" A NEW MATHEMATICAL ANALYSIS OF THE INFLUENCE OF MAIN DIMENSIONS OF SMALL POWER THREE PHASE INDUCTION MOTORS ON THEIR CHARACTERISTICS"

BY

Sayed A. Hassan* and Mohamed Helmy El-Maghraby**.

ABSTRACT

The ratio of stator length to the bore diameter (ξ) of small power poly-phase induction motors affects to some extent the technical and economical characteristics. The value of(ξ) has usally a ratio between 0.6 and 1.4, however, it is shown in the present analysis that increasing this ratio to a value around 2 improves these characteristics. On the other hand, decreasing his ratio less than 1.0 worsens these characteristics. Therefore, in designing small power polyphase induction motors, it is favourable to take the ratio (ξ) between 1 to 2, to obtain a lighter motor with improved characteristics.

I. INTRODUCTION

In designing electrical machines, even with known values of mognetic induction in air gap (B_g) and specific electric loading (A_1), there are many choices for the selection of the relation or ratio between the stator length (L_1) and the stator bore diameter (D_1). In (2), for example, this ratio eccurs in the region from 0.6 to 1.4, and in (1) from 0.7 to 1.6 for $2p=2$ and from 0.6 up to 1.2 for $2p = 4$.

To the authors's knowledge, the effect of (ξ) on the characteristics of electric motors has-tillnow-not enough explanation, therefore the object of this paper is to study and make a complete mathematical analysis of his effect.

* Assistant Professor, Faculty of Engineering and Technology, Shebin El-Kom, Monofia University.

** Lecturer, Faculty of Engineering, El-Mansoura University
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II. THEORETICAL ANALYSIS

2.1. Weight of induction motor:

The weight of an induction motor is approximatly the sum of the following weights: Stator and rotor windings G_{w1}; G_{w2} ; stator and rotor iron cores G_i; motor frame G_f, and motor end-covers Ge.c. These weights can be calculated by the following equations:

$$G_{w1} = 8{,}9\ (2.l_{mc} \cdot m_1 \cdot w_1 \cdot a_{c1})\ 10^{-5} = a_2 \xi^{1/3} + a_3 \xi^{-2/3} \quad \ldots\ldots\ (kg) \quad (1\text{-}1)$$

$$G_{w2} = G_b + G_r = 2{,}7 \cdot a_b \cdot L_1 \cdot S_2 \cdot 10^{-5} + 2.2{,}7 \cdot a_r \cdot \pi \cdot D_r \cdot 10^{-5}$$
$$= a_4 \xi^{1/3} + a_5 \xi^{-2/3} \quad \ldots\ldots\ (kg) \quad (1\text{-}2)$$

$$G_i = G_{t1} + G_{c1} + G_{t2} + G_{c2} = 7{,}8 \cdot S_1 \cdot b_{t1} \cdot h_{s1} \cdot L_1 \cdot 10^{-3} + 5{,}5\ (D_{o1}^2 - D_{c1}^2)\ L_1 \cdot 10^{-3} + 5{,}5 \cdot 10^{-3} (D_{o2}^2 - D_{c2}^2 - S_2 d_{s2})\ L_1$$
$$+ 5{,}5 \cdot 10^{-3} \cdot D_{c2}^2\ L_1 = a_6 + a_7 + a_8 + a_9 = a_{10} \quad \ldots\ldots\ (kg) \quad (1\text{-}3)$$

$$G_f = 2{,}7 \cdot 10^{-3} \cdot \pi \cdot D_f \cdot L_f \cdot f = a_{11} \quad \ldots\ldots\ (kg) \quad (1\text{-}4)$$

$$G_{ec} = 2.2{,}7 \cdot 10^{-3} \cdot \frac{\pi}{4} \cdot D_f^2 \cdot \Delta f = a_{12}/\xi \quad \ldots.\ (kg) \quad (1\text{-}5)$$

Where:

$a_1 = D_1^2\ L_1 = D_1^3 \xi = CP_s/n_1$; $a_2 = 27{,}95 \cdot 10^{-5}\ (k_2\ A_1/j_1) a_1^{2/3}$;

$a_3 = 87{,}75 \cdot 10^{-5}\ (k_1 k_2\ A_1/j_1 \cdot 2p)\ a_1^{2/3}$;

$a_4 = 8{,}48 \cdot 10^{-5} \cdot k_{w1}\ k_2\ k_3\ (A_1/j_b)\ a_1^{2/3}$;

$a_5 = 26{,}62 \cdot 10^{-5}\ (k_{w1}\ k_2\ k_3\ A_1/j_r \cdot S_2 \cdot \sin \frac{\pi \varphi}{S_2})(1-2\ \lambda_1 - \lambda_2) a_1^{2/3}$;

$a_6 = 0{,}0263 \cdot \beta_1 \cdot \lambda_3 \cdot a_1$;

$a_7 = 0{,}022\ ((\lambda_1 + \lambda_4) + (\lambda_1^2 + \lambda_4^2) + 2\lambda_1(\lambda_3 + \lambda_4))\ a_1;$

$a_8 = 5{,}5.10^{-3}\ (4\lambda_2(1-2\lambda_1 - \lambda_2) - S_2\lambda_2^2)\ a_1;$

$a_9 = 5{,}5.10^{-3}(1-2\lambda_1 - 2\lambda_2)^2\ a_1;$

$a_{11} = 0{,}017.\ \lambda_5\ (1+2\lambda_1 + 2\lambda_3 + 2\lambda_4 + \lambda_5)\ a_1;$

$a_{12} = 4{,}25.10^{-3}\ .\lambda_5(1 + 2\lambda_1 + 2\lambda_3 + 2\lambda_4 + \lambda_5)^2\ a_1,$

$k_1 \simeq 1{,}8$ for $2P = 2;4;6$ and full-pitch winding and equal to $1{,}5$ for $2P = 2;4;6$ and short pitch winding,

$k_2 = I_1\Phi/I_1 = 1$ for star connection and $= 1/\sqrt{3}$ for delta connection;

k_3 = coefficient equal to 0,3 - 0,6;

k_{w1} = stator winding factor,

$\lambda_1 = l_g/D_1;$

$\lambda_2 = h_{s2}/D_1;$

$\lambda_3 = h_{s1}/D_1;$

$\lambda_4 = h_{c1}/D_1;$

$\lambda_5 = \Delta f/D_1$ - ratio between air gap length, rotor slot depth; stator core depth and frame thickness to stator bore diameter;

$\beta_1 = B_g/B_{t1\ max}$ - relation between air gap flux density to max. flux density in stator teeth;

$D_{o1} = D_1 + 2\ l_g + 2\ h_{s1} + 2\ h_{c1}$ = stator outer diameter, cm;

$D_{c1} = D_1 + 2\ h_{s1}$; cm;

$D_{o2} = D_1 - 2\ l_g$ = rotor outer diameter, cm;

