STABILITY CHECKS OF COLUMNS AND BEAMS N BRACED FRAMES USING ELASTIC EFFECTIVE LENGTHS.

اختبارات الأتزان للأعمدة والكمرات فى الأطارات المسنودة مستخدما الأطوال الفعالة المرنة

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ملخص البحث :

فى هذا البحث تم دراسة اختبارات الأتزان للأعمدة والكمرات فى الأطارات المسنودة جانبيا على اساس استخدام الأطوال الفعالة المرنة. وقد تم استنتاج معادلة تصميمية بسيطة لأختبارات اتزان الأعمدة المفردة ذات النهايات المفصلية حيث يكون الطول الفعال مساويا لطول المنشأ الأساسى. أيضا فى هذا البحث قد تم الأخذ فى الأعتبار دراسة مشكلة اختبارات الأتزان للأطارات المفردة والممنوعة من الأنحراف جانبيا باستخدام العمود المكافئ المفرد ' وفى هذة الطر يقة يتم استبدال اختيارات الأتزان للأطارات المفردة و المسنود جانبيا باختبارات الأتزان لأعمدة مكافئة مفردة مع الأخذ فى الأعتبار المتطلبات الضرورية للكمرات فى هذة الأطارات. ومن الدراسة تبين أن استخدام طريقة العمود المكافئ المفرد قد أدى الى تبسيط مشكلة اتزان الأطارات المفردة و المسنودة جانبيا حيث تم استنتاج معادلة تقريبية بسيطة للعزم السائد الضرورى لعمل اختبارات الأتزان على هذا النوع من الأطارات و المؤثر عليها باحمال مركزة على الأعمدة.

ABSTRACT

Checking for stability of columns and beams in bracec frames is based on the elastic effective lengths. Simple design formulae. for checking the stability of the individual Pin-ended columns, are derived, where the effective length is equal to the line structure length. In this paper the problem of checking the stability of single braced frames, by using the equivalent column approach is considered. i.e the stability check of the total braced frame is replaced by stability cheeks of the individual Columns. Requirements necessary for the beams of a simple braced frame are sought, when the column is checked for stability using the elastic effective length .

Notations

A	cross sectional area
b	beam length
E	Young's modulus
I	moment of inertia
K	rotational stiffness
l_b	buckling length
l_e	elastic effective length
l_s	system length or line structure length
P	concentrated load
P_A	column capacity based on $l_b = l_e$
P_B	column capacity based on $l_b = l_s$
P_E	Euler buckling load

M_1	end-moment with the smallest absolute value
M_2	end-moment with the largest absolute value
M	absolute value of M_2
M_p	Plastic moment of a column section
$M_{p,red}$	reduced plastic moment
M_r	restraining moment
N_p	squash load
w	unifomly distributed load
Δ	displacement
X, Y	rectangular coordinates
δ	displacement
δ_o	initial imperfection
δ_o^*	amplitude of imperfection
λ	slenderness ratio
λ_e	elasic effective slenderness ratio
λ_s	system slenderness ratio

INTRODUCTION

A definition of structural stability is given in terms of the sensitivity of structures to variations of the design parameters. However, the overall stability of structures is a fundamental engineering concept which, nevertheless, has only been rather loosely defined in the past. In an attempt to improve the understanding of this concept the main factors governing stability are discussed below.

In general, the stability of a braced frame is checked using the first order elastic force distribution. Stability check of the total braced frame is replaced by stability checks of the individual columns. The columns are 'cut' out of the frame and the resultant pin-ended columns, with the imposed bending moments and axial forces, are checked for stability, Fig. (1), [10]. The problem of the stability of a braced frame is thus simplified to that of an individual columns .

