TIME AND MEMORY STORAGE SAVING IN THE ANALYSIS OF SYMMETRIC PLANAR MICROWAVE CIRCUITS

اختصار زمن الحساب وحجم الذاكرة عند تحليل دوائر الموجات الدقيقة المستوية

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الخلاصة – يقدم هذا البحث طريقة لتحليل دوائر الموجات الميكروويية الرقيقة المتماثلة وذلك بتطبيق طريقة العزوم ثم حل المعادلات الخطية الناتجة – والتي تتميز بالتماثل التكراري لمصفوف المعاملات – بالاستفادة بخواص المصفوفات التكرارية وباستعمال تحويل فورير للمتتابعات الرقمية المنفصلة والطرق السريعة لتنفيذه . ويتضح ذلك وفرا في الوقت وفي حجم الذاكرة مما يجعل الطريقة مناسبة للحاسبات الصغيرة .

ABSTRACT- Conventional moment method with subsectional bases and Dirac testing functions reduces the analysis of symmetric planar microwave circuits to solving a system of linear equations with block circulant coefficient matrix. A method is presented for diagonalizing this matrix using discrete Fourier transforms (DFT). It is shown that much economy in memory space and computation time can be achieved by making use of the properties of block circulants and by implementing the DFT's using fast Fourier transform (FFT) techniques.

I. INTRODUCTION

Planar circuits considered in this work are microwave junctions having dimensions comparable to the wavelength in two directions but much less thickness in the perpendicular direction. The commonly used technique for analyzing these circuits is based on a contour integral representation of the Helmholtz wave equation which is reduced to a matrix equation by the conventional method of moments; usually using pulse functions as subsectional basis functions and Dirac delta functions as testing functions [1-5].

A drawback of this method is that it involves the inversion of large order matrices to get the matrix-impedance description of the junction. The situation is worse when the method is used for the analysis of planar resonators, for even with an efficient root finding algorithm like, for instance, the Muller algorithm, a big determinant has to be evaluated repeatedly in order to determine the resonant frequency. Matrix inversion and determinant evaluation are time and memory-space consuming operations, especially with limited computer resources.

This paper presents a proposal to reduce the memory-space and computation time in the analysis of planar circuits with rotational symmetry, where an appropriate discretization of the contour integral is shown to result in a system of linear equations with a block-circulant coefficient matrix. This is a matrix in which a basic row of blocks is reper again and again but with a shift in position. In practical computations, therefore, only basic row need to be computed and stored in the computer memory. Besides, the bu periodicity means that block circulants tie in with Fourier analysis and in the present we show how block circulant equations can be solved using FFT techniques which p considerable saving of computation time.

In a recent work, a technique is suggested to use FFT to speed analysis of sy cal planar junctions with circular boundaries characterized by a system of linear e with circulant coefficient matrix [5]. The present work considers m-fold symmetric junctions and is, therefore, a generalization from which the special case treated in [5] is readily deduced.

II. THE CONTOUR INTEGRAL METHOD

Consider an m-fold symmetric planar junction made up of a center conductor sandwiched by two substrates. For the sake of generality, the substrates are assumed to be of ferrite material with a magnetic field acting perpendicular to the ground conductors as shown in Fig. 1a. When the thickness d is much smaller than the wavelength and the ferrite spacers are homogeneous and linear, only the field components E_z, H_x, and H_y do exist and are independent of z. It is deduced from Maxwell's equations that the RF-voltage $V=dE_z$ satisfies the Helmholtz equation

$$(\nabla_t^2 + k^2)\, V = 0 \qquad \ldots (1)$$

where

$k = (\omega/c)\,(\mu_e\, \epsilon_f)^{1/2}$

ϵ_f = relative dielectric constant of the ferrite

μ_e = effective permeability of the ferrite

$= (\mu^2 - k^2)/\mu$

μ, k = diagonal and off-diagonal elements of the permeability tensor.