$D_{c2} = D_{o2} - 2\ h_{s2}$, cm;

$D_f = D_{o1} + \Delta f$ = frame diameter, cm;

$L_f = 2L_1 + (0,4 - 1,0)$ = frame length, cm;

S_1 - number of stator slots;

S_2 - number of rotor slots;

m_1 - number of stator phases;

W_1 - number of turns/phase;

A_1 - specific electric loading, A.C/cm;

C - machine constant;

P_a - electro magnetic power, VA;

n_1 - synchronous speed, r.p.m.;

$j_1; j_b; j_r$ - Stator, bar and ring current density, A/mm^2;

2P - number of poles;

$a_{c1}; a_b; a_r$ - stator conductor, bar and ring cross-section area, mm^2;

l_{mc} - length of stator mean conductor, cm;

L_1 - stator core length, cm;

D_r - ring mean diameter, cm;

d_{s2} - rotor slot diameter, cm;

G_b, G_r - weight of rotor bars and cage rings, k_g;

G_{t1}, G_{c1} - weight of stator teeth and core, kg;

G_{t2}, G_{c2} - weight of rotor teeth and core, kg.

Therefore, total weight of inductien motor as a function of the ratio (ξ), will be in the form:

$$G_M = a_{13}\,\xi^{1/3} + a_{14}\,\xi^{-2/3} + a_{12}\,\xi^{-1} + a_{15} \quad \ldots (kg) \qquad (1\text{-}6)$$

Where:

$a_{13} = a_2 + a_4$; $a_{14} = a_3 + a_5$; $a_{15} = a_{10} + a_{11}$;

and the specific weight of the motor

$$\gamma_H = G_M/P_2 \qquad \ldots\ldots \text{(kg/watt)} \qquad (1\text{-}7)$$

2.2. Losses and efficiency of induction motor:

The losses in induction motor consist of: copper losses in stator and rotor windings P_{cu1}, P_{cu2}; iron losses in stator core P_{i1}; friction and windage losses P_{fr+w} or simply mechanical losses P_m. Additional or stray losses - as in usual practice - are taken as a percentage of the total losses

$$P_{cu1} = m_1 \cdot I_1^2 \phi \cdot r_1 = a_{16}\, \xi^{1/3} + a_{17}\, \xi^{-2/3} \qquad \ldots\text{(watts)} \quad (2\text{-}1)$$

$$P_{cu2} = m_1 \cdot k_3^2 \cdot I_1^2 \phi \cdot r'_2 = a_{18}\, \xi^{1/3} + a_{19}\, \xi^{-2/3} \qquad \ldots\text{(watts)} \quad (2\text{-}2)$$

$$P_{i1} = P_{t1} + P_{c1} = p_i\, B_{t1}^2\, (f_1/50)^{1,3}\, G_{t1} + p_i \cdot B_{c1}^2 (f_1/50)^{1,3} \cdot G_{c1} = a_{20} + a_{21} = a_{22} \qquad \ldots\text{(watts)} \quad (2\text{-}3)$$

$$P_m \Big|_{n_1 \leqslant 15000\ \text{r.p.m}} = P_{fr} + P_w = k_m \cdot G_r \cdot n_1 \cdot 10^{-3} + 2 D_{o2}^3\, L_1 n^3 \cdot 10^{-14} = a_{23} + a_{24}/\xi^{1/3} \qquad \ldots\text{(watts)} \quad (2\text{-}4.a)$$

$$P_m \Big|_{n_1 > 15000\ \text{r.p.m}} = P_{fr} + P_w = k_m \cdot G_r \cdot n_1 \cdot 10^{-3} + 0{,}3\, D_{o2}^5 \left(1 + 5\,\frac{L_1}{D_{o2}}\right) n^3 \cdot 10^{-16} = a_{23} + a_{25}/\xi^{5/3} + a_{26}/\xi^{2/3} \qquad \ldots\ldots\text{(watts)} \quad (2\text{-}4.b)$$

Therefore the total losses of induction motor are:

a - in the case of $n_1 \leqslant 15000$ r.p.m.

$$\sum P = \zeta_o\,(P_{cu1} + P_{cu2} + P_{i1} + P_m) = \zeta_o\,(a_{27}\,\xi^{1/3} + a_{28}\,\xi^{-2/3} + a_{29} + a_{24}\,\xi^{-1/3}) \quad \text{.....(watts) (2-5.a)}$$

b- in the case of n_1 15000 r.p.m

$$\sum P = \zeta_o\,(P_{cu1} + P_{cu2} + P_{i1} + P_m) = \zeta_o\,(a_{27}\,\xi^{1/3} + a_{30}\,\xi^{-2/3} + a_{29} + a_{25}\,\xi^{-5/3}) \quad \text{.....(watts) (2-5.b)}$$

Efficiency of induction motor in case of n 15000 r.p.m

$$\eta = \frac{P_2}{P_2 + \sum P} = \frac{P_2^{\;2/3}}{P_2\,\xi^{2/3} + a_{27}\,\zeta_o\,\xi + a_{28}\,\zeta_o + a_{29}\,\zeta_o\,\xi^{2/3} + a_{24}\,\zeta_o\,\xi^{1/3}} \quad \text{........ (2-6.a)}$$

and in the case of $n > 15000$ r.p.m

$$\eta = \frac{P_2}{P_2 + \sum P} = \frac{P_2^{\;5/3}}{P_2\,\xi^{5/3} + a_{27}\,\zeta_o\,\xi^2 + a_{30}\,\zeta_o\,\xi + a_{29}\,\zeta_o\,\xi^{5/3} + a_{25}\,\zeta_o} \quad \text{........ (2-6.b)}$$

Where:

$a_{16} = \pi\, k_\theta\, k_2\, j_1\, \Delta_1\, a_1^{2/3}/5700$; $a_{17} = \pi^2\, k_\theta\, k_1\, k_2\, j_1 \Delta_1 a_1^{2/3}/5700.2p$;

$a_{18} = \pi\, k_\theta\, k_{w_1} k_2\, k_3\, j_b\, \Delta_1\, a_1^{2/3}/100.\gamma$;

$a_{19} = \pi^2\, k_\theta . k_{w1} . k_2 . k_3 (1-2\lambda_1 - \lambda_2)\, j_r\, \Delta_1 a_1^{2/3}/100\,\gamma\, S_2\, \sin(\frac{\pi\Phi}{S_2})$;

$a_{20} = 0{,}0263.p_i .\, \beta_1 . B_g^2\, (f_1/50)^{1,3}\, \lambda_3\, a_1$;

$a_{21} = 0{,}022.\, p_i .\, \beta_2^2 . B_g^2\, (f_1/50)^{1,3}\, ((\lambda_1 + \lambda_4) + (\lambda_1^2 + \lambda_4^2) + 2\lambda_1\,(\lambda_3 + \lambda_4))\, a_1$;

