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STIFFNESS OF THE FLAT JOINT LOADED BY SHEAR AND TORSIONAL LOADS.

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ABSTRACT

This paper is concerned with the flat joint subject to the shear and torsional loads.The objective is to approach as closely as possible to some appropriate mathematical analysis for the most important parameters . These parameters are;Loads,Quality,Geometry and Dimensions of the mating surfaces.Several assumptions have been taken into consideration to simplify the proposed mathematical model.The advantage of the model is to get the optimal flat joint design in a minimum time and less effort.Calculation of the joint stiffness and some basic optimum values would then be within the reach of the designers.

INTRODUCTION

The relationship between the normal stress" σ_n "and the normal deformation" δ_n" of the two mating surfaces can be written as:

$$\delta_n = \alpha \circ \sigma_n^m \qquad (1)$$

Where α and m are constants depending upon the type of materials and the surface finish. [6]

The shear compliance was analysed by Kirsanova [5] and for repeated loading;the relationship between shear stress" σ_s"and shear deformation" δ_s"was given by:

$$\delta_s = K_s \cdot \sigma_s \qquad (2)$$

Where K_s is the shear factor in μm mm^2/N.Also, he reported that the shear compliance of the machined surfaces is dependent upon the surface finish and that it decreases with the increase of the normal stress. From the results reported in Refs, [1,5] it can be seen that the relationship between the shear compliance and the normal stress can be written as;

$$K_s = R / \sigma_n^s \qquad (3)$$

Where R and s are constants dependent on the joint materials and surface finish. To determine the static stiffness properties of the flat joint, it is necessary to know all design parameters influencing the stiffness magnitude. These parameters my be :

- Material properties of the parts to be connected.
- Roughness and flatness of the machined mating surfaces.
- Dimensions and geometry of the joint contact surface.

The parameters affecting the stiffness of the machined surfaces have been analysed before in Refs. [1,4,7,8] It was found that, there is no previous work to compute the static joint stiffness due to the normal, shear and torsional loads. Thererfore, the objective of this paper is mainly to approach as closely as possible to some appropriate mathematical model to calculate the stiffness using design parameters of the joint.

NORMAL STIFFNESS OF THE JOINT

1- Factors Affecting the Stiffness.

From Eq.(1) it can be seen that the magnitudes of the constants α and m influence the normal static stiffness. These constants depends on;

a-The mating surface material : In general, when different materials are used, the constant α of Eq.(1) varies inversely with the modulus of elasticity of the material, while the constant m remains at approximatelly 0.5 [1] .

b-Surface finish and lay orientation: The roughness of the surface of a given material depends on the machining process and cutting conditions. Theoretical models of contact conditions have been considered and the results showed that the surface stiffness is increased with the reduction in surface roughness. Experimental investigation [2] using mild steel specimens showed that, the stiffness is inversely proportional to the surface roughness.

c-Hardness: The effect of hardness on stiffness has been investigated using C.I specimens at relatively low interface pressures. The results indicated that the hardness had no effect on surface stiffness. A similar investigation was carried out [3] which included different surface finish and contact stress up to 160 N/mm^2. It appears that elastic stiffness increases with decreased hardness.

d- Flatness deviation and surface size: In practice the flatness deviation of a surface will increase with the size of the surface. An investigation [4] on mild steel specimens of constant area in the form of an annulus, with different outside dimeters showed that the flatness deviation is increased with the specimen. Recent work by Schofield [7] suggested that the stiffness of large surface which were subjected to flatness deviation, could be increased by the use of rougher surfaces becouse it will be less sensitive to flatness deviation.