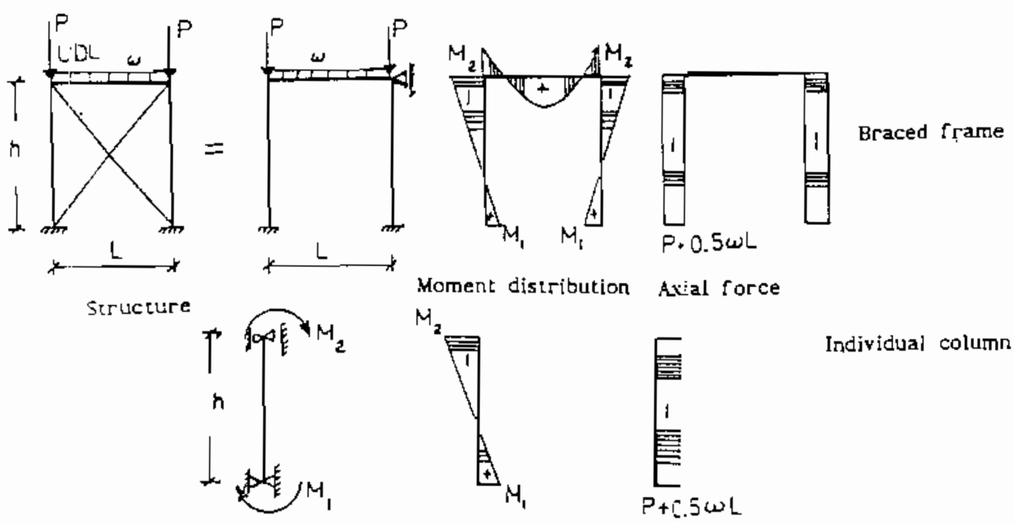


FIG. (1) The individual column approach

Non-linear analyses of such columns have led to interaction curves that describe the collapse of columns under combined bending and axial compression. Interaction formulae are derived from these interaction curves.

These formulae [11] can be written in the following forms;

$$\frac{N}{A} + \mu^{*} . \frac{\beta M + N e^{*}}{Z_e} \leq \sigma_y \quad (1)$$

where :

$$\mu^{*} = \frac{\mu}{\mu - 1} \quad (2)$$

- A is the cross-sectional area,
- N is the axial load,
- M is the absolute value of largest end bending moment,
- μ is the ratio between Euler buckling load and axial force,
- e^{*} is the imperfection parameter,
- Z_e is the elastic section modulus,
- β is the equivalent moment factor, and
- σ_y is the yield stress.

also μ and e^{*} are based on the buckling length.

according to ECCS [11]

$$\beta = 0.6 + 0.4 \frac{M_1}{M_2} \geq 0.4 \quad (3)$$

Equation (1) can be modified as follows,

$$\rho . \frac{N}{A} + \mu^{*} \frac{\beta M}{Z_p} \leq \sigma_y \quad (4)$$

where:

ρ :the buckling coefficient,

Z_p : the plastic section Modulus,

ρ and μ are based on the buckling length and β is given in Eq. (3).

Eq.(4) can be written in the following form,

$$\rho \frac{N}{N_p} + \mu^{*} \frac{\beta M}{M_p} \leq 1 \quad (5)$$

The β Factor accounts for the shape of the moment distribution and is a measure of the magnitude of the bending moment in a critical section of the column in the deformed situation [1] . In fact, the stability check of the individual column is reduced to a check on the characteristic moment .

1 : THE BUCKLING LENGTH CONCEPT, AND THE PROBLEM DESCRIPTION

When the individual column approach is consistently applied, the buckling length should be taken equal to the system length, Fig.(1). However the buckling length concept is used in Eqns. (1-4). In fact, the columns form part of the braced frame and it can be shown that the use of the system length as buckling length is conservative in many cases. In the extreme case of a column in conjunction with a beam with infinite stiffness and no bending moments, the elastic carrying capacity is four times that of a pin-ended column, Fig.(2). When the end moments obtained by linear elastic moment distribution, are small, the use of the individual column approach with $l_b = l_s$ gives conservative results. This is also shown by geometric and material non-linear analyses of braced frames with initially imperfect columns, Fig.(3). These analyses are carried out with the finite element method (f.e.m.) using computer program DIANA [3].

