2\sqrt{\pi}}\left[\ 1 - \kappa' - \sqrt{\pi}\ \delta_1 \left(\frac{1-\varepsilon}{2-\varepsilon}\right)\ \right] = \kappa . \tag{47}$$

Making use of the relations

$$\frac{H^2}{4\pi n_o kT} = \frac{2}{\beta} , \quad \frac{m}{2\alpha} = \frac{mv_T^2}{2} = KT ,$$

we get after some manipulations that

$$\kappa = \frac{\beta}{2}\ \varepsilon\ MM^*\left[\ 1 - \kappa' - \sqrt{\pi}\ \delta_1\left(\frac{1-\varepsilon}{2-\varepsilon}\right)\ \right]. \tag{48}$$

If $\beta=2$, then as was discussed before $\kappa = -\kappa'$, so from eq.(48), and the above numerical values

$$\kappa' = \frac{\varepsilon MM^*}{\varepsilon MM^* - 1}\left[\ 1 - \sqrt{\pi}\ \delta_1\left(\frac{1-\varepsilon}{2-\varepsilon}\right)\ \right]$$

$$= -\ 7{,}45 \times 10^{-4} \tag{49}$$

Thus we see that four independent quantities namely M, M^*, δ, and ε control the relative gradient κ'.

3- a) Two factors affect the coefficient of thermal viscosity. First, in the denomenator of eq.(46) as $\kappa' Kn_1 = -\frac{\ell}{n}\frac{dn}{dy}$ increases, μ decreases. Second, as the quantity $\kappa Kn_1 = \frac{\ell}{H}\frac{dH}{dy}$ increases, μ decreases also. This is consistent with the notion of "magnetic tray" which is in our case the magnetic confinment of plasma between the two boundaries.

b) On the other hand, according to the above numerical values, we get that $\mu = 8 \times 10^{-9}$ poise, where as |Shedlovskiy, 1967|

$$\mu_c = \frac{5}{8A_2(2)} \sqrt{\frac{mKT}{\pi}} \left(\frac{2KT}{e^2}\right)^2 = 6 \times 10^{-11} \text{ poise ; here}$$

$$A_2(2) = 2\left[\ln(1+v_{01}^2) - \frac{v_{01}^2}{2+v_{01}^2} \right] \simeq 24 \; ; \; v_{ol} = \frac{4aKT}{e^2} = 642.$$

Hence

$$\mu >> \mu_c$$

i.e. in agreement with Debye approximation.

4) From eq.(44), as plasma electrons possess considerable randum thermal motion, so firstly the enitial electron velocity is caused by the movement of the boundary walls, and secondly by the external magnetic field. This result is analogous to that presented by |Chakraborty,1978| in the case of Couette flow of electrically conducting incompressible and viscous fluids.

5) Due to the inhomogeneous magnetic field, a transverse drift gradient for the individual electrons is produced |Frank-Kamenetskiy, 1968| , its velocity equals to

$$\underline{V}_d = c \frac{mv_{\perp}^2}{2eH_o^2} \left|\frac{dH}{dy}\right| \hat{\underline{x}} . \qquad (50)$$

This expression can be simplified if we introduce, without loss of generality, v_T instead of $v_\perp$ and making suitable substitutions we get:

$$V_d = \frac{\kappa d}{4M^*} v_T$$
$$= 3,7 \times 10^4 \text{ cm/sec} << v_T, \qquad (51)$$

in agreement with the condition of weak variation of magnetic field mentioned by |Arsimovitch et al,1972|. comparing this result with the mean velocity (42) at $y = 0$ and $y = d/2$:

$$U_x(0) = 3,77 \times 10^4 \text{ cm/sec} \quad , \quad U_x(d/2) = 4,5 \times 10^4 \text{ cm/sec}$$

which indicate that

$$V_d < U_x(0) < U_x(d/2) .$$

On the other hand the slip velocity at $y = d/2$

$$V_s = \frac{W}{2} - U_x(d/2) = 5 \times 10^3 \text{ cm/sec}$$

is less than the drift velocity by one order of magnitude.

In conclusion , from the physical point of view the gained results are a step towards a deeper understanding of the interaction of plasma with boundary surfaces.

References

Abdel-Gaid M.A.: Ph.D. Thesis- Assuit University. EGYPT.(1972).

Akhiezer A.I. et al:"Plasma electrodynamics". Nauka, Moscow(1974). In Russian.

Arsimovitch L.A. et al:"The motion of charged particles in electric and magnetic fields". Nauka, Moscow (1972). In Russian.

Bhatnagar P.L. et al: Phys. Rev. 94,511,(1954).

Chakraborty B.:"Principles of plasma mechanics",Wiley eastern, New Delhi, (1978).

Frank-Kamenetskiy D.A.:"Lectures in plasma physics".Atomizdat, Moscow (1968). In Russian.

Gross E.P. et al: Ann. Phys., 1,141,(1957).

Isihara A.: "Statistical physics", Academic P.,N.Y. (1971).

Johnson E.A.: AERE Harwell-R 10461,(1982).

Johnson E.A. et al: Phys. D.: Appl. Phys.,16,1207,(1983).

Kogan M.N.:"Principles of gas dynamics", Plenum, N.Y. (1969).

Krall N.A. et al:"Principles of plasma physics", McGraw-Hill, N.Y.(1973).

Mahmoud M.A.: Ph.D. Thesis, Menoufia University, EGYPT (1985).

Marshall T.C.:"Free electron lasers", Macmillan, N.Y. (1985).

Pomraning G.C.: General Electric Technical Information Series, Rept. No APED-4374, 63 APE 22.

Shidlovskiy V.P.:"Introduction to the dynamics of rarefied gases". Elsevier, N.Y. (1967).