2-Mathematical Model :

It is apparent from the factors discussed previously that the stiffness would be proportional to the apparent area of the contact surfaces provided that the surfaces are flat. Therefore, the stiffness of a flat joint can be determined directly from the dimensions and the geometry of the joint contact surfaces. The exponential relationship (1) between the normal stress and the normal deflection may be used to compute the normal stiffness, where;

$$C_n = \partial F / \partial \delta = \sigma_n^{1-m} . A_F / \alpha \cdot m \qquad (4)$$

For hand-scraped contact joint surface (cast-iron) with h=6-8 um and z=2-3 Spots/cm^2 as usually used in machine tools, the constant m may be taken about 0.5 and when one of the joint surfaces recessed (see Fig.1), the normal stiffness may be calculated by the following equation.

$$C_n = 2\,\sigma_n^{0.5} . a_f . b_f (1-R_a . R_b) / \alpha \qquad (5)$$

JOINT STIFFNESS UNDER SHEAR LOAD

Figure 1 shows the geometrical model of the joint. To derive the proposed mathematical model, it may be assumed that, the deflection of the elements to be connected is smaller than the joint shear deformation. It may, therefore, be neglected. Equation (2) presents the linear relationship between the deformation and applied shear stress, which may be calculated as;

$$\sigma_s = F_s / A_F \qquad (6)$$

Therefore, the shear stiffness of the joint may be calculated with the following linear equation;

$$C_s = F_s / \delta_s = A_F / K_s \qquad (7)$$

The factor K_s is dependent on the normal stress, therefore, the shear stiffness may be calculated with the following equation.

$$C_s = a_f . b_f . \sigma_n^{0.5} (1-R_a . R_b)/R \qquad (8\text{-a})$$

or,

$$C_s = \left[a_f . b_f . F(1-R_a . R_b) \right]^{0.5} /R \qquad (8\text{-b})$$

Figures 2and 3 represent the relationship between the shear stiffness and the joint recess ratios(R_a, R_b).It can be seen that, the shear stiffness of the connected elements increases with the reduction of the joint recess ratio.If the normal stress is constant, the relatio is linear.It is clear also that, the recess ratios R_a and R_b have no position effect on the stiffness value.When the normal stress in the joint increases through the decreasing of the joint area, the stiffness decreases gradually.At the constant Force and $R_a = R_b = 0.6$ the decrease in the stiffness value may be about 20% with respect to the stiffness of a closed joint.

TORSIONAL STIFFNESS OF THE JOINT

To derive the mathematical model which gives the value of the torsional stiffness, the following assumptions have been taken into consideration:

- No plastic deformation occurred in the cross-section of the jointing surface due to the applied torsional stress, i.e.all deformation are elastic.
- The resultant normal stress in the jointing surface is equally distributed.
- The torsional deformation due to the shearing stress in the jointing surface can be computed by the relation(1) after Kirsanova [5] i.e. $\delta_t = \delta_s$.

  Where, δ_t is the tangential deformation due to the shear stress in the jointing surface(μm.)

Figure 4 shows the elementary area($dA_F = r.dr.d\phi$) of the jointing surface subject to the torsional stress; where, r is the radial distance of the elementary area w.r.t. the torsional centroidal point.The torsional moment about point O may be derived from Fig.4 using the following expression;

$$dM_t = \tau_t . r^2 . dr . d\phi \qquad (9)$$

The magnitude of the torsional stress τ_t may depend on the value of the tangential joint stiffness and the displacement of the elementary area which subject to the shearing stress. This displacement is dependent on the radius r i.e.,

$$\delta_t (r) = r. \ \phi_t \tag{10}$$

Thus, the torsional stress can be computed as:

$$\tau_t = r. \ \phi_t \ / \ K_s \tag{11}$$

The values K_s and ϕ_t are independent of the radius r and the polar angle ϕ. Therefore, by substituting the value of τ_t into Eq.(2) and integrating the equation on the jointing surface area, we have;

$$M_t = \phi_t / K_s \ . \int_0^{2\pi} \int_0^{\phi} r^3 . dr . d\phi \tag{12}$$

The integration represents a polar moment of inertia of the joint area. For the rectangular jointing surface (as usually used in the machine tool joints) with length a_f and width b_f (see Fig.1), the polar moment of inertia can easily be computed as:

$$I_p = a_f . b_f (a_f^2 + b_f^2) \ /12 \tag{13}$$

Accordingly, the torsional stiffness may be computed by the following linear equation.

$$C_t = M_t \ / \ \phi_t = I_p \ / \ K_s \tag{14}$$

For the rectangular jointing surface with a rectangular recess (see Fig.1), the following mathematical model may be useful to compute the stiffness.