$a_{23} = 2.\pi.10^{-6}.k_m\, n_1(1-2\lambda_1)^2\, a_1;$

$a_{24} = 2.10^{-14}.n^3\,(1-2\lambda_1)^3\;\; a_1^{4/3}\;;$

$a_{25} = 0{,}3.10^{-16}.\; n^3(1-2\lambda_1)^5\;\; a_1^{5/3};$

$a_{26} = 1{,}5.10^{-16}\, n^3\,(1-2\lambda_1)^4\, a_1^{5/3};$

$a_{27} = a_{16} + a_{18};$

$a_{28} = a_{17} + a_{19};$

$a_{29} = a_{22} + a_{23};$

$a_{30} = a_{26} + a_{28};$

r_1 - Stator resistance, ohm; $\grave{r}_2$ - rotor resistance referred to the stator, ohm;

p_i - Specific iron loss, w/kg; f_1 - Supply frequency, Hz;

n - rotor speed, r.p.m;

P_2 - useful power at the motor shaft, watts; P_{t1} - iron losses in stator teeth, watts;

P_{c1}- iron losses in stator core, watts; B_{t1}, B_{c1} - flux density in stator teeth and core, wb/m^2; $\beta_2 = B_{c1}/B_g$ - relation between flux density in stator core and air-gap; Gr -weight of rotor with Cage, kg; k_m = 1 - 3 - imperical oonstant;

k_θ - temperature coefficient; ζ_o - stray load losses coefficient; γ- Specific conductivity.

2.3. Power factor of induction motor:

Power factor of a poly - phase induction motor as a function of (ξ), is determined by the following equations:

$$\cos\phi\Big|_{n\leqslant 15000\ r.p.m} = \frac{P_1}{m_1\cdot V_{1\varphi}\, I_{1\varphi}} = a_{31}\cdot P_2 + a_{32}\cdot \xi^{1/3} + a_{33}\,\xi^{-2/3} + a_{34} + a_{35}\,\xi^{-1/3} \quad (3\text{-a});$$

$$\cos\phi\Big|_{n>15000\ r.p.m} = \frac{P_1}{m_1\cdot V_{1\varphi}\, I_{1\varphi}} = a_{31}\cdot P_2 + a_{32}\,\xi^{1/3} + a_{36}\,\xi^{-2/3} + a_{34} + a_{37}\,\xi^{-5/3} \quad (3\text{-b}).$$

where:

$a_{31} = 5,48,10^4\ \overline{\delta}/k_{w1}\cdot\ \alpha\cdot\ A_1\cdot B_g\cdot n_1\cdot a_1;$

$a_{32} = a_{31}\cdot\ a_{27}\cdot\ \overline{\delta}_o;$

$a_{33} = a_{31}\cdot\ a_{28}\cdot\ \overline{\delta}_o;$

$a_{34} = a_{31}\cdot\ a_{29}\cdot\ \overline{\delta}_o;$

$a_{35} = a_{31}\cdot\ a_{24}\cdot\ \overline{\delta}_o;$

$a_{36} = a_{31}\cdot\ a_{30}\ \overline{\delta}_o;$

$a_{37} = a_{31}\cdot\ a_{25}\cdot\ \overline{\delta}_o;$

P_1 - input power to the motor, watts; $V_{1\varphi}$ - applied phase voltage, volts;

$\alpha = B_{av}/B_g \cong 0,64$ - relation between average and total flux density in the air-gap; $\overline{\delta} = E_1/V_{1\varphi} = 0,8 - 0,94$ - for induction motors vating $\leqslant$600 watts.

2.4. Thermal characteristics of induction motors:

Thermal characteristics of induction motors in this work means the specific thermal loading and the average temperature rise for the frame surface area. Specific thermal loading of poly-phase induction motors is the quotient of total losses at nominal load $\sum P$ to the total frome and the end-covers surface area, i.e.

$$q_{th} = \Sigma p/S_f = \frac{S_o\,(a_{27}\xi^{1/3} + a_{28}\xi^{-2/3} + a_{29} + a_{24}\xi^{-1/3})}{a_{38}\xi^{1/3} + a_{39}\xi^{-2/3}} \Bigg|_{\text{at } n \leq 15000 \text{ r.p.m}}$$

(w/cm^2) (4-1.a);

$$q_{th} = \Sigma p/S_f = \frac{S_o(a_{27}\xi^{1/3} + a_{30}\xi^{-2/3} + a_{29} + a_{25}\xi^{-5/3})}{a_{38}\xi^{1/3} + a_{39}\xi^{-2/3}} \Bigg|_{\text{at } n > 15000 \text{ r.p.m}}$$

(w/cm^2) (4-1.b)

Average temperature rise of the frame surface

$$\vartheta = \frac{\Sigma p}{\grave{\alpha}\, S_f} = q_{th}/\grave{\alpha} = f\,(\xi) \qquad (°C) \qquad \ldots\ldots.(4\text{-}2)$$

where:

$$S_f = a_{38}\,\xi^{1/3} + a_{39}\,\xi^{-2/3} \quad (cm^2)$$

$$a_{38} = 2\,\pi(1+2\lambda_1 + 2\lambda_3 + 2\lambda_4 + \lambda_5)\,a_1^{2/3};$$
$$a_{39} = \frac{\pi}{2}\,(1+2\lambda_1 + 2\lambda_3 + 2\lambda_4 + \lambda_5)^2 a_1^{2/3};$$

$\grave{\alpha}$ - heat transfer coefficient of the frame surface.

2.5. Parameters of induction motor:

The dependance of poly phase induction motor parameters on the ratio (ξ) is analysed and explained as follows.

$$r_1 = k_\upsilon \cdot \frac{2.W_1.l_{mc}}{5700.\;a_{cl}} = a_{40}\,\xi^{1/3} = a_{41}\,\xi^{-2/3} \quad \text{ohm} \qquad (5\text{-}1)$$

$$\grave{r}_2 = \frac{4m1(k_{w1}\;W_1)^2}{S_2}\,(r_b + \frac{r_r}{2\sin^2\frac{\pi\Phi}{S_2}}) = a_{42}\,\xi^{1/3} + a_{43}\,\xi^{-2/3}$$

ohm (5-2)

$$r_o = \frac{P_i}{m_1 \; I_o^2} = a_{44} \; \xi^{2/3} \qquad \text{ohm} \quad (5\text{-}3)$$

$$x_1 = \frac{4\pi f_1 \; W_1^2 \; L_1}{Pq_1} \grave{\lambda}_1 = a_{45} + a_{46} + a_{47} \; \xi^{-1} \qquad \text{ohm} \quad (5\text{-}4)$$

$$\grave{x}_2 = 2\pi f_1 \; L_1 \; \frac{4m1(k_{w1} \; W_1)^2}{S_2} \; \grave{\lambda}_2 = a_{48} + a_{49} + a_{50} \; \xi^{-1}$$

