Mansoura University Faculty of Engineerig First year Elec. power	(اولى كهرباء قوى) Time 3 hr. (Date: Jan., 2013)	Final Semester Exam First semester: 2012-2013 Math. 3

Answer the following questions [Full Marks 110]

Question 1 [30 Marks]

(a) Find the orthogonal trajectory to the family of curves $y = c\,e^{\tan^{-1} x}$ [5 marks]

(b) Are the two functions e^{2x} *and* e^{-2x} linearly independent? Explain why? [3 marks]

(c) Find the general solution to the Bernoulli D.E. $xy' - 4y = x^2\sqrt{y}$ [5 marks]

(f) Find the general solution to the Cauchy – Euler equation $xy'' - xy' = x^2$. [7 marks]

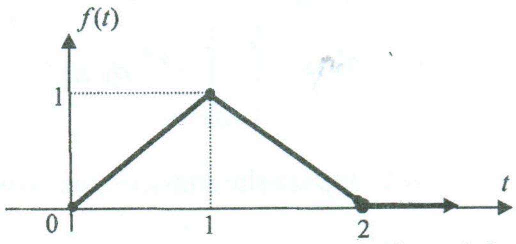
(d) Find the particular solution (y_p) to the fourth order D.E. $(D^4 + 3D^3 - 5D^2 + D - 4)y = 8x + 1$

[5 marks]

(e) Find the complementary solution (y_c) to the third order D.E. $(D^3 + 2D^2 + 4D)y = \tanh^{-1} x$.

[5 marks]

Question 2 [25 Marks]



(a) Find Laplace transform to: (i) The signal shown in Fig.

(ii) $f(t) = t\,e^{-2t}\sin 3t$ [6 marks]

(b) Find the inverse Laplace transform to: (i) $F(s) = \dfrac{s^2}{s^2+9}$ (ii) $F(s) = \ln\left(\dfrac{s^2}{s^2+9}\right)$ [5 marks]

(c) Evaluate the improper integral: $\int_0^\infty \frac{1}{t} e^{-\sqrt{3}\,t} \sin t \, dt$. [5 marks]

(d) Prove that if $L(f(t)) = F(s)$ then, $L(f'(t)) = sF(s) - f(0)$. [4 marks]

(e) Use Laplace transform to obtain $u(t)$ and $v(t)$ satisfying the system of equations:

$$\frac{du(t)}{dt} - v(t) = 0, \qquad u(0) = 0,$$

$$2u(t) + \int_0^t v(t-\tau)\,d\tau = 3t^2.$$

[5 marks]

Good luck (أنظر خلف الورقة)

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Question (3) [25 marks]

(a) Complete the following sentences:

(i) The quantity $\lim\limits_{\alpha \to 0} \dfrac{f(x, y+\alpha) - f(x, y)}{\alpha}$ is called ...... and its physical meaning is ......

(ii) At a point P, $\nabla f\,|_P \odot (\cos\theta\, \boldsymbol{i} + \sin\theta\, \boldsymbol{j})$ is called...... and its physical meaning is ......

(iii) The zero change of a differentiable function at point P occurs in the direction ....

(iv) The quantity $\lim\limits_{\Delta x \to 0, \Delta y \to 0} \sum\limits_{i=1}^{n} \sum\limits_{j=1}^{m} f(x_i, y_j)\, \Delta x\, \Delta y$ is ....

(v) In Figure(1), the curve tangent to the family circles is called ....

(vi) The sign of the divergence of the vector field shown in Figure (2) at Point P is ...... and the point is called ...... and the sign at point M is .... and the point is called........

(vii) The vector field shown in Figure (2) is irrotational at point......... because.........

(b) Find an approximate solution of the equation $y = 0.01\cos(1+y) + x$

(c) Find the extreme values of the function $f(x, y) = x^2 + 2y^2$ on the circle $x^2 + y^2 = 1$.