At a coupling port, the following boundary condition should be satisfied

$$\left(j\,\frac{k}{\mu}\,\frac{\partial V}{\partial t} + \frac{\partial V}{\partial n} \right) = -j\omega\mu_e\, d\, i_n \qquad \ldots (2)$$

where i_n is the surface current density normal to the boundary and ∂t and ∂n are, respectively, the derivative tangential and normal to the boundary. At parts of the boundary where there are no coupling ports we may assume, neglecting fringing fields, a perfect magnetic wall, i.e. $i_n = 0$.

Following Miyoshi et al. [2,4] and using Weber's solution for the cylindrical Green's functions, equation (1) with the boundary condition (2) is reduced to the contour integral equation

$$V_p = \frac{1}{2j} \int_c \left[-j\omega\mu_e\, d\, H_0^{(2)}(kr)\, i_q + k\,(\cos\theta - j\,\frac{K}{\mu}\sin\theta)\, H_1^{(2)}(kr)\, V_q \right] dt \qquad \ldots (3)$$

where p and q are points on the boundary c of the junction and the symbol $\int$ denotes Cauchy's principal value. The variables r and θ are as indicated in Fig. 1.

The integral equation (3) has been solved by first discretizing the contour into N-uniform elements. The conventional method of moments is then applied with N-pulse functions defined at N-sampling points at the centres of the elements as testing functions. In this way the integral equation is reduced to the matrix equation

$$U\ V = H\ I \qquad \ldots (4)$$

where V and I are column vectors made up, respectively, of the voltages and current densities at the sampling points. The elements of matrices U and H are [5]

$$u_{ij} = 1 \qquad i = j$$
$$= \frac{-kW}{2j} (\cos\theta - j\frac{k}{u}\sin\theta)\, H_1^{(2)}(kr) \quad i \neq j \qquad \dots (5)$$

$$h_{ij} = \frac{-\omega\mu_e d W}{2} H_o^{(2)}(kr) \qquad i \neq j$$
$$= \frac{-\mu_e d W}{2} \left(1 - \frac{2}{j}\left(\log\frac{kW}{4} - 1 + \gamma\right)\right) \quad i = j \qquad \dots (6)$$

Due to the m-fold symmetry of the junction, the values of the variables r and Θ are repeated every n = N/m sampling points so that

$$r_{ij} = r_{kl} \text{ and } \Theta_{ij} = \Theta_{kl} \qquad \dots (7)$$

provided

$$k = [\, i+p\,n \,] \text{ and } l = [\, j+p\,n \,] \qquad \dots (8)$$

Where p is an integer and the brackets denote residue module N. Since the matrix elements u and h are functions of r and Θ only (equations 5 and 6), these elements are periodic in the same manner as the independent variables so that the matrix U, for instance, has the from

$$U = \begin{bmatrix} u_{11} & u_{12} \cdots u_{1n} & u_{1n+1} & u_{1n+2} \cdots & \cdots u_{12n} & \cdots u_{1N-n+1} & \cdots u_{1N} \\ u_{21} & u_{22} \cdots u_{2n} & u_{2n+1} & \cdots & \cdots u_{22n} & \cdots u_{2N-n+1} & \cdots u_{2N} \\ \cdots & \cdots U_1 \cdots & \cdots & U_2 & \cdots & \cdots & U_m \cdots \\ u_{n1} & u_{n2} \cdots u_{nn} & u_{nn+1} & \cdots & \cdots u_{n2n} & \cdots u_{nN-n+1} & \cdots u_{nN} \\ & U_m & & U_1 & \cdots & & U_{m-1} \\ & \vdots & & \vdots & & & \vdots \\ & U_2 & & U_3 & \cdots & & U_1 \end{bmatrix} \qquad \dots (9)$$