$$C_t = \sigma_n^{0.5} . a_f^2 \ . b_f^2 \ / \ 12 \ R \ . \left[a_f / b_f + b_f / a_f - R_a^2 . R_b^2 \ (R_a . a_f / R_b . b_f + R_b . b_f / R_a . a_f) \right] \tag{15}$$

Or, at the constant normal force F is ;

$$C_t = \left[F \cdot a_f^3 . b_f^3 / (1 - R_a . R_b) \right]^{0.5} \left[a_f / b_f + b_f / a_f - R_a^2 . R_b^2 \ (R_a . a_f / R_b . b_f + R_b . b_f / R_a . a_f) \right] /12R \tag{16}$$

Where; $R_a = a_{fi} / a_f$ and $R_b = b_{fi} / b_f$.

DISCUSSION OF THE MODEL

Figure 5 shows the analysis of Eq.(15). It can be seen that:

- At the ratio $b_f / a_f = 1$ (square jointing surface), the torsional stiffness is minimum.
- At the ratio $b_f / a_f = 0.64$ to 1.56 the torsional stiffness is improved only 10 % than the minimum, i.e. at $b_f = a_f$.
- The size of the recess has no effect on the position of the minimum torsional stiffness, but it affects the stiffness magnitude .

The relationship between the torsional stiffness, torsional moment and the geometrical parameters of the jointing surface (b_f, a_f, R_a and R_b) can also be represented asshowen in Fig.6 for constant normal stress and in Fig.7 for constant normal force. At constant normal stress and the ratio b_f/a_f equal one or two, the torsional stiffness decreases when the size of the joint recess increases. Also, it can be seen that, at the high ratios of the R_a with low ratios of the R_b, the torsional stiffness decreases slowly than with high ratios of the R_b .

At constant normal force F (Fig.7), the decrease of the contact surface of the joint with the increasing of the size of jointing recess produecs high normal stress in the joint and therefore, the tangential contact stiffness and the torsional stiffness are increased. But, during the continued decreasing of the torsional stiffness is decreased due to the decrease of the polar moment of inertia. It can be seen that, the torsional stiffness has an optimum condition occurs at the geometric ratio $b_f/a_f = 1$ with the jointing recess $R_a = R_b \approx 0.6$. This optimum condition has also torsional stiffness magnitude which is greater than that of the complete jointing surface around 8.8% . At the high ratios of $b_f/a_f > 1$, the optimum conditions of the torsional stiffness may be still at the small ratios of R_b and the high ratios of R_a. It can be seen that, the optimum condition for the case $b_f/a_f = 1,5$ occurred at $R_a \approx 0,8$ and $R_b \approx 0,5$ and for the case $b_f/a_f = 2$ occurred at $R_a = 1,0$ and $R_b \approx 0,4$.

CONCLUSION

The stiffness of the flat joint is greatly affected by joint parameters; they are the loads, quality, geometry and dimensions of the jointing surface. The joint recess has a pronounced effect on the joint stiffness, it increases as the recess size increases. Based on the results of this work, the following optimum conditions may be recommended :

- For the joint subject to the shear stress only; The recess ratios R_a and R_b have no position effect on the stiffness magnitude. A closed joint has the maximum stiffness value, but the designers may better use $R_a = R_b = 0{,}6$ to reduce the difficulties in the joint production. In this case , the stiffness is decreased about 20 % .
- For the joint subject to the torsional moment;
  1. Rectangular jointing surface gives better values of the torsional stiffness at the ratio $0{,}64 > b_f/a_f > 1{,}56$.
  2. At a constant normal stress with the ratio $b_f/a_f=2$, the torsional stiffness decreases with the increase in the size of jointing recess.
  3. At a constant normal force, the torsional stiffness has an optimum conditions, w.r.t. the recess size. They are:
     - at $b_f/a_f=1$, the recess ratio is $R_a=R_b \approx 0{,}6$
     - at $b_f/a_f=1{,}5$, the recess ratio is $R_a \approx 0{,}8$ and $R_b \approx 0.5$.
     - at $b_f/a_f=2$, the reces ratio is $R_a \approx 1; R_b \approx 0{,}4$.

