PARTICLE SWARM OPTIMIZATION FOR SOLVING THE PROBLEM OF TRANSMISSION SYSTEMS AND GENERATION EXPANSION

الانتشار الأمثل لاسراب الجزيئات لحل مشكلة التوسع فى نظم النقل الكهربية و التوليد

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ملخص:

فى هذه الورقة البحثية يتم تقديم طريقة فعالة لحل مشكلة التخطيط على عدة مراحل منظمة للتوليد و نظم النقل الكهربية و ذلك باستخدام الانتشار الأمثل لاسراب الجزيئات. هذه المشكلة هى مشكلة لا خطية و صعبة الحل خاصة للنظم التى تتميز بالتعقيد و كبر الحجم. طريقة الانتشار الأمثل لاسراب الجزيئات المستخدمة هى طريقة قادرة على ايجاد الشكل الأمثل للنظم المعقدة. حل مشكلة التخطيط على عدة مراحل منظمة للتوليد و نظم النقل الكهربية باستخدام الانتشار الأمثل لاسراب الجزيئات تم بتكلفة أقل من تلك التى يمكن تحقيقها باستخدام طرق الحل التقليدية. تم عرض نتائج اختبار طريقة الحل المقترحة على نظامين صغيرى الحجم متوسطى التعقيد و على نظام ثالث كبير الحجم و عالى التعقيد.

ABSTRACT:

This paper proposes an efficient algorithm to solve the problem of multistage and coordinated planning of generation and transmission systems using particle swarm optimization. This problem is a mixed integer nonlinear programming one, which is difficult for systems of medium and large size and high complexity. The particle swarm optimization algorithm used is capable of finding high quality and optimal topologies for large size and high complexity systems. The multistage and coordinated planning with the aid of particle swarm optimization technique present a lower investment cost than static planning. Test results are applied to two small size medium complexity systems and one large size high complexity system.

KEYWORDS:

Particle Swarm Optimization (PSO) – Multistage and Coordinated Planning – Transmission Network Expansion.

1. INTRODUCTION:

Transmission network expansion planning (TNEP) problem is a large, complex mixed integer nonlinear programming problem, which has presented a major challenge to all known heuristic, optimization approaches. In this problem the number of options to be analyzed increases exponentially with the size of the network. A solution to a planning problem specifies where, how many, and when new equipment must be installed in an electric system, so that it operates adequately within a specified planning interval. The purpose of optimizing the expansion planning is to determine the optimal topology of the power system that can meet the forecast demand with minimum cost while satisfying certain technical and financial constraints. This complex problem is often simplified by the planners by a mathematical model for solving a static transmission network expansion planning (STNEP) problem, which consists of

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minimizing the investment costs in a new transmission facilities subjected to operational constraints, to meet the power system requirement for only one planning interval. Investment is carried out at the beginning of the planning interval time. Another way of solving the TNEP problem is multistage and coordinated planning. In this way of solution, not only the optimal locations and type of investments are defined, but also the most appropriate time to carry out such investments so that the continuing growth of the demand and generation is always considered by the system in an optimized way. The planning interval is divided into several stages and the circuits must be added to each stage of the planning interval. Investment calculations are carried out at the beginning of each stage. When multiple stages in the optimization process are considered, the objective is the minimization of the present value of the sum of all the investments carried out throughout the years corresponding to the simulated periods.

Four main types of mathematical models have been used in literature for representing the transmission network in the transmission expansion planning studies: the transposition model, the hybrid model, the disjunctive model, and the DC power flow model. The best accepted model is the DC model. The AC models are considered only at later stages of the planning process when the most optimal topologies have been determined. The solution techniques used in the past were diverse but may be classified in three groups: 1) constructive heuristic method; 2) classic optimization; and 3) intelligent systems. The latter has been used in recent years because it has the capability to find good or suboptimal solutions in large complex systems. Algorithms such as simulated annealing (SA) [1], genetic algorithm (GA) [2,3], tabu search (TS) [4], greedy randomized adaptive search procedure (GRASP) [5], and hybrid versions belong to this group of methods. In this paper, a particle swarm optimization (PSO) technique is developed for the multistage and coordinated planning problem.