The matrix U is made up of a row of blocks $(U_1, U_2, \dots, U_m)$ which repeats itself but with a shift to the right. Such a matrix is called "block circulant" matrix. Hereafter we shall use the notation

$$U = \text{bcirc}\,(U_1, U_2, \dots, U_m) \qquad \dots (10)$$

to denote block circulants. Accordingly, equation (4) is rewritten as

$$\text{bcirc}\,(U_1, U_2, \dots, U_m)V = \text{bcirc}\,(H_1, H_2, \dots, H_m)\, I \qquad \dots (11)$$

From this last equation the impedance matrix of the equivalent N port is given by [2,3]

$$Z = U^{-1}\, H \qquad \dots (12)$$

where U^{-1} denotes the inverse of the matrix U. When the circuit has but only m ports, then Z can readily be reduced to the corresponding m x m terminal impedance matrix based on knowledge of either the electric or the magnetic field distribution on the ports. One usually adopted approximation is to assume that the magnetic field distribution is uniform and identical with the lowest order TEM stripline mode. The impedance matrix entries are then obtained from the elements of the Z matrix on the basis of an average electric field across the striplines [1-3].

When the junction has no coupling ports, then

$$\det U = 0 \qquad \ldots (13)$$

gives the proper frequency for which equation (11) has a non-trivial solution, that is the resonant frequency of the planar structure.

III. Solution Of The Block-Circulant Matrix Equation

As readily seen from the previous section, the majority of the computational effort with the contour integral method is devoted to computing the entries of the matrices U and H and to inverting the matrix U or solving equation (11). The block circulant structure of U and H for an m-fold symmetric junction reduces the number of matrix entries to be computed and stored by a factor of 1/m. Further saving in computation time would be realized if the number of basic operations required to solve the block circulant equation could be reduced. Indeed, this is possible by making use of the intimate connection of block-circulants with Fourier transforms. Thus, it is shown in the appendix how a block circulant is diagonalized using discrete Fourier transforms so that equation (11) may be transformed into.

$$(F_m \Theta F_n)^* \operatorname{diag}(A_1, \ldots, A_m)(F_m \Theta F_n) V = C \qquad \ldots (14)$$

where C stands for the right side of the original equation, F_m and F_n are Fourier matrices of order m and n, respectively, and Θ denotes tensor or Kroncker products. The square blocks $A_1, \ldots, A_m$ are derived from the corresponding blocks of the U matrix as follows. Compute

$$B_i = F_n U_i F_n^* = (F_n (F_n U_i)^*) \quad i = 1, \ldots, m \qquad \ldots (15)$$

then

$$(A_1 \ldots A_m)^T = m \; (F_m^* \Theta I_n)(B_1 \ldots B_m)^T \qquad \ldots (16)$$

Next, the following substitutions are made

$$X = (F_m \Theta F_n) V \qquad \ldots (17)$$

$$Y = (F_m \Theta F_n) C \qquad \ldots (18)$$

Equation (14) then becomes

$$\operatorname{diag}(A_1, \ldots, A_m) X = Y \qquad \ldots (19)$$

or

$$A_i X_i = Y_i \qquad i = j, \ldots, m \qquad \ldots (20)$$

where X_i and Y_i are vectors obtained by partioning X and Y, respectively, into m subvectors each of n elements. The potential V is obtained by solving the m systems for X and inverting the transformation in equation (17).

In this way the original matrix equation (11) of order mn degenerates into m seperate systems each of order n. The whole process can be programmed using only two square arrays of dimension nxn. One of these arrays is used as working space for computing the U blocks and the other for storing the current A block. On the other hand, a direct solution of equation (11) would require at least mn x mn memory spaces. Beside this save in memory space, the present technique provides a significant reduction in computation time. Thus, it is shown in Table 1 that the number of multiply-add operations required to implement the proposed method is of the order of mn^3, compared with $m^3 n^3$ operations for solving the original equation (11) by conventional Gaussian elimination or Crout-LP factorization. It is assumed that the DFT's involved are carried out using fast transform algorithms instead of convertional matrix multiplication. This reduces the number of multiply-add operations required to transform a sequence of length n from n^2 to $n \log_2 (n)$ [6].