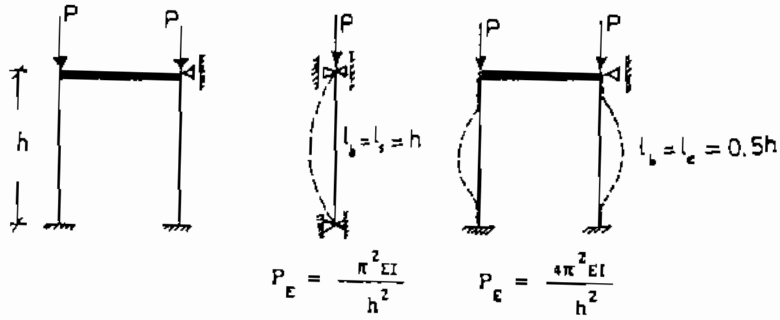


FIG. (2) Braced frame where $l_b = l_s$ is conservative

The imperfections in the columns are so determined that a pin-ended axially loaded column attains collapse[*] at the load carrying capacity resulting from the ECCS buckling curves using $l_b = l_s$.

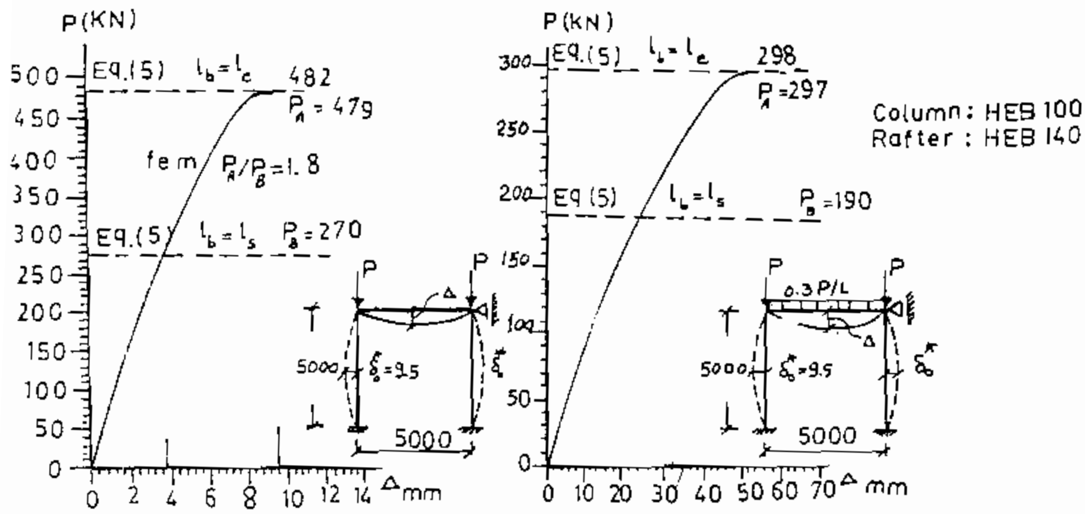
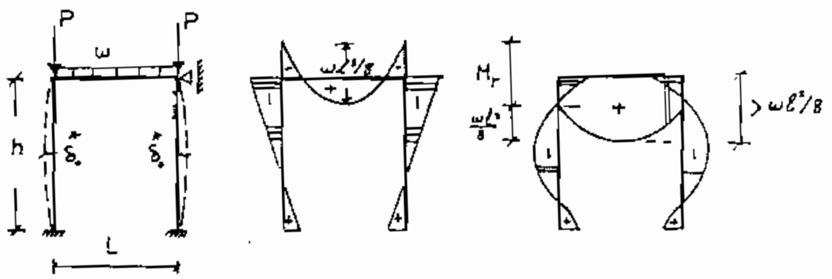


FIG.(3) Comparison of finite element method (f.e.m.) results, [3] with Eq.(5)