A more detailed model for the TNEP problem was proposed in [3], which allows an integrated multistage planning of generation and transmission systems dealing with operation costs. However, in that work only the transmission planning is addressed. In this paper, the detailed analysis of an integrated planning of generation and transmission systems is introduced. In this paper both the investment costs of new transmission circuits and new generation units are considered. Also, the operation costs of the existing and newly added generation units are taken into consideration. The proposed algorithm is tested using three test systems. Two small size medium complexity systems and one large size high complexity system.

The paper is organized as follows. Initially, an overview of the particle swarm optimization is presented. Then, the mathematical formulation of the multistage and coordinated planning problem is introduced. Next, the proposed algorithm to solve the TNEP problem using PSO is explained. Finally, the results obtained for the test systems and a number of conclusions are presented.

2. PARTICLE SWARM OPTIMIZATION:

PSO is one of the optimization techniques and belongs to evolutionary computation techniques [6-8]. The method has been developed through a simulation of simplified social models. The features of the method are as follows:

(1) The method is based on researches on swarms such as fish schooling and bird flocking.

(2) It is based on a simple concept. Therefore, the computation time is short and it requires few memories.

According to the research results for bird flocking, birds are finding food by flocking (not by each individual). It leaded the assumption that information is owned jointly in flocking. According to observation of behavior of human groups, behavior pattern on each individual is based on several behavior patterns authorized by the groups such as customs and the experiences by each individual (agent). The assumptions are basic concepts of PSO.

individual (agent). The assumptions are basic concepts of PSO.

PSO is basically developed through simulation of bird flocking in two-dimension space. The position of each individual (agent) is represented by XY axis position and also the velocity is expressed by vx (the velocity of X axis) and vy (the velocity of Y axis). Modification of the agent position is realized by the position and velocity information.

An optimization technique based on the above concept can be described as follows: namely, bird flocking optimizes a certain objective function. Each agent knows its best value so far (pbest) and its XY position. Moreover, each agent knows the best value so far in the group (gbest) among pbests. Each agent tries to modify its position using the following information:

- the current positions (x,y),
- the current velocities (vx,vy),
- the distance between the current position, and pbest and gbest.

This modification can be represented by the concept of velocity. Velocity of each agent can be modified by the following equation:

$$v_i^{k+1} = w v_i^k + c_1 rand \times (pbest_i - s_i^k) + c_2 rand \times (gbest - s_i^k) \qquad (1)$$

where,

v_i^k : velocity of agent i at iteration k,
w : weighting function,
c_j : weighting factor,
rand : random number between 0 and 1,
s_i^k: current position of agent i at iteration k,
$pbest_i$: pbest of agent i,
gbest : gbest of group.

Using the above equation, a certain velocity, which gradually gets close to pbest and gbest can be calculated. The current position (searching point in the solution space) can be modified by the following equation:

$$s_i^{k+1} = s_i^k + v_i^{k+1} \qquad (2)$$

Fig.1 shows a searching concept with agents in a solution space and Fig.2 shows a concept of modification of a searching point by PSO.

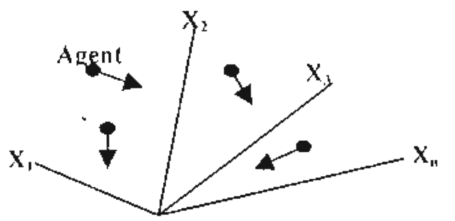
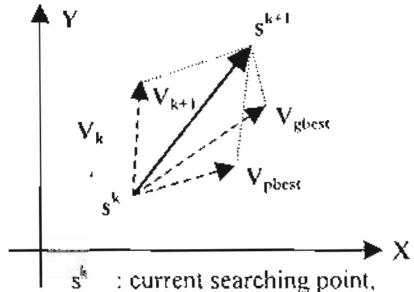


Fig.1 Searching concept with agents in a solution space by PSO.



s^k : current searching point,
s^{k+1} : modified searching point,
V^k : current velocity,
V^{k+1}: modified velocity,
V_{pbest} : velocity based on pbest,
V_{gbest} : velocity based on gbest

Fig.2 Concept of modification of a searching point by PSO.

3. MATHEMATICAL FORMULATION OF THE MULTISTAGE AND COORDINATED PLANNING PROBLEM:

In this section, the mathematical model of the multistage and coordinated planning problem is presented. In this model, the planning interval is divided into several stages and in that context the equipment that should be installed in every planning stage should be determined.