By the way of illustration, we applied the present method to the analysis of the planar Y-junction circulator shown in Fig. 1b. The results plotted in Fig. 2 have been obtained with a total of 48 nodes and are in good agreement with the corresponding results of reference [4] The computed elements of the scattering matrix of the junction are found to satisfy the unitary condition to within 1 percent, which indicates the accuracy of computations. The computation time to solve the problem, i.e. to determine the scattering and loss parameters at a specific frequency, is less than 2 minutes on an NCR-TOWER minicomputer. Performing the same computations, but using Gaussian elimination algorithm to solve the system of linear equations, is

Table 1: Steps and approximate number of multiply-add operations for solving the block-circulant equation (11).

Step	Notes	Approximate number of multiply-add operations
Form m B blocks equation (15)	2 mn FFT	$3mn^2 \log n$
Form m A blocks equation (16)	nm FFT of sequences of length m	$nm^2 \log (m)$
Transform the right-side $Y = (F_m \odot F_n) C$	FFT of m sequences of length n followed by n FFT's of sequences of length m	$mn \log (n) + nm \log (m)$
Solve the systems $A_i X_i = Y_i$ $i = 1, \ldots, m$	solution by Crout-LP factorization	mn^3
Transform X to obtain V $V = (F_m \odot F_n) * X$	2 mn FFT	$2 mn^2 \log (n)$
		Total : $O (mn^3)$

found to take 6.2 minutes on the same machine. This indicates the save in computation time provided by the present technique. However, this save is less than would be expected

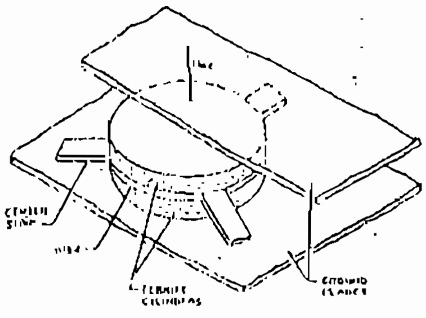

Fig. 1-a A ferrite planar circuit.

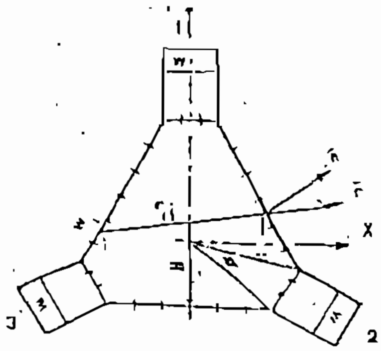

Fig. 1-b Center conductor and coordinate system for a planar triangular circulator.