The finite element method analyses (f.e.m.)[3] show that the interaction formula, using $l_b = l_e$, gives a good approximation to the load carrying capacity of the frame, if the collapse of the columns characterizes the frame behaviour, Fig.(3). Therefore, checks on column stability in braced frames are acceptable on the basis of $l_b = l_e$, [11]. This, however has consequences for checking of the beams. The moment distribution at collapse is different from the moment distribution using the first order elastic analysis. At collapse the sum of the bending moments on the beam is greater than $\omega l^2/8$, Fig.(4). This is caused by the top restraining moment M_r that has to stabilize the column. At the top of the column, the bending moment can even change sign, Fig.(5). Figures (4) and (5) show that knowledge of the moment distribution at collapse is indispensible for checking of the beams. In view of this it is clear that checking the beams on the basis of the first order moment distribution is insufficient .



(a) Structure (b) B.M. 1st order-elastic (c) B.M. at collapse

FIG. (4) **Comparison of bending moment diagrams**

It is suggested [11,15] that the first order elastic moment distribution should be magnified by multiplication with the amplification factor $(\mu/(\mu-1))$ of the column, Fig.(4-b). This approach does not satisfy equilibrium. In addition, a possible change of sign of the moment at the top of the column is not brought into account. Therefore, other methods are discussed in this paper.

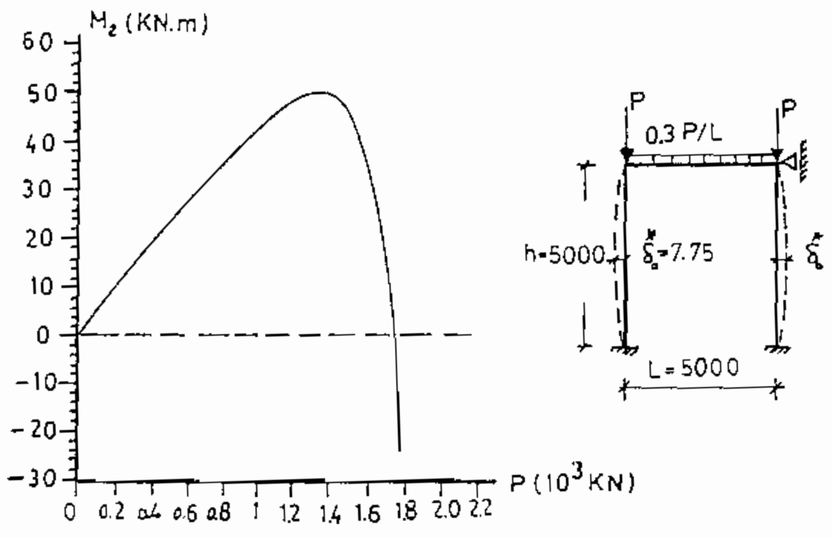


FIG. (5) **Change of sign of bending moment at the top of the column**

Only by shifting the first order elastic bending moment diagram of the beam is it possible to satisfy equilibrium and to account for a possible change in the sign of the moment, Fig..(4-c). The bending moment diagram is shifted by a value of M_r. The restraining moment M_r is the moment carried over from the column to the beam when the column achieves collapse. In other words, M_r is the moment that the beam should provide in order to stabilize the column. The restraining moment M_r, that the beam has to offer to the column at the moment of column collapse, will be attempted to be determined in a simple manner.

2: DETERMINATION OF THE RESTRAINING MOMENT, M_r

A simple braced frame is considered for determining the restraining moment. Two methods are described for determining the restraining moment for a single storey, single bay braced frame with concentrated loads on the column.