Considering an annual discount rate I, the present values of the investment costs and the operation costs, for the reference year t_0, with an initial year t_1, with an interval of t_T-t_1 years, and with (T-1) stages, are the following:

$$c(x) = (1-I)^{t_1-t_0} c_1(x) + (1-I)^{t_2-t_0} c_2(x) + ...$$
$$+ (1-I)^{t_T-t_0} c_T(x)$$
$$= \delta^1_{inv} c_1(x) + \delta^2_{inv} c_2(x) + ... + \delta^T_{inv} c_T(x) \quad (3)$$

$$b(y) = \sum_{t=t_1}^{t_2-1} (1-I)^{t-t_0} b_1(y) + \sum_{t=t_2}^{t_3-1} (1-I)^{t-t_0} b_2(y) + ...$$
$$+ \sum_{t=t_T}^{t_{T+1}-1} (1-I)^{t-t_0} b_T(y)$$
$$= \delta^1_{oper} b_1(y) + \delta^2_{oper} b_2(y) + ... + \delta^T_{oper} b_T(y) \quad (4)$$

where

$$\delta^t_{inv} = (1-I)^{t_t-t_0} \qquad \delta^t_{oper} = \sum_{p=t_t-t_0}^{t_{t+1}-1-t_0} (1-I)^p$$

and where $c_i(x)$ is the investment cost and $b_i(y)$ is the yearly operation cost of each stage.

With the previous equations, the DC model for the multistage and coordinated planning problem, considering generation and transmission expansion as well as operation costs, assumes the following form [3]:

$$\min v = \sum_{t=1}^{t} \left[\delta^t_{inv} \left(\sum_{(i,j)} c_{ij} n^t_{ij} + \sum_i C_i N^t_i \right) + \delta^t_{oper} \left(\sum_i OC_i G^t_i + \sum_j oc^t_j g^t_j + \alpha \sum_k r^t_k \right) \right] \quad (5)$$

s.t.

$$B^t \theta^t + G^t + g^t + r^t = d^t$$

$$\left(n^o_{ij} + \sum_{m=1}^{t} n^m_{ij} \right) \left| \theta^t_i - \theta^t_j \right| \le \left(n^o_{ij} + \sum_{m=1}^{t} n^m_{ij} \right) \bar{\Phi}_{ij}$$

$$\sum_{m=1}^{t} N^m_i \underline{G}_i \le G^t_i \le \sum_{m=1}^{t} N^m_i \bar{G}_i$$

$$\underline{g}^t_j \le g^t_j \le \bar{g}^t_j$$

$$0 \le r^t \le d^t$$

$$\underline{n}^t_{ij} \le n^t_{ij} \le \bar{n}^t_{ij}$$

$$\underline{N}^t_i \le N^t_i \le \bar{N}^t_i$$

$$\sum_{t=1}^{T} n^t_{ij} \le \bar{n}^t_{ij}$$

$$\sum_{t=1}^{T} N^t_i \le \bar{N}_i$$

n^t_{ij} and N^t_i integer
θ^t_i unbounded
t=1,2,….,T

where

- v : the present value of the expansion and operation cost (generation and transmission) of the system.
- δ_{inv} : the discount factor to find the investment value.
- δ_{oper} : the discount factor modified to take into account the duration (in years) of the planning interval.
- c_{ij} : the cost of the circuit in the i-j branch.
- n_{ij} : the number of circuits added in the path i-j.
- n_{ij}^o : the number of branches in path i-j in the initial configuration.
- C_i : the installation cost of the candidate generator i.
- N_i : the number of generators added at node i.
- OC_i : the annual operation cost of the candidate generator i.
- G_i : the active power injected by the candidate generator i.
- oc_j : the annual operation cost of the generator j already installed.
- g_j : the active power injected by the generator j already installed.
- α : the factor to compatiblize cost units with loss of load.
- r : the loss of load vector in buses with elements r_k.
- B^t : the initial topology and the candidate circuits in the stage t susceptance matrix.
- G^t : the vector of active power injections of the candidate generators in the stage t.
- g^t : the vector of the active power injections of the generators already installed in the stage t.
- θ^t : the vector of angles of the buses in the stage t with the components θ_i^t for the node i.
- $\bar{\Phi}_{ij}$: the maximum angle between the i and j nodes.
- d^t : the demand vector in the stage t.

$\bar{N}_i, \bar{n}_{ij}, \bar{g}_j$ *and* $\bar{G}_i$ represent these variables upper limits and

$\underline{N}_i, \underline{n}_{ij}, \underline{g}_j$ *and* $\underline{G}_i$ represent these variables lower limits.