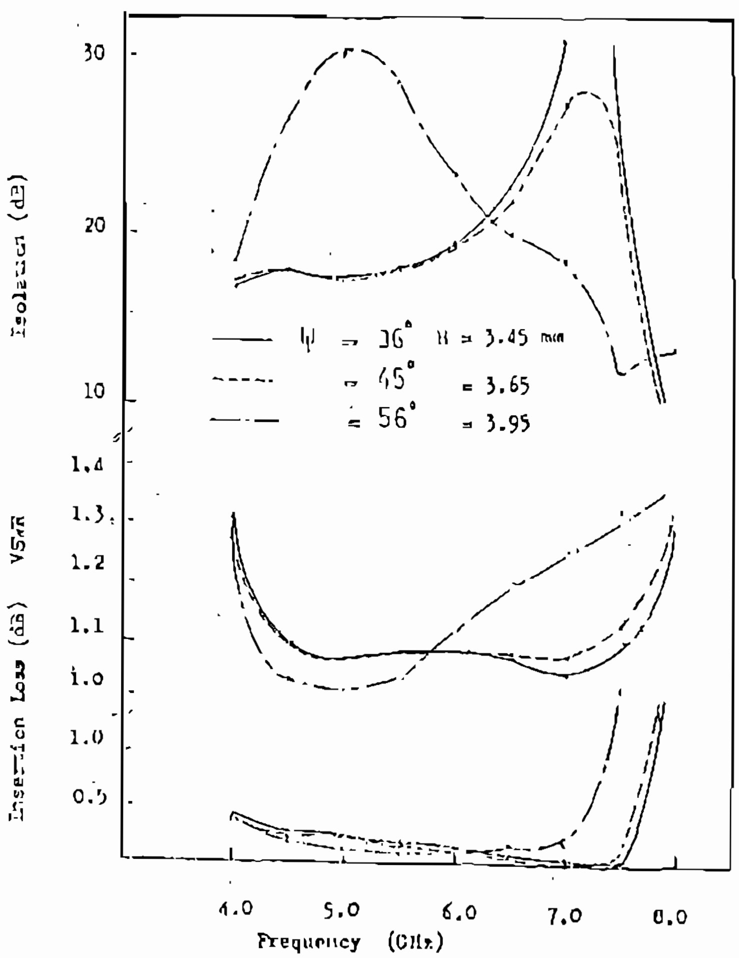

Fig. 2 Computed performance of the triangular circulators.

from Table 1, which compares only the numbers of arithmetic operations necessary to solve the system of linear equations. The overall computation time includes also the time consumed in forming the matrix elements and other computations.

IV. CONCLUDING REMARKS

Moment-method analysis of symmetric microwave planar junctions is shown to result in matrix equations with block circulant structure. These matrices are intimately related to Fourier analysis: the eigen-vectors of the basic circulant are the columns of the discrete Fourier transform matrix.

A similarity transformation for diagonalizing block circulants has been presented and its implementation using FFT techniques has been demonstrated. When incorporated with the moment method , the proposed transform provides great economy in computation time which adds to the memory-space saving distinguishing block circulants. This time saving would be particularly useful when the solution is iterated in order, for instance, to determine the resonant frequency of a planar resonator or the optimum circuit pattern of a symmetric planar structure [7].

The present technique for the manipulation of block circulants is readily applicable in other electromagnetic field problems involving m-fold symmetric structures analysed by moment methods.

APPENDIX: Diagonalization of Circulants and Block - Circulants

In this appendix we develop similarity transformations for diagonalizing circulant and block-circulant matrices we begin by introducing some basic definitions.

Definition D1 : If A and B are, respectively, mxn and pxq matrices, the Kroncker or tensor product of A and B is the mp x nq matrix.

$$A \times B = \begin{bmatrix} a_{11} B & a_{12} B & \cdots & a_{1n} B \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} B & a_{m2} B & \cdots & a_{mn} B \end{bmatrix} \qquad (A1)$$

Definition D2: The basic circulant P_n is the square matrix of order n defined by

$$P_n = \mathrm{circ}\,(0, 1, 0, \ldots, 0) = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & 0 & 0 & \cdots & 0 \end{bmatrix} \qquad (A2)$$

It is readily seen that P is a permutation matrix in the sense that post-multiplication of an arbitrary matrix A by P amounts to a right-shift of the columns of A while pre-multiplication of A by P amounts to an upward shift of the rows of A. It follows that

$$\begin{aligned} P^2 &= P \times P = \mathrm{circ}\,(0, 0, 1, 0, \ldots, 0) \\ P^3 &= \mathrm{circ}\,(0, 0, 0, 1, \ldots, 0) \\ \cdots & \quad \cdots \quad \cdots \quad \cdots \\ P^n &= \mathrm{circ}\,(1, 0, 0, \ldots, 0) = I_n \end{aligned} \qquad (A3)$$

where I_n is the identity (unit) matrix of order n. This last result expresses the fact that repeated multiplication of A by P n times maps A into itself.