Results obtained from this work require an experimental study to justify and confirm the computed stiffness values. The effect of the elasticity, shear mudulus and the surface quality on the shear constant should be experimentally investigated.

NOMENCLATURE

A_F	Joint area (mm^2).
a_f	Length of the mating surface(mm).
a_{fi}	Length of the jointing recess(mm).
b_f	Width of the mating surface (mm).
b_{fi}	Width of the jointing recess (mm).
C_n	Normal stiffness (N/μm).

C_s Shear stiffness ($N/\mu m$).
C_t Torsional stiffness($N.m / \mu rad.$)
K_s Shear constant (mm^3/N).
M_t Torsional moment (N.m).
r Radial distance of the elementary area(mm.).
ϕ Polar angle of the elementary area(rad.)
ϕ_t Torsional angle of the jointing surface(rad.).
δ_n Normal deformation(μm.).
δ_s Shear deformation (μm.).
δ_t Torsional deformation (μm.).
σ_s Shear stress (N/mm^2).
σ_n Normal stress (N/mm^2).
τ_t Torsional stress (N/mm^2).

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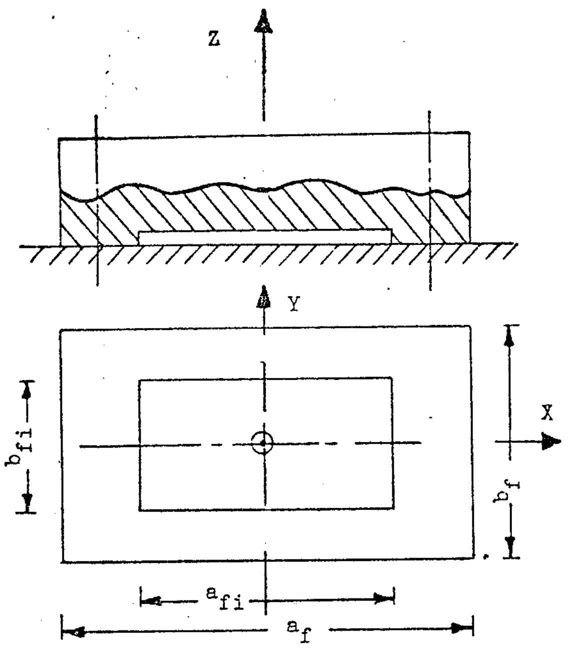

$R_a = a_{fi}/a_f$, $R_b = b_{fi}/b_f$

$A_F = a_f \cdot b_f \ (1 - R_a . R_b)$

Fig. 1 Flat Joint Model.

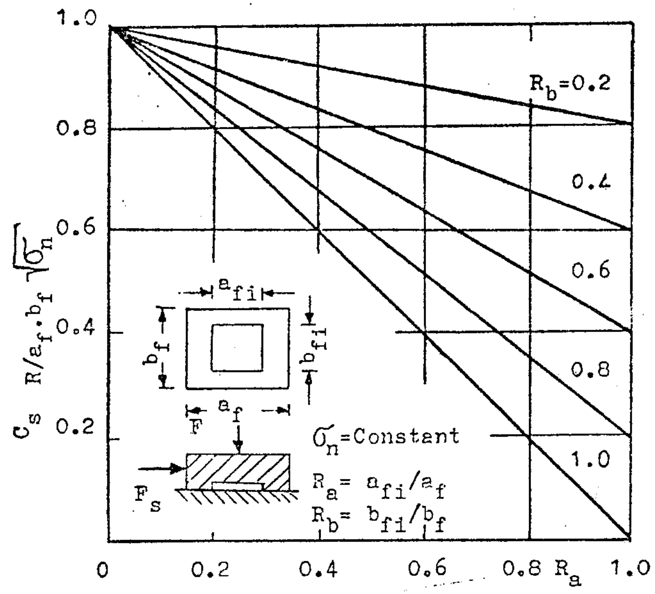

Fig.2. Effect of the Joint Recess.