In system (5), the investment variables are represented by the number of generators N_i and by the number of circuits (transmission lines and transformers) n_{ij} that must be installed. The operation variables are represented by the existent generators injection if active power g_i and candidates G_i. The variables r_k are artificial generation variables that may be zero and can be interpreted as costs by loss of load. Obviously, the mathematical model presented in (5) is very difficult to solve for large and medium complexity systems. For such problems, only metaheuristics like GA, SA, TS, GRASP, PSO, etc., have the capability to find high quality solutions of real systems. In this paper, a PSO algorithm is proposed.

4. PROPOSED SOLUTION ALGORITHM:

In this paper, the particle swarm optimization is used to solve the TNEP problem. The proposed algorithm proceeds as follows:

1. Input data: the following data are input
   - The network configuration (line data).
   - The load values at each bus in every stage of the planning interval.
   - The generation values in every stage of the planning interval.
   - The upper and lower limits of the bounded variables.
2. Initialize the swarm with random positions and velocities.
3. Evaluate the fitness of each particle (objective function).
4. Determine the personal best position (pbest) and global best position (gbest).
5. Update the velocity of agents using eqn.(1).
6. Update the position of agents using eqn.(2).
7. Perform the position check (the boundaries of each parameter). If violated then repair the algorithm then go to step 8. If not violated go to step 8.
8. Check the stopping criterion. If met go to step 9 and if not met go back to step 3.
9. Output the optimal solution, which is the optimal configuration of the system and the optimal values and locations of generation added in each stage to meet the load growth and keep the system constraints not violated.

5. TEST RESULTS:

The proposed algorithm was applied to several electric systems. The results of the following test systems are presented: 1) the Garver's 6-bus network of small size low complexity, 2) the IEEE 24-bus system of small size medium complexity, and 3) a practical 87-bus electric system of large size high complexity. In the test results, only the added circuits and generation during the optimization process are presented. The loss of load (w) is assumed to be zero for all test systems as given in [9-11].

A- Garver's 6-Bus System:

The system data and single line diagram are given in [9]. The best topology found, with the actual value of investment and operation costs projected to the base year 2002 equal to v = 2.9168 million dollars and loss of load w = 0 MW, for the two operation stages, is the following:

- Stage P1: 2002-2004

$n_{1\text{-}2}=2$, $n_{1\text{-}4}=2$, $n_{1\text{-}5}=2$, $n_{1\text{-}6}=1$, $n_{2\text{-}3}=1$, $n_{3\text{-}4}=2$, $n_{3\text{-}5}=1$, $n_{4\text{-}5}=2$, $n_{4\text{-}6}=$
$G_6 = 55$ MW $N_6 = 2$.

- Stage P2: 2004-2006

$n_{1\text{-}2}=2$, $n_{1\text{-}4}=2$, $n_{1\text{-}6}=2$, $n_{2\text{-}3}=1$, $n_{2\text{-}5}=1$ $n_{2\text{-}6}=2$, $n_{3\text{-}6}=2$, $n_{4\text{-}5}=2$, $n_{5\text{-}6}=2$
$G_3 = 130$ MW $N_3 = 1$
$G_6 = 60$ MW $N_6 = 2$.

The following parameters were used: α=[1200-1400]\$/MW, annual discount rate I=10%, swarmsize = 20, maximum number of iterations = 100, maximum number of circuits per path $n_{max} = 5$, $\overline{G}_3$=360 MW and $\overline{G}_6$=600 MW. The circuits and generations added to P1 appear in the objective function with their nominal costs and those added to P2 are multiplied by a cost factor of 0.656. The convergence process for the 6-bus system is shown in Fig.(3).

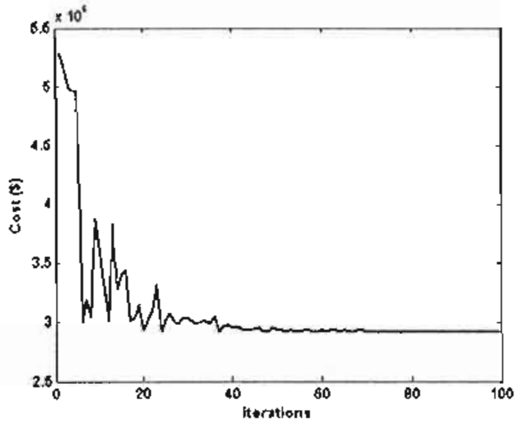


Fig.3 Convergence process of the 6 bus test system

B- The IEEE 24-Bus System:

The data and the single line diagram of this test system are presented [10]. The number of added circuits in the best topology found, with the actual value of investment and operation costs projected to the base year 2002 equal to v = 10.1386 million dollars and loss of load w = 0 MW, for the one operation stage, is given in Table1.