<u>Definition D3</u> : By the Fourier matrix of order n is meant the matrix $F = F_n$ where

$$F_n^* = \frac{1}{\sqrt{n}} \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & w & w^2 & \cdots & w^{n-1} \\ 1 & w^2 & w^4 & \cdots & w^{2n-2} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & w^{n-1} & w^{2(n-1)} & \cdots & w^{(n-1)(n-1)} \end{bmatrix} \qquad (A4)$$

the star means conjugate-transpose

and $1, w, \ldots, w^{n-1}$ are the n primitive roots of unity. Since $w^n = 1$, $w^{-k} = w^{n-k}$ and F^* can be written alternatively as

$$F_n^* = \frac{1}{\sqrt{n}} \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & w & w^2 & \cdots & w^{n-1} \\ 1 & w^2 & w^4 & \cdots & w^{n-2} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & w^{n-1} & w^{n-2} & \cdots & w \end{bmatrix} \qquad (A5)$$

Both F and F* are symmetric and it can be easily established that

$$F\, F^* = I_n \quad \text{or} \quad F^{-1} = F^* = \bar{F} \qquad (A6)$$

where the bar denotes complex conjugate. From the definition of F_l it follows that if $Z = (Z_1, Z_2, \ldots, Z_n)^T$ is a sequence of complex numbers, then $\hat{Z} = FZ$ is the usual discrete Fourier transform of Z.

The following theorem establishes the relation between the basic circulant and other circulants or block-circulants.

<u>Theorem T1</u> Let A be block-circulant
$A = \text{bcirc}\,(A_1, A_2, \ldots, A_m)$

where the A_k's are square matrices of order n. Then

$$A = \sum_{k=0}^{m-1} P_m^k \otimes A_{k+1} \qquad (A7)$$

<u>Proof</u> *From definitions D1 and D2 and equations A3,*

$$P_m^0 \quad A_1 = I_m \ominus A_1 = \begin{bmatrix} A_1 & 0 & 0 & \dots & 0 \\ 0 & A_1 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & & A_1 \end{bmatrix}$$

$$P_m^1 \ominus A_2 = \begin{bmatrix} 0 & A_2 & 0 & \dots & 0 \\ 0 & 0 & A_2 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ A_2 & 0 & 0 & \dots & 0 \end{bmatrix}$$

$$P_m^2 \ominus A_3 = \begin{bmatrix} 0 & 0 & A_3 & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & A_3 & 0 & \dots & 0 \end{bmatrix}$$

etc.

The theorem follows by summing up the above equations. In the special case-when the A_ks are ordinary scalers, A is a circulant and the theorem reduces to

$$\text{Circ}\,(a_1, a_2, \dots, a_n) = \sum_{k=1}^{m-1} a_{k+1}\, P_m^k \qquad \text{(A8)}$$

Next we prove the following theorem concerning the diagonelization of the basic circulant P.

<u>Theorem T2</u> $P_n = F_n^* \; W_n \; F_n$ (A9)

where W_n is the diagonal matrix of order n defined by

$$W_n = \text{diag}\,(1, w, w^2, \dots, w^{n-1}) \qquad \text{(A10)}$$

<u>Proof</u> The theorem can be proved by evaluating the matrix product F*WF following the rules of conventional matrix multiplication. Another approach, which will make clear the relation between the basic circulant P and the Fourier matrix F, is to consider the eigenvalue problem associated with P, namely

$$P \;\; p_i = \lambda_i \, p_i \qquad \text{(A11)}$$

Multiplying both sides by P (n-1) times and using (A3), it is readily seen that

$P^n\, p = Ip = \lambda^n\, p$ or $\lambda^n = 1$

The solution of this last equation is $\lambda = 1, w, \dots, w^{n-1}$. In other words, the eigenvalues of P are the diagonal elements of W so that

$$\begin{bmatrix} \lambda_1 & 0 & \dots & 0 \\ 0 & \lambda_2 & \dots & \dots \\ 0 & 0 & \dots & \dots \\ \dots & \dots & \dots & \dots \end{bmatrix} = W \qquad \text{(A.2)}$$