$$\text{ohm} \quad (5\text{-}5)$$

$$x_o = 1{,}6 \cdot m_1 \cdot f_1 \cdot \frac{(k_{w1} \cdot W_1)^2}{P} \cdot \frac{\tau \; L_1}{k_s \cdot k_g \cdot l_g} \cdot 10^{-8} = a_{51} \; \xi^{2/3}$$

$$\text{ohm} \quad (5\text{-}6)$$

where:

$$a_{40} = 28{,}9.10^5 \; \frac{k_\theta \; k_3 \; j_1 (k_3 \zeta V_{1\varphi})^2}{k_2 \; (k_{w1} \cdot \alpha \cdot B_g \cdot n_1)^2 A_1 \; a_1^{4/3}}$$

$$a_{41} = a_{40} \; \frac{\pi k_1}{2p} \; ;$$

$$a_{42} = 1{,}63.10^8 \; \frac{k_\theta (k_3 \zeta V_{1\varphi})^2 \; j_b}{\gamma \cdot k_{w1} \cdot k_2 \cdot A_1 (\alpha \cdot B_g \cdot n_1)^2 \; a_1^{4/3}}$$

$$a_{43} = 5{,}18.10^8 \; \frac{K_\theta \; (k_3 \zeta V_{1\varphi})^2 (1 - 2\lambda_1 - \lambda_3) j_r}{\gamma \cdot k_{w1} \cdot k_2 \cdot A_1 (\alpha \cdot B_g \cdot n_1)^2 \sin \frac{\pi \varphi}{S_2} \cdot S_2 \cdot a_1^{4/3}} \; ;$$

$$a_{44} = \frac{\pi \cdot m_1 \cdot p_i (k_{w1} \cdot k_3)^2 (f_1/S_o)^{1/3} (B_1^2 \; a_6 + B_2^2 \; a_7)}{(P \cdot k_g \cdot k_s \cdot \; \lambda_1)^2 \cdot a_1^{2/3}} \cdot 10^7$$

$$a_{45} = 15{,}51.10^8 \; \frac{2p \cdot m_1 \; (k_3 \; S \; V_{1\varphi})^2}{(k_{w1} \cdot \alpha \; B_g)^2 \cdot S_1 \cdot n_1 \cdot a_1} \; \grave{\lambda}_{s1} ;$$

$$a_{46} = a_{45} \; (\grave{\lambda}_{t1} / \grave{\lambda}_{s1}) ;$$

$$a_{47} = a_{45}\,(\lambda_{o1}/\lambda_{s1});$$

$$a_{48} = 1861.10^{8}\ \frac{m_1(k_3\,\tau\,V_{1\varphi})^2\,f_1}{S_2\,(\alpha\cdot B_g\cdot n_1)^2 a_1}\ \lambda_{s2};$$

$$a_{49} = a_{48}(\lambda_{t2}/\lambda_{s2}); \quad a_{50} = a_{48}(\lambda_{o2}/\lambda_{s2});$$

$$a_{51} = 0{,}42.10^{-9}\ \frac{k_{w1}^2\,k_3\,m_1\,n_1\,\sqrt[3]{a_1}}{p\cdot k_s\cdot k_g\cdot\lambda_1};$$

r_b, r_r - bar and ring resistance, ohm; r_o - core loss resistance ohm; I_o - no-load carrent, a; x_1 - stator leakage reactance, ohm; x_2' - rotor leakage reactance referred to the stator, ohm; x_o - magnetizing reactance, ohm; λ_1 - total permeance coefficient of the stator; λ_{s1}, λ_{t1}, λ_{o1} - permeance coefficient of stator slot, teeth and orer hange; q_1 - number of slots/pole/phase; λ_2 - total permenace coefficient of the rotor; λ_{s2}, λ_{t2}, λ_{o2} - permeance coefficient of rotor slot, teeth and around the cage rings; τ - polepitch, cm; k_g - air gap coefficient, k_s - saturation coefficient.

2.6. Starting and working characteristics:

A good design of poly-phase induction motor has to fulfil the following requirements: minimum working slip (S_n), maximum over load capacity (M_{max}/M_n); maximum starting torque (M_{st}) or maximum ratio of (M_{st}/M_n); and minimum starting current, i.e. minimum (I_{st}/I_n). The following equations describe the above requirements in relation of (ξ):

$$S_n = \frac{P_{cu2}}{P_{cm}} = \frac{P_{cu2}}{P_2 + P_{cu2} + P_m} = \frac{a_{18}\,\xi^{1/3} + a_{19}\,\xi^{-2/3}}{P_2 + a_{18}\,\xi^{1/3} + a_{19}\,\xi^{-2/3} + a_{23} + a_{24}\,\xi^{-2/3}}\Bigg|_{n\leqslant 15000\ \text{r.p.m}} \qquad (6.1.a)$$

$$S_n = \frac{a_{18}\xi^{1/3} + a_{19}\xi^{-2/3}}{P_2 + a_{18}\xi^{1/3} + a_{19}\xi^{-2/3} + a_{23}+a_{25}\xi^{-5/3} + a_{26}\xi^{-2/3}} \Bigg|_{n > 15000 \text{ r.p.m}} \quad (6.1.b)$$

$$S_m \simeq \frac{r_2'}{x_1 + x_2'} \simeq \frac{a_{42}\xi^{1/3} + a_{43}\xi^{-2/3}}{a_{52} + a_{53}\xi^{-1}} \quad (6.2)$$

$$M_{max}/M_n = \tfrac{1}{2}\left(\frac{S_n}{S_m} + \frac{S_m}{S_n}\right) a + b = f(\xi) \quad (6.3)$$

$$M_{st}/M_n = \frac{(r_1 S_n + C r_2')^2 + (x_1 + C x_2')^2}{((r_1 + C r_2')^2 + (x_1 + C x_2')^2) S_n} = f(\xi) \quad (6.4)$$

$$I_{st}/I_n = \sqrt{\frac{M_{st}/M_n}{S_n}} = f(\xi) \quad (6.5)$$

where:

$$a_{52} = a_{45} + a_{46} + a_{48} + a_{49}; \quad a_{53} = a_{47} + a_{50};$$

$$a = \frac{\sqrt{1+d^2}}{1 + \sqrt{1 + d^2}}; \quad d = \frac{x_1 + x_2'}{r_1}; \quad c = 1 + \frac{x_1}{x_0}.$$