As a result of generation planning, the generation added was as follows:

G_{23} = 224 MWN_{23} = 1

The following parameters were used: α=[1200-1400]$/MW, annual discount rate I=10%, swarmsize = 20, maximum number of iterations = 100, maximum number of circuits per path n_{max} = 5. In order to carry out the expansion planning with generation redispatch the maximum generation levels are determined using the following relationship $\overline{g}_k = 1.3 g_k$ [11].

The convergence process for the 24-bus system is shown in Fig.(4).

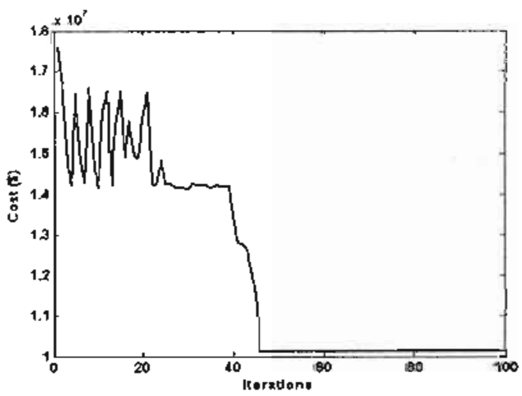


Fig.4 Convergence process of the 24 bus test system

Table 1: The best topology for the 24-bus system

From - To		n_{ij}	From - To		n_{ij}
1	2	4	14	16	2
1	3	1	15	21	1
1	5	4	15	24	4
2	4	2	16	17	2
2	6	4	16	19	1
3	9	2	20	23	2
3	24	2	1	8	1
4	9	1	2	8	1
13	23	1	6	7	1
6	10	2	14	23	2
7	8	4	16	23	1
8	9	1	19	23	2
8	10	1	11	13	2
9	11	2	11	14	1
9	12	2	12	13	1
10	11	1	12	23	2
10	12	1			

C- The 87-Bus Practical Test System:

The single line diagram of the system is presented in [11] and the system data are given in [9]. The number of added circuits in the best topology found, with the actual value of investment and operation costs projected to the base year 1998 equal to v = 101.85 million dollars and loss of load w = 0 MW, for the two operation stage, is given in table 2.

As a result of generation planning, the generation added was as follows:

- Stage P1: 1998 – 2002

Bus No.	37	67	68	69
G_i (MW)	109	1304	512	582
N_i	2	1	2	1

- Stage P2: 2002 – 2008

Bus No.	2	14	35	37	67	68	69
G_i (MW)	1722	533	966	56	84	238	603
N_i	1	2	2	1	1	1	2

The following parameters were used: α=[1200-1400]$/MW, annual discount rate I=10%, swarmsize = 20, maximum number of iterations = 100, maximum number of circuits per path n_{max} = 5. In order to carry out the expansion planning with generation redispatch the maximum generation levels are determined using the following relationship $\overline{g}_k = 1.3 g_k$ [11].

The convergence process for the 87-bus system is shown in Fig.(5).