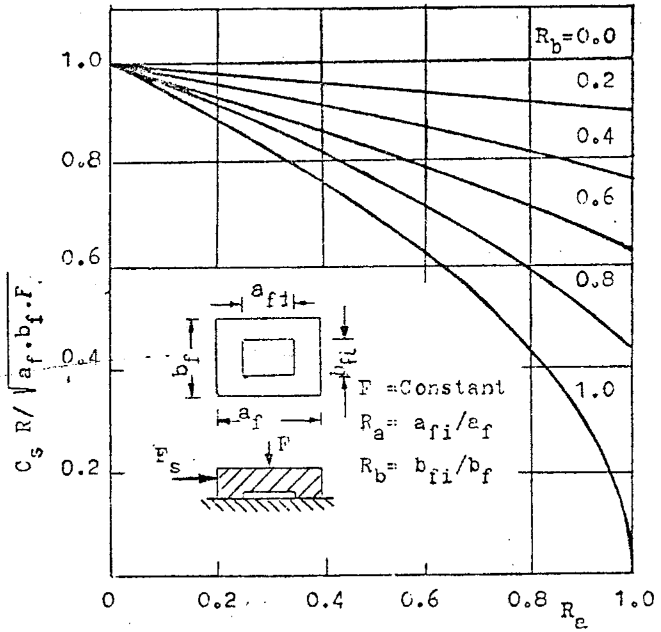

Fig.3. Effect of the Joint Recess.

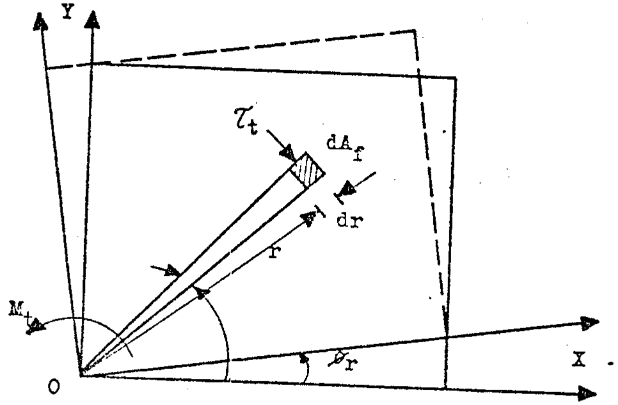

Fig. 4 Deformation of the elementary joint area due to the torsional moment .

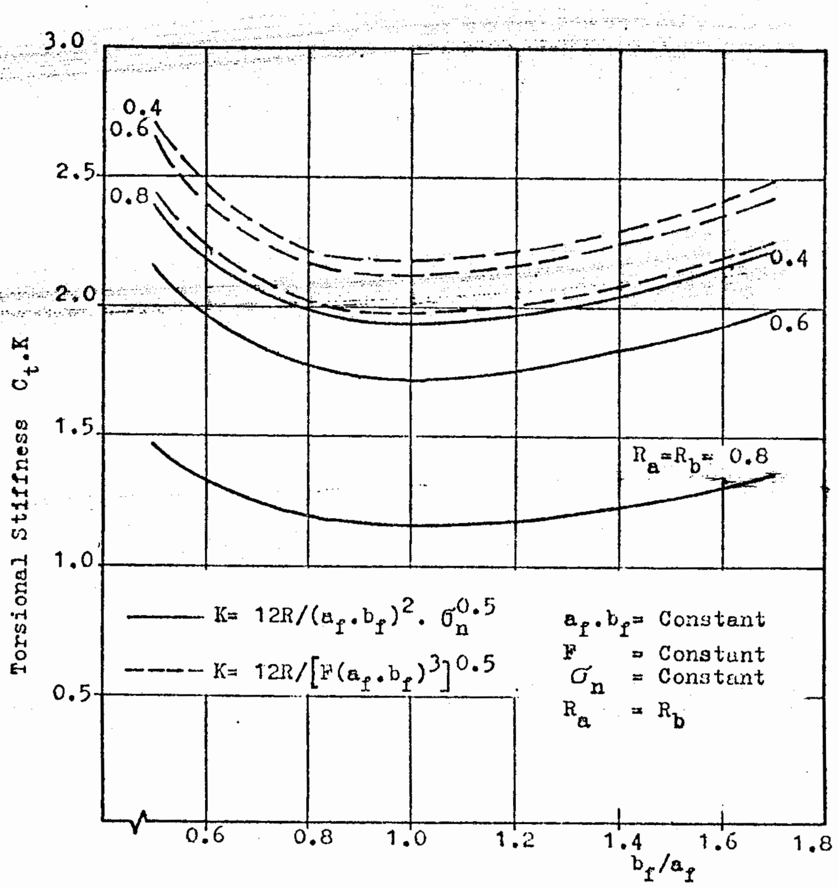

Fig.5 The relationship between the torsional stiffness and the ratio b_f/a_f .