2:1 The First Method : The Individual Column Approach

The column capacity for the frame A shown in Fig.(6-a) is P_A, based on $I_b = I_e$. If the column is isolated from the frame ($I_b = I_s$) it can only carry load P_B, Figs.(6-b) & (6-c). The carrying capacity of the individual column can be increased to P_A which is greater than P_B by the application of restraining moment M_r, Fig.(6-d). The value of M_r, required to increase the carrying capacity has to be determined. The magnitude of the restraining moment decides the strength requirement for the beam . The following assumptions are considered for the calculations of the restraining moment M_r, Fig. (6-d) :-

a- The sections possess a bilinear moment-curvature diagram, which is a reasonable approximation for I-sections.

b- The yield criterion used, Ref. [6], is as given in Eq.(6),

$$\left.\begin{array}{ll} N < 0.15\, N_p & M_{p,red} = M_p \\ N \geq 0.15\, N_p & M_{p,red} = 1.18\, M_p \left(1 - \dfrac{N}{N_p}\right) \end{array}\right\} \qquad (6)$$

c- The calculations are geometric non-linear computations based upon equilibrium in the deformed situation [17,18].

d- The columns have parabolic initial deformations, δ_o which include geometrical imperfections and residual stresses, in which ;

$$\delta_o = \frac{-4\,\delta_o^*}{h^2}\,(-x^2 + xh) \qquad (7)$$

where; x is the horizontal coordinate.
h is the column length.

e- For all calculations, the ECCS [11] buckling curve b is used

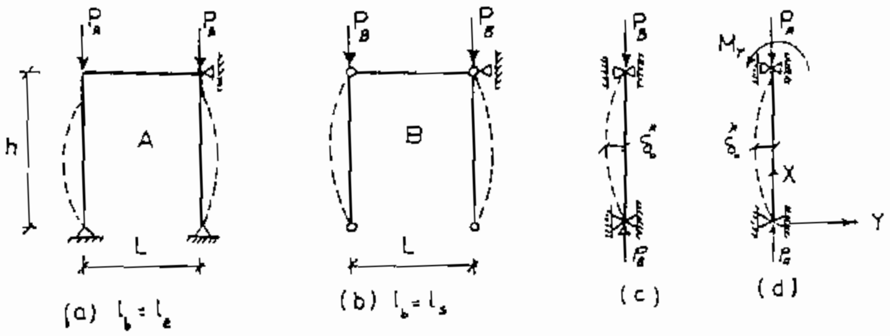


FIG. (6) The first method : The individual column approach

2.1.1: Procedure For Determining The Amplitude of Initial Imperfection δ_o^*

For a pin-ended column with a parabolic initial imperfection, the following differential equation can be derived :

$$\frac{d^2\delta}{dx^2} + \alpha^2 \delta = -\frac{8\,\delta_o^*}{h^2} \qquad \text{with} \quad \alpha^2 = \frac{P}{EI} \qquad (8)$$

for $x = 0$ $\delta = 0$
for $x = h$ $\delta = 0$

P is taken equal to P_B , on the basis of $l_b = l_s$. Then δ_o^* is determined so that the column just attains collapse. A plastic hinge then occurs in the middle of the column.

2.1.2 Procedure For Determining The Required Restraining Moment M_r If The Column is Loaded with P_A.

From the column of Fig. (6-d) the following differential equation can be formulated ,

$$\frac{d^2\delta}{dx^2} + \alpha^2 \delta = -\frac{8\delta_o^*}{h^2} - \frac{M_r . x}{EI.h} \qquad \text{with } \alpha^2 = \frac{P}{EI} \qquad (9)$$

for $x = 0$ $\delta = 0$
for $x = h$ $\delta = 0$

P is taken equal to P_A, on the basis of $l_b = l_e$.The restraining moment M_r is then determined so that the column just attains collapse. A plastic hinge occurs somewhere in the column.

2.1.3 The Results of The First Method

The Figures (7-a) and (7-b) give the results for two system slendernesses. The solid line gives the necessary restraining moment as a function of the amount of partial restraint of the column. The dashed line gives the reduced plastic moment as a function of the amount of partial restraint of the column. Both figures show that the restraining moment increases. Because P_A also increases, the reduced plastic moment decrease.