The eigenvectors are obtained by solving the eigenvalue equation (A11). It is easily seen that

$$P_j = (1, \lambda_j, \lambda_j^2, \ldots, \lambda_j^{n-1})^T \qquad (A13)$$

where the first component has been arbitrarily set equal to unity as the eigenvalues are always determined up to a constant multiplier. Substituting $\lambda_j = 1, w, \ldots w^{n-1}$, respectively, we see that the eigenvectors of P are the columns of the Fourier matrix F*, or

$$[\, P_1 \quad P_2 \quad \ldots \quad P_n \,] = F^* \qquad (A14)$$

Combining the eigenvalue equations (A11) for all eigenvalues and corresponding eigenvectors of the matrix P in a single matrix equation, we get

$$P\,[\, P_1 \; P_2 \ldots P_n \,] = [\, P_1 \; P_2 \ldots P_n \,] \begin{bmatrix} \lambda_1 & 0 & \ldots & 0 \\ 0 & \lambda_2 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & \lambda_n \end{bmatrix}$$

or, using (A12) and (A14)

$$PF^* = F^* \Lambda \qquad (A15)$$

Premultiplying by F and using (A6) we see that the basic circulant P is diageonalized by the following similarity transformation

$$F^{-1} P F = \Lambda \qquad (A16)$$

and the theorem follows. We now make use of this theorem and the relation between the basic circulant and block circulant matrices establisheo in theorem T1 to develop a similarity translormation for diagonalizing these latter matrices. The result is stated in the following theorem.

Theorem T3 If A is an arbitrary block circulant made up of m basic blocks of order n, then there are m square matrices $M_1, \ldots M_m$ of order n such that

$$A = \mathrm{bcirc}\,(A_1, \ldots, A_m)$$
$$= (F_m \otimes F_n)^* \,\mathrm{diag}\,(M_1, \ldots, M_m)\,(F_m \otimes F_n) \qquad (A17)$$

Proof From theorem T1 we have

$$A = \sum_{k=n}^{n-1} (P_m^k \otimes A_{k+1}) \qquad (A18)$$

But from theorem T2 and identities (A6), it follows that

$$P_m^k \otimes A_{k+1} = (F_m^* W^k F_m) \otimes F_n^* (F_n A_{k-1} F_n^*) F_n \qquad (A19)$$

Letting $B_k = F_n A_{k+1} F_n^*$ and using the tensor product identity $UX \otimes VY = (U \otimes V)(X \otimes Y)$, the line above becomes

$$(F_m^* \odot F_n^*)(W^k \odot B_k)(F_m \odot F_n)$$

Therefore,

$$A = (F_m \odot F_n)^* \left(\sum_{k=0}^{m-1} W^k \odot B_k \right) (F_m \odot F_n) \quad \text{(A20)}$$

Now, by an explicit computation, it is seen from the definition of W that

$$\sum_{k=0}^{m-1} W^k \odot B_k = \text{diag}\,(M_1, M_2, \ldots, M_m) \quad \text{(A21)}$$

where

$$(M_1\ M_2\ \ldots\ M_m)^T = m^{1/2}\,(F_m^* \odot I_n)\,(B_0\ B_1\ \ldots\ B_{m-1})^T \quad \text{(A22)}$$

Thus $$A = (F_m \odot F_n)^* \text{diag}\,(M_1, M_2, \ldots M_m)\,(F_m \odot F_n) \quad \text{(A23)}$$

and the theorem is proved.

If n = 1, the block circulant degenerates into an ordinary circulant, and from (A23) we see that a circulant of order m may be represented as

$$A = \text{Circ}\,(a_1, a_2, \ldots, a_m) = F^* M F,$$

$$M = m^{1/2}\,\text{diag}\,(F^*(a_1\ a_2\ \ldots\ a_m)^T) \quad \text{(A24)}$$

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