Question (4) [30 marks]

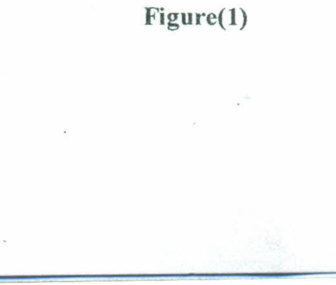
(a) Evaluate $\displaystyle\int_0^1 \int_{\sqrt{x}}^1 e^{y^3}\, dy\, dx$

(b) Compute the mass of the solid bounded by the planes $x = y^2 + z^2$ and $x = 16$ if the density function is given by $\rho(x, y, z) = \sqrt{y^2 + z^2}$

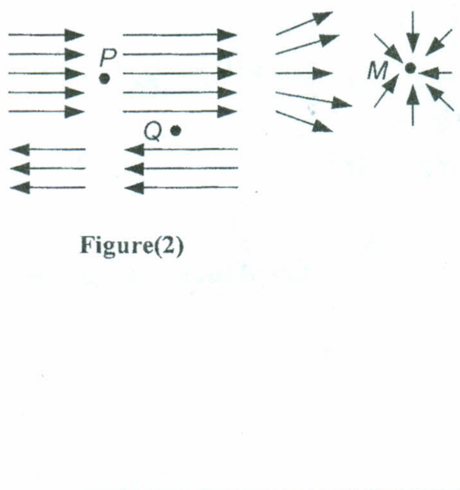
(c) Find the flux outward of the surface of the sphere $x^2 + y^2 + z^2 = 9$ of the vector filed

$$\vec{F}(x, y, z) = \left(x + z\frac{y^x}{\ln(y)}\right)i - \frac{zy^{x+1}}{x+1}j + (4z + yx)k \cdot$$

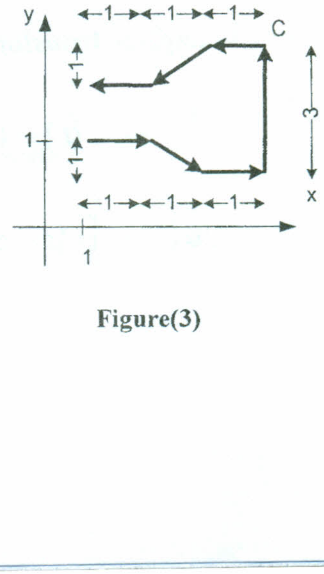
(d) Evaluate the work done by the vector field $\vec{F}(x, y) = (e^x \tan^{-1} x + y)i + 4xj$ in moving a particle along the curve C shown in Figure (3).



Figure(1)



Figure(2)



Figure(3)

Mansoura University		First Semester
Faculty of Engineering	حمر اكارج	Jan.-2013
Department of Engg. Math. and Phys.		Time: 3 hr
First year	Math(3)	Full mark(110)

[1]-(a) [**10 pts**] Solve by any method

1. $x^2\, y'' + x\, y' + y = 0$
2. $(6\, x\, y - y^3)\; d\, x + (4\, y + 3\, x^2 - 3\, x\, y^2)\; d\, y = 0$

(b) [**5 pts**] Find the orthogonal trajectories of

$$x^2 + y^2 = a\, y, \quad a \quad \text{is an arbitrary constant}$$

(c) [**5 pts**] Set up the appropriate form of a particular solution y_p (**Undetermined Coefficients**), but **DO NOT** determine the values of the coefficients

$$y^{(5)} - 3\, y^{(4)} - y^{(3)} + 11\, y'' - 12\, y' + 4\, y = \left(e^x + e^{-3\,x} + x\right)\; e^x$$

(d) [**5 pts**]Determine the shape of the deflection curve of a uniform horizontal beam of length L and weight w per unit length and fixed at $x = 0$ and free at its other end.