Figure (7-a) is characteristic for system slendernesses $\lambda_s < 100$. To the left of the intersection point in Fig.(7-a), the necessary restraining moment is greater than the reduced plastic moment. Therefore, the required restraining moment cannot be achieved at the top of the column. In this area, the first method cannot be used with $l_b = l_e$ for checking the column.

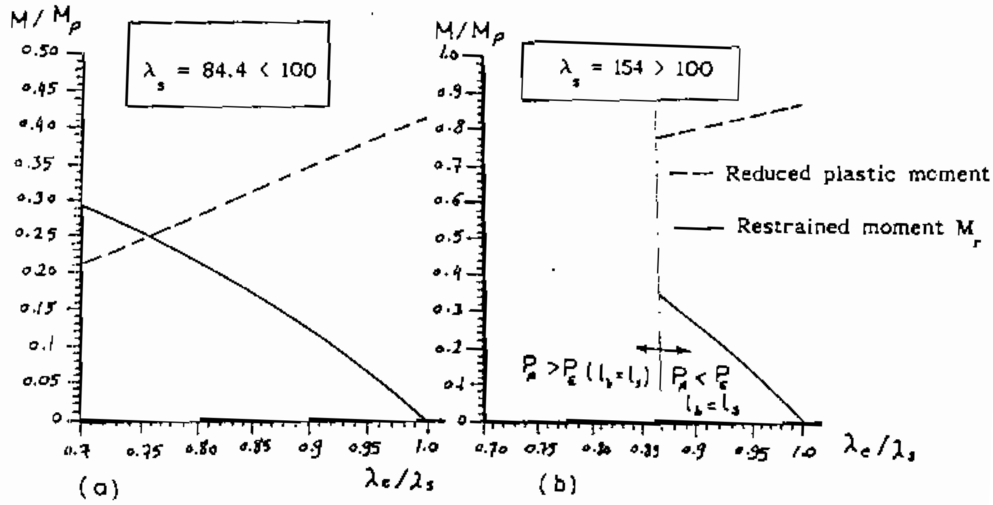


FIG.(7) Results for two characteristic system slendernesses (The first method)

Figure (7-b) is characteristic for system slendernesses $\lambda_s \geq 100$ and in comparison to Fig.(7-a) is discontinued before an intersection point is achieved. This is because the model in Fig.(6-d) cannot carry loads greater than the Euler buckling load based on $l_b = l_e$. Also see Fig.(8).

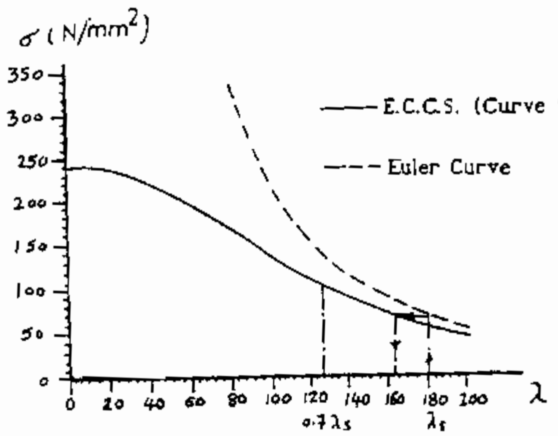


FIG. (8) P_A cannot exceed P_E ($l_b = l_s$)

For areas in Fig.(7) where the restraining moment is smaller than the reduced plastic moment, $l_b = l_e$, may be used for checking the column stability. The strength requirement necessary for the beam can be obtained from Fig.(7). Because of compatibility requirements at the column / beam junctions, a stiffness requirement for the beams can be derived. This stiffness requirement is , however, not discussed further in this paper. The individual column approach for $\lambda_s \geq 100$ gives a relatively large range where checks on column stability cannot be carried out using $l_b = l_e$. It is shown with a finite element method analysis, [3], that the load carrying capacity calculated with the interaction formula, using $l_b = l_e$, underestimates the load carrying capacity , Fig.(9). The bound $P_A = P_E$ in Fig. (7-b) has no physical meaning but is a result of the first method, the individual column approach. Therefore the second method has been developed.