$$b = \frac{1}{1 + \sqrt{1 + d^2}}$$

III. COMMENTS AND DISCUSSIONS

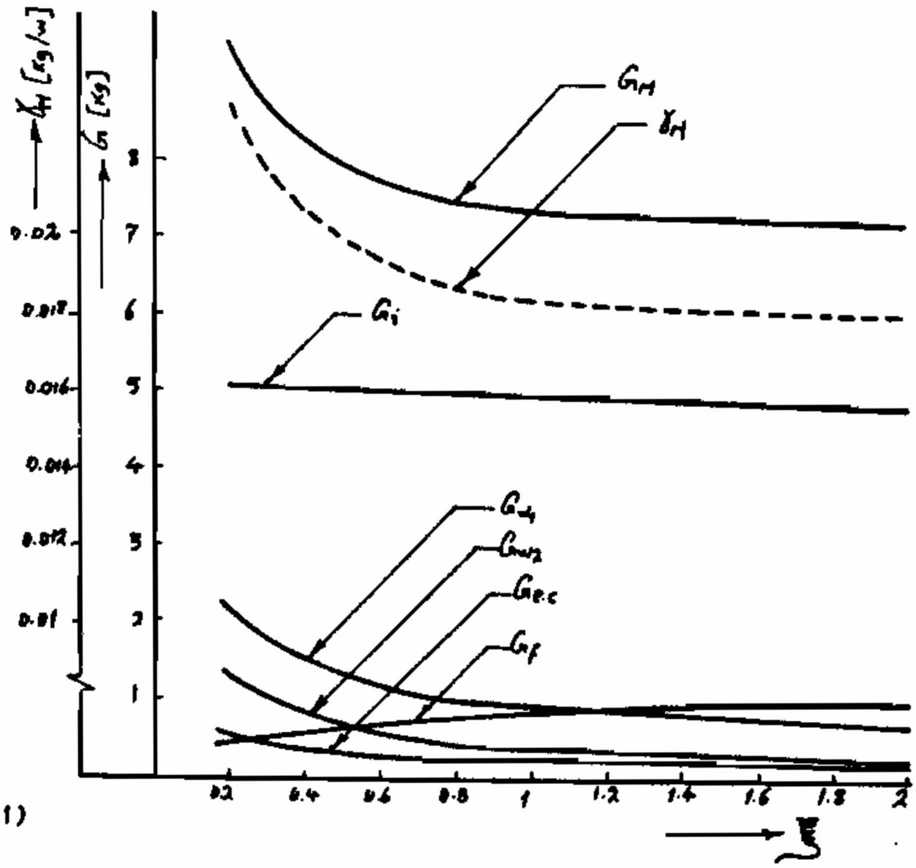
The obtained equations are used to calculate the characteristics for a number of small power polyphase induction motor. As a reference data the following motors are taken[1]: AOL 011/2; AOL 11/2; AOL 21/2 and AOL 22/2, the results are plotted as curves in Figs. (1 ÷ 7).

Figs. (1,5,6,7) show the effect of (ξ) on the motor weight from which it could be seen that increasing ratio (ξ) is accomplished by: decreasing the weight of stator windings due to the decrease in the over hang length; decreasing the weight of rotor windings due to the decrease in the mean diameter of the short circuit rings; decreasing the weight of the motor end-covers due to the small outer diameter of the motor; and increasing the weight of the motor frame due to the increasing of motor length. The effect of ratio (ξ) on the weight of iron is not pronounced due to the fact that the stator volume is constant. Therefore the total weight and specific weight of the motor is decreased with increasing ratio (ξ), it should be notic ed that decreasing of the motor weight is very pronounced when the ratio (ξ) equals 0,2 up to 1.

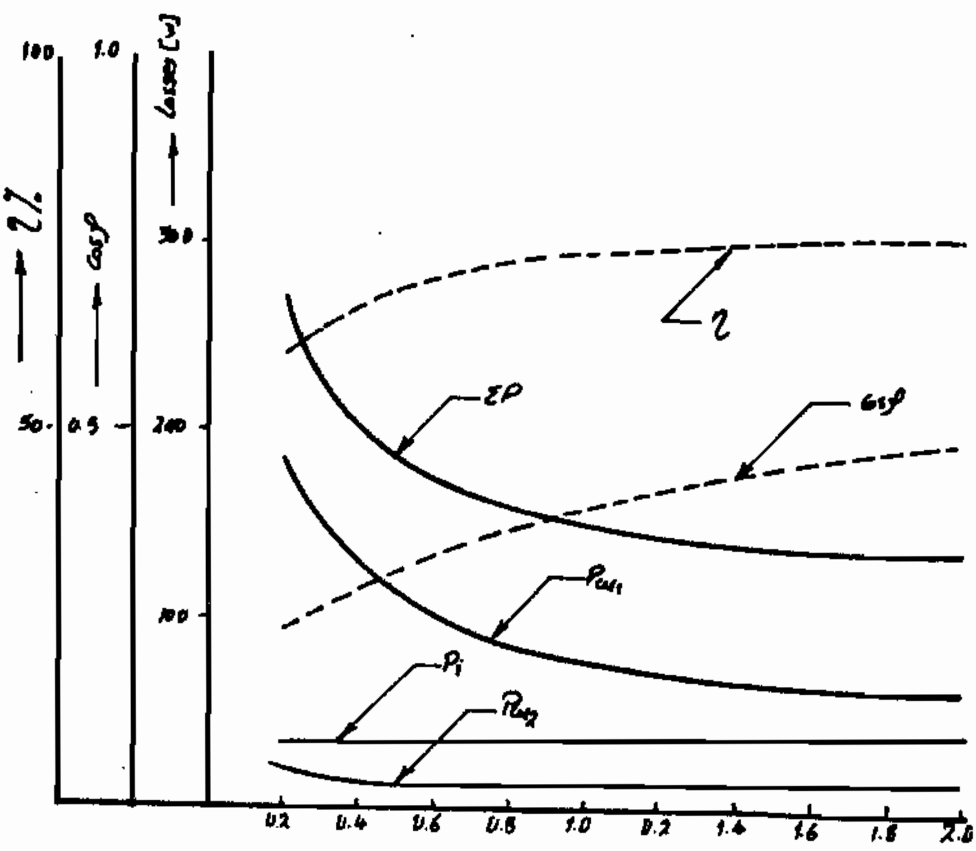
Increasing (ξ) decreased the losses in the stator and the rotor winding due to the decrease in stater and rotor active resistances while the iron losses are nearly constant. Therefore, the total losses decrease and the efficiency increased with the increasing of (ξ). The power factor also increases due to the decrease of reactive power and the motor leakage ractances. Specific thermal loading and average temperature rise decrease with the increasing of (ξ) due to the decreasing of total losses and increasing of the frame surface area (Figs. 2,3,5,6,7).

The starting and working characteristics with respect to the ratio (ξ) are shown in Figs. (4,5,6,7) from which it could be seen that increasing (ξ) improves this characteristics with the exception of the starting current.