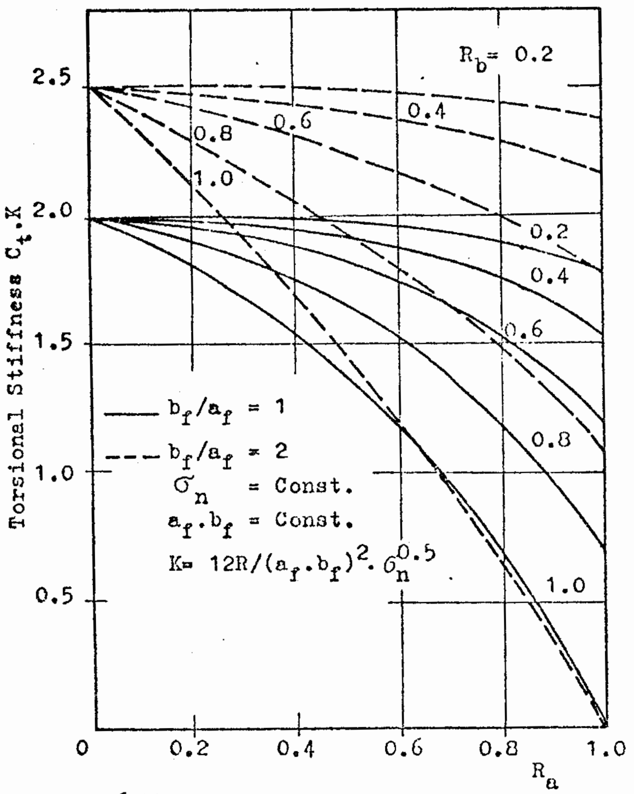

Fig. 6 The effect of recess size on the torsional stiffness at the constant normal stress.

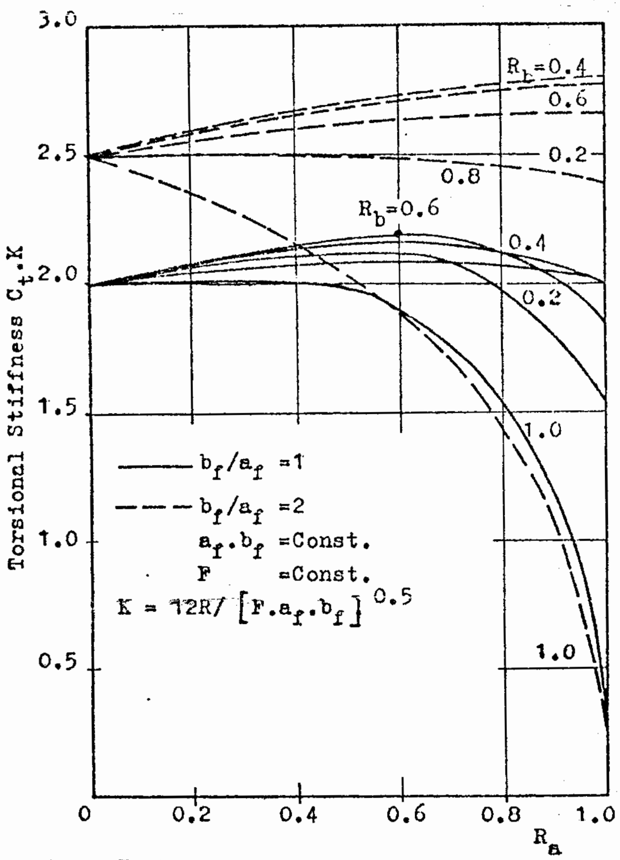

Fig. 7 The effect of recess size on the torsional stiffness at constant normal force .