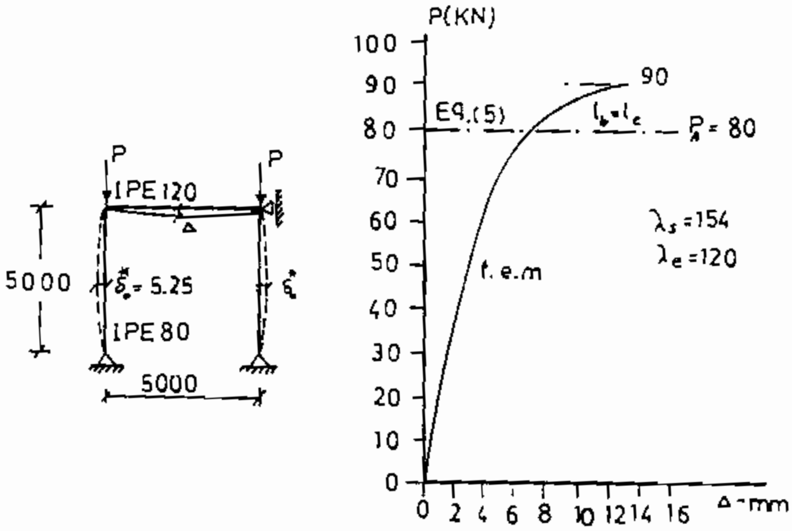


FIG. (9) **Comparison of finite element method (f.e.m.) results with Eq.(5)**

2-2 THE SECOND METHOD : The Column With Rotational End Restraint

The column of Fig.(6-d) is schematised as given in Fig.(10). The restraining moment is replaced by an elastic rotational spring at the top of the column. The moment M_r at the top of the column will again be calculated. The assumptions given in 2.1 are again valid. The procedure for determining the amplitude of the initial imperfection δ_o is as described in section 2.1.1.

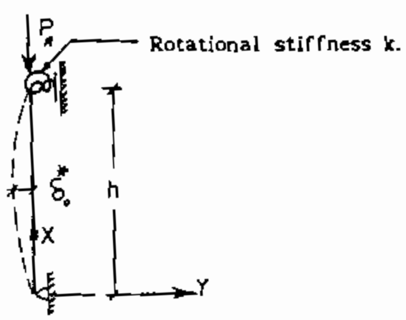


FIG. (10) **The second method : The column with rotational end restraint**

2.2.1 Procedure For Determining The Required Restraining Moment M_r When The Column is Loaded with P_A.

The differential equation for the column shown in Fig.(10) can be derived as follows;

$$\frac{d^2\delta}{dx^2} + \alpha^2 \delta = - \frac{8\delta_o^*}{h^2} - \frac{M_r . x}{EI.h} \qquad \text{with } \alpha^2 = \frac{P}{EI} \tag{10}$$

for $x = 0$ $\delta = o$

for $x = h$ $\delta = 0$ and $M_r = k \left(\frac{d(\delta + \delta_o)}{dx} \right)_r$

Where ; k is the rotational stiffness of the rotational end restraint.

P is taken equal to P_A on the basis of $l_b = l_e$. It is then determined whether the yield criterion is exceeded somewhere in the column. If not, P_A is supported elastically by the column. If the yield criterion is exceeded, then a value of P smaller than P_A is determined so that the column remains elastic. The restraining moment M_r is determined from Eqn (10).

2.2.2 The Results of The Second Method

Figs.(11-a) and (11-b) give the results for two system slendernesses. The strength requirement for the beam can be obtained from these figures. Because compatibility between the top of the column and the end of the beam is included in the calculations, a stiffness requirement for the beam is no longer necessary.