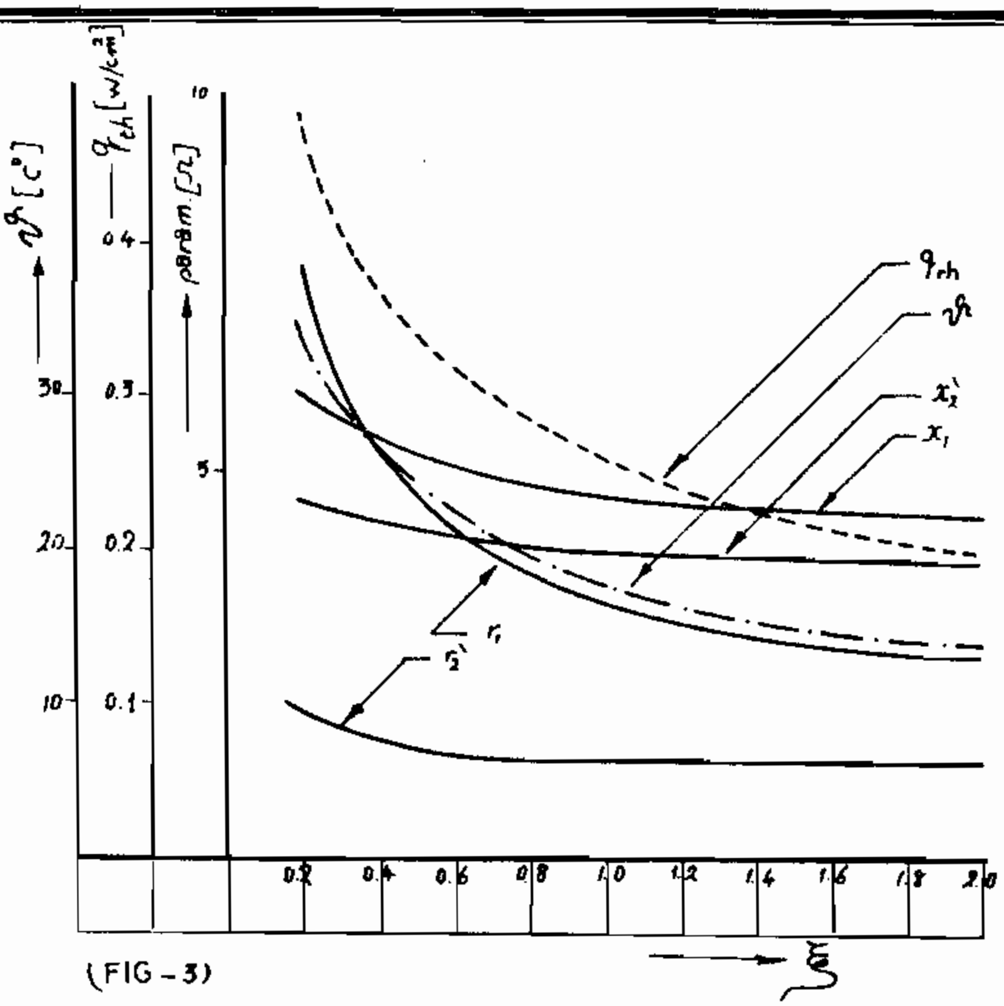
However, it is clear from the above discussion that increasing ratio (ξ) decreases the motor weight and hence it gives a cheaper motor. Also, improving the motor characteristics could be obtained by increasing the motor length and decreasing the motor diameter.


G [Kg]
8
7
6
5
4
3
2
1
0.02
0.017
0.016
0.014
0.012
0.01
G_H
γ_H
G_i
G_{w1}
G_{w2}
$G_{e.c}$
G_f
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
ξ

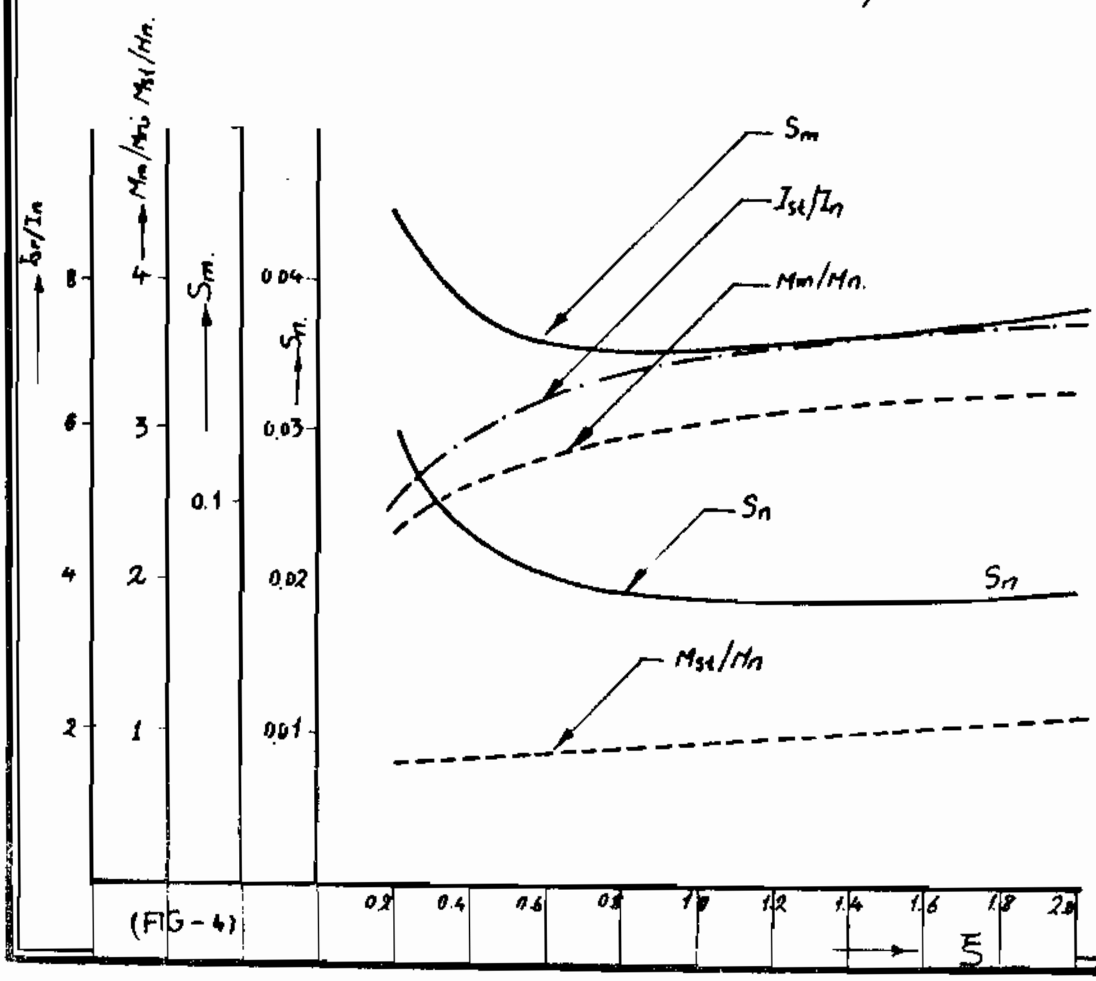
(FIG - 1)