[2]-(a) [**12 pts**] Find the Laplace transform of the following functions

$$f_1(t) = \frac{\cos\, 3\, t - \cos\, 2\, t}{t}, \qquad f_2(t) = t\, e^{-t}\, \sin^2\, t,$$

$$f_3(t) = e^{-t}\, \sinh\, t\, \cos\, 2\, t$$

(b) [**8 pts**] Find the inverse Laplace transform of the functions

$$F_1(s) = \frac{2\, s\, (e^{-3\, s} - e^{-2\, \pi\, s})}{s^2 + 10}, \qquad F_2(s) = \ln\, \frac{s^2 + 1}{s^2 + 9}$$

(c) [**5 pts**] Find the current for all values of $t \geq 0$ in the **LC** circuit

$$L\, \frac{d\, I}{d\, t} + \frac{Q}{C} = E(t)$$

with $I(0) = I'(0) = 0$. Using the given data

$$L = 1\, H, \qquad C = 0.04\, f, \quad \text{and} \quad E(t) = 100\, v$$

Assoc. Prof. Dr. El-Gamel

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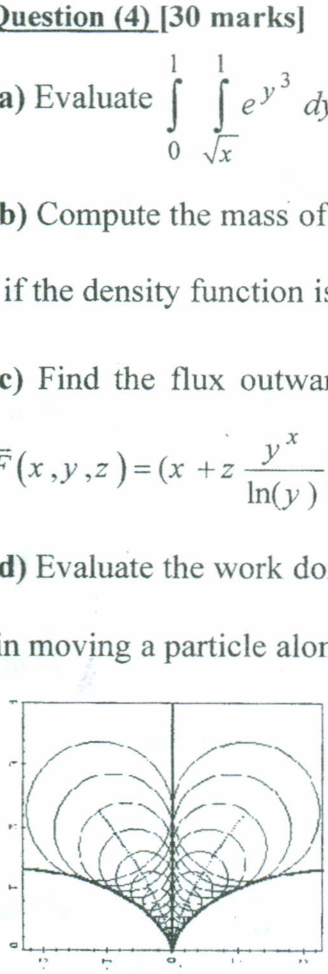
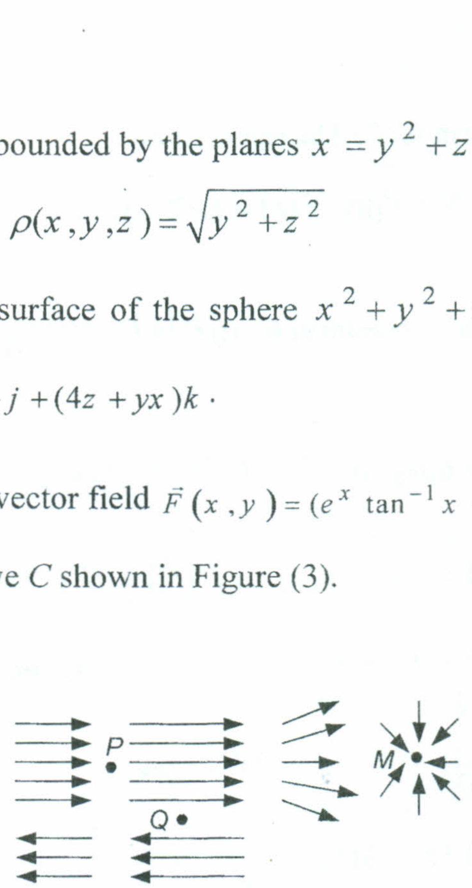
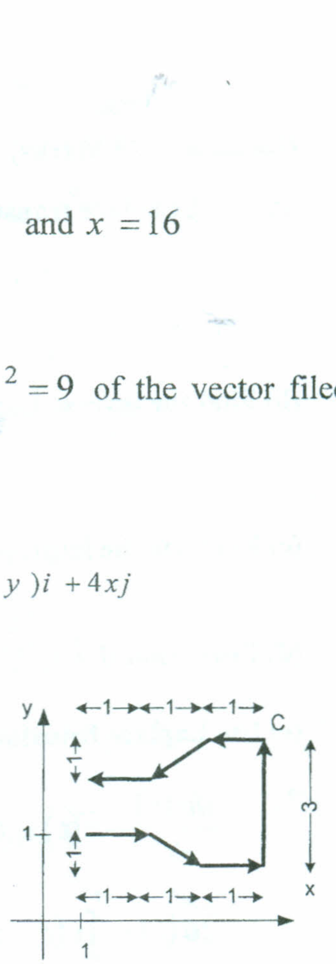


Figure(1)



Figure(2)



Figure(3)