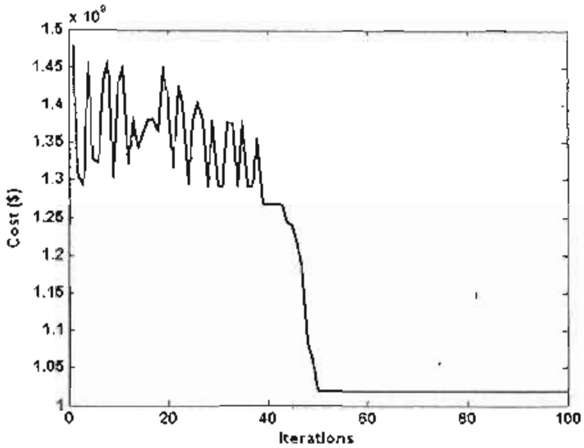


Fig.5 Convergence process of the 87 bus test system

Table 2: The best topology for the 87-bus system

Plan 1									Plan 2								
From - To		n_{ij}	From - To		n_{ij}	From - To		n_{ij}	From - To		n_{ij}	From - To		n_{ij}	From - To		n_{ij}
1	2	2	27	35	2	43	58	2	1	2	1	33	67	1	14	45	1
2	4	1	28	35	1	44	46	2	2	4	1	34	39	1	14	59	1
2	60	1	29	30	2	48	49	2	2	60	2	34	39	1	15	16	2
3	71	2	30	31	1	48	50	2	3	83	1	34	41	1	15	45	2
3	87	1	30	63	1	42	44	1	3	87	2	36	39	1	15	46	1
4	5	2	31	34	1	22	23	1	4	6	1	36	46	1	15	53	2
4	68	1	32	33	1	22	37	1	4	60	2	39	42	2	16	44	1
4	81	1	33	67	2	22	58	1	4	68	2	39	86	2	16	61	1
4	87	1	34	39	2	23	24	2	4	69	1	5	68	2	16	77	1
5	58	1	34	39	1	24	43	1	4	81	2	5	70	1	17	18	2
5	60	1	34	41	2	25	26	2	4	87	1	5	80	1	17	59	1
5	68	2	35	46	1	25	55	2	5	6	2	6	68	1	18	74	1
5	80	1	35	47	1	26	27	2	5	38	1	6	70	1	20	21	2
6	7	1	36	46	2	67	69	2	5	56	2	6	75	1	20	21	1
6	37	1	39	42	2	67	71	1	5	58	2	7	8	1	20	56	2
6	68	1	8	9	1	68	69	1	6	67	1	7	53	2	20	66	2
6	70	1	8	12	1	68	87	1	26	54	2	8	9	2	21	57	2
6	75	1	8	17	2	75	76	1	27	28	2	8	17	1	22	23	2
7	8	1	8	53	2	75	81	1	27	35	1	8	53	1	22	37	1
7	53	1	8	62	2	75	82	1	28	35	1	8	73	2	23	24	2
7	62	1	10	11	1	76	82	1	30	31	2	9	10	2	24	25	1
40	45	1	13	45	1	77	84	1	10	11	1	40	45	1	24	43	1
40	46	1	14	17	2	78	79	2	11	12	2	40	46	1	25	26	1
41	64	1	14	45	2	81	83	2	12	13	1	41	64	1	26	29	2
12	35	1	14	59	1	62	68	1	12	15	1	42	44	1	71	72	1
12	84	2	15	46	2	62	72	1	12	35	1	42	85	1	71	75	1
13	15	1	15	53	1	65	66	1	12	84	1	43	55	2	71	83	2
11	15	2	16	45	1	67	68	1	13	14	1	48	50	1	72	73	1
11	17	1	16	61	1	70	82	1	13	15	1	48	51	2	73	74	2
11	53	1	16	77	2	71	72	1	13	17	1	49	50	1	74	84	2
54	55	2	17	59	2	71	75	2	13	45	1	52	59	1	75	81	1
54	58	1	18	50	1	71	83	1	13	59	1	53	86	1	75	82	1
54	63	1	18	59	2	72	73	2	14	17	2	54	58	2	75	83	1
49	50	2	18	74	1	72	83	1	76	77	2	54	63	1	77	79	2
51	52	2	19	20	2	75	83	2	76	82	1	54	79	1	77	84	1
52	59	1	19	22	2	79	82	1	78	80	1	56	57	1	80	82	1
53	54	2	20	21	2	80	81	2	79	82	1	58	78	1	80	83	1
53	76	1	20	21	1	80	82	2	68	87	1	60	66	1	67	68	2
43	55	2	20	38	2	80	83	1	69	87	1	60	87	2	67	71	1
61	85	2	21	57	1	60	66	1	70	82	1	61	85	2	68	69	1
61	86	1	54	79	1	60	87	1	65	87	1	65	66	1			
62	67	1	56	57	1	61	64	2									

6. CONCLUSIONS

A particle swarm optimization algorithm was proposed for solving the transmission network expansion planning problem with minimum cost. The multistage and coordinated planning of expansion of transmission systems has the capability of adapting to continuous growth of the demand and generation, in contrast with the static planning. Static planning considers only the initial and final years demand and generation. The proposed algorithm also takes into consideration the operation cost of generation unlike the simplified algorithms used in previous works. Results show that the proposed algorithm is not only suitable, but a promising technique for solving such a problem.

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