100
1.0
Losses [w]
η %
Cos φ
300
50
0.5
200
100
η
ΣP
Cos φ
P_{cu1}
P_i
P_{cu2}
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
ξ

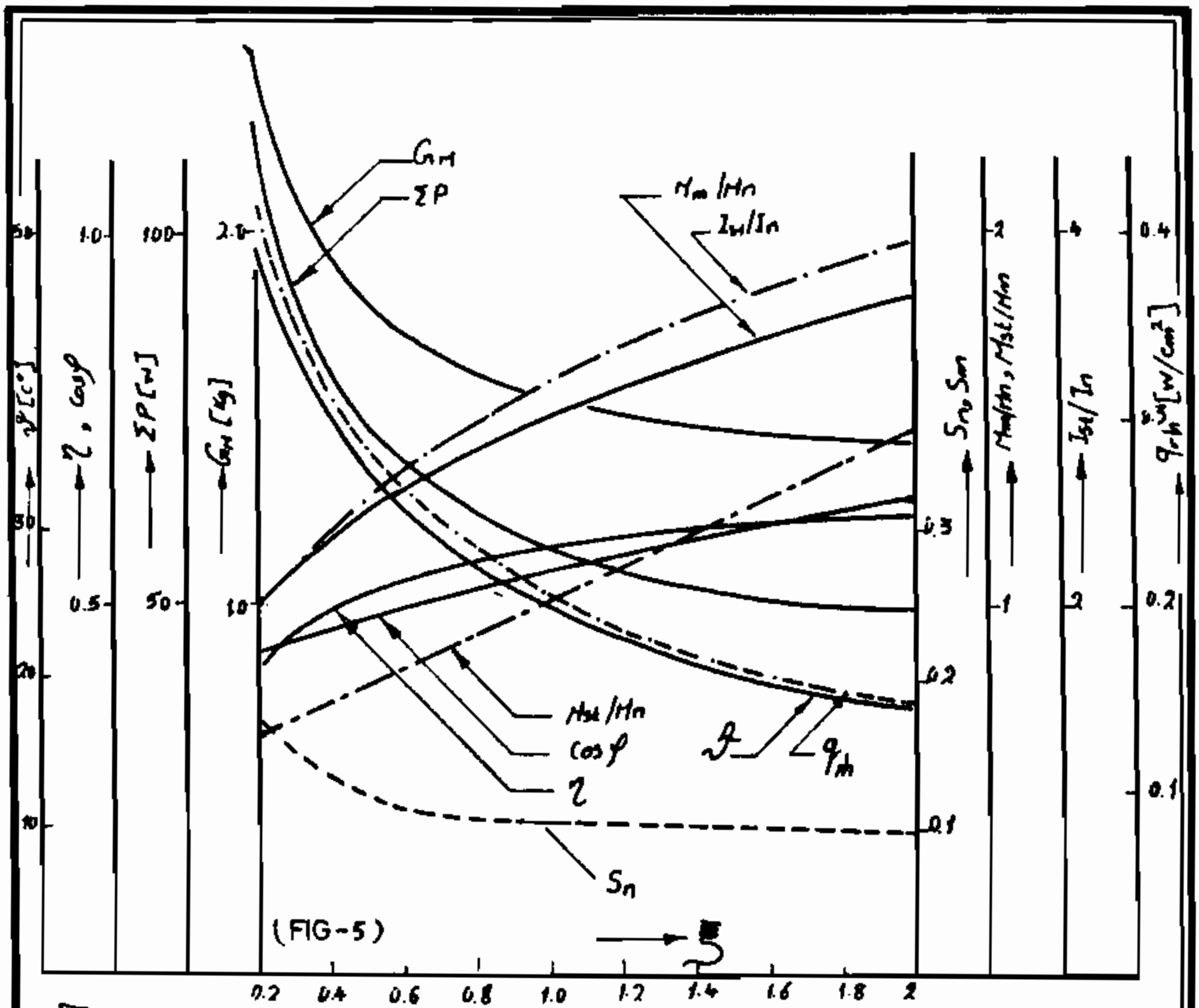
(FIG-2)


q_ch
x_2'
x_1
r_1
r_2'
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0

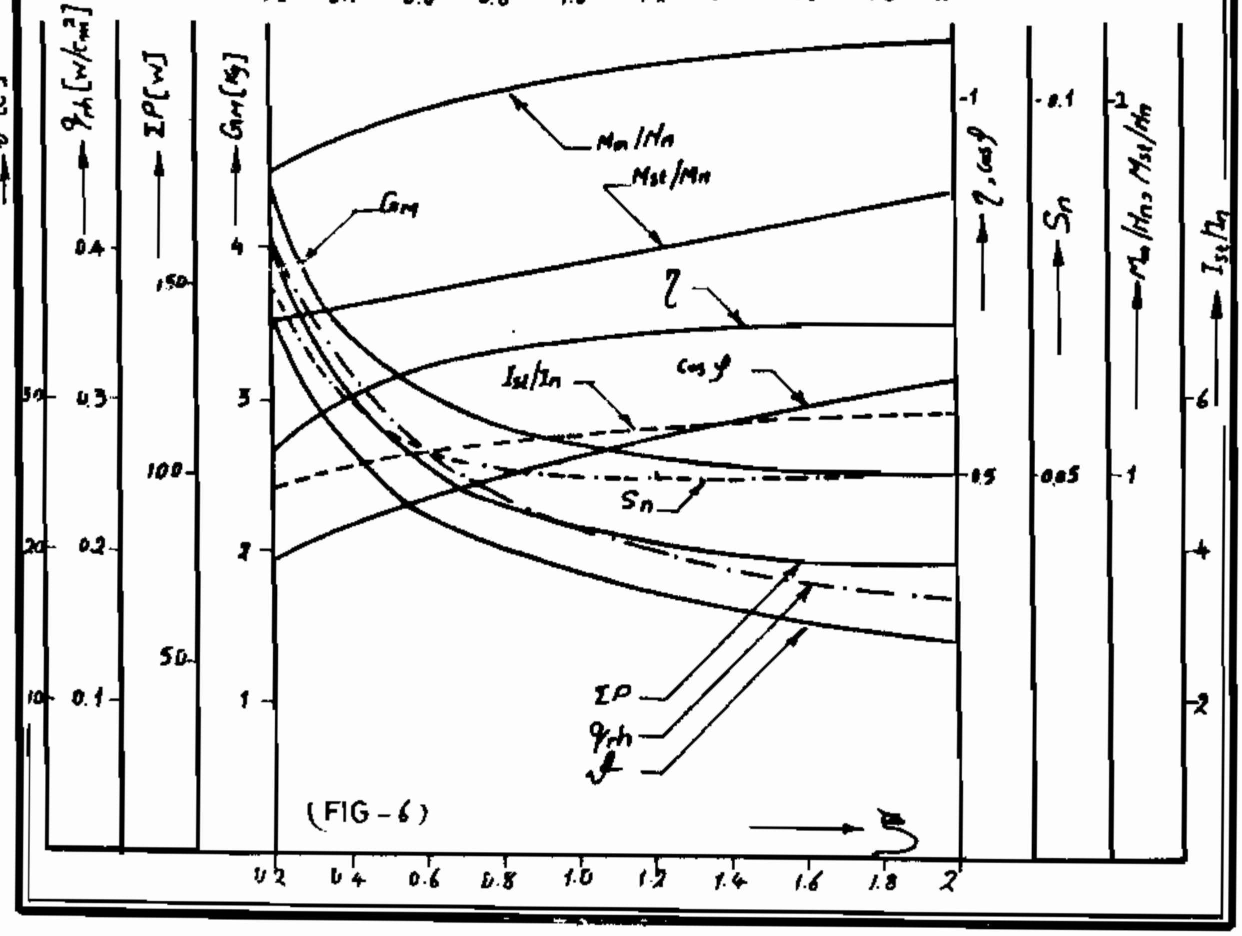
(FIG - 3)


S_m
I_st/I_n
Mm/Mn.
S_n
Mst/Mn
0.04
0.03
0.02
0.01
0.1
0.2
0.4
0.6
1.2
1.4
1.6
1.8
2.0

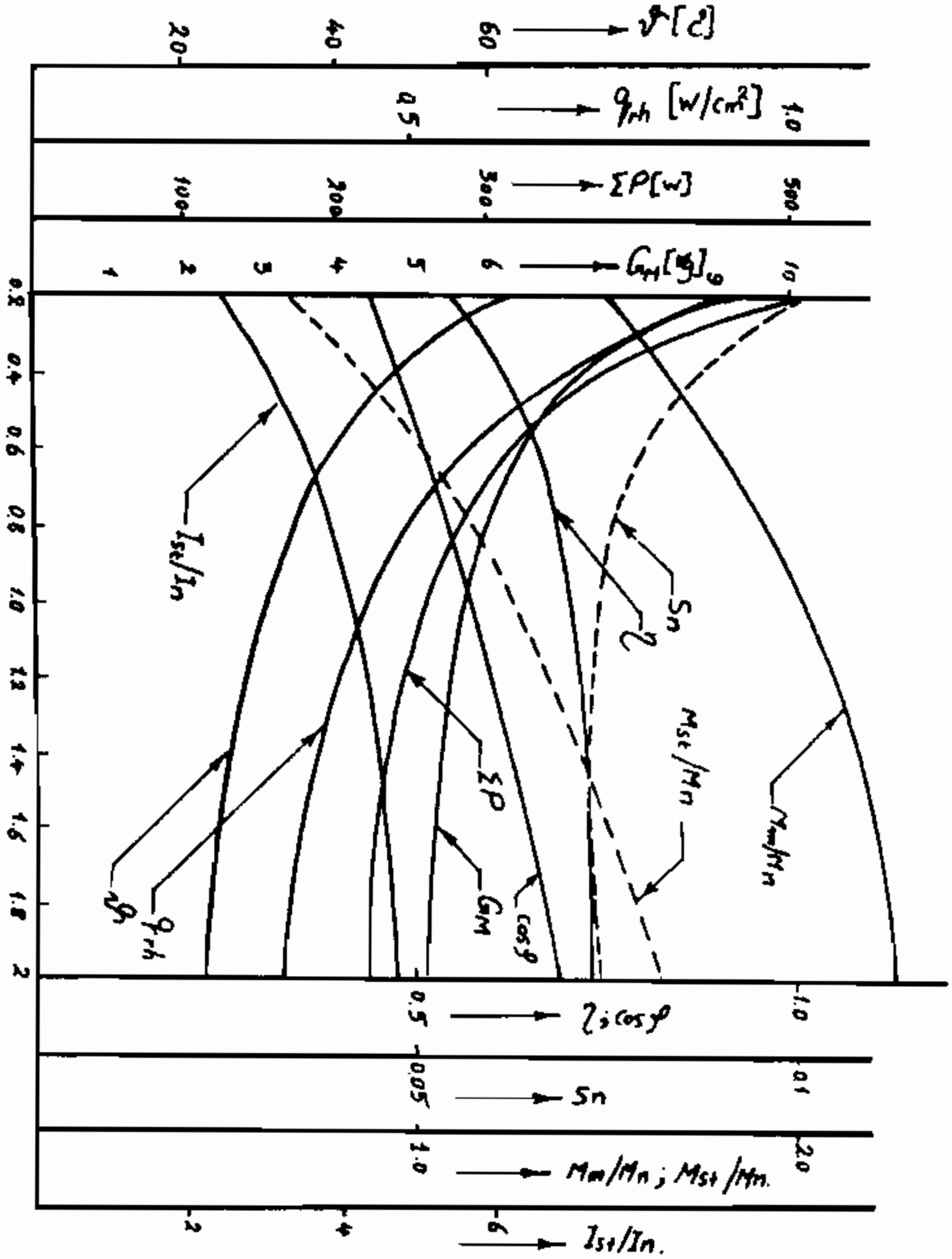
(FIG - 4)



(FIG-5)
Gm
ΣP
Mm/Mn
Ist/In
Mst/Mn
cos φ
η
ϑ
qrh
Sn
0.2 0.4 0.6 0.8 1.0 1.2 1.4 1.6 1.8 2



(FIG-6)
Mm/Mn
Mst/Mn
Gm
η
Ist/In
cos φ
Sn
ΣP
qrh
ϑ
0.2 0.4 0.6 0.8 1.0 1.2 1.4 1.6 1.8 2


ϑ [°C]
q_{rh} [W/cm²]
ΣP [W]
G_M [kg]
I_{st}/I_n
η
Sn
M_{st}/M_n
M_m/M_n
ΣP
G_M
cos φ
q_{rh}
ϑ
η ; cos φ
Sn
M_m/M_n ; M_{st}/M_n
I_{st}/I_n
ξ

(FIG-9)

CONCLUSIONS

In designing small power poly-phase induction motors, it is favourable to take the ratio (ξ) between (1 ÷ 2) to obtain a lighter motor with improved characteristics. Comparison between the new values of (ξ) and the old ones (0,6- 1,4) show that the new values - suggested in this work - give small weight and hence small cost of material. Also it gives low losses, high efficiency, high power factor, small thermal loading, lower temperature rise, lower nominal and maximum slip and higher starting and maximum torque.

REFERENCES

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"Design of single and three-phase small power induction motors" (book) Energy publishers, Moscow, 1961.

2. N.P. ERMOLEN
"Small-power electric machines"
(book) Highschool publishers, Moscow, 1967.

LIST OF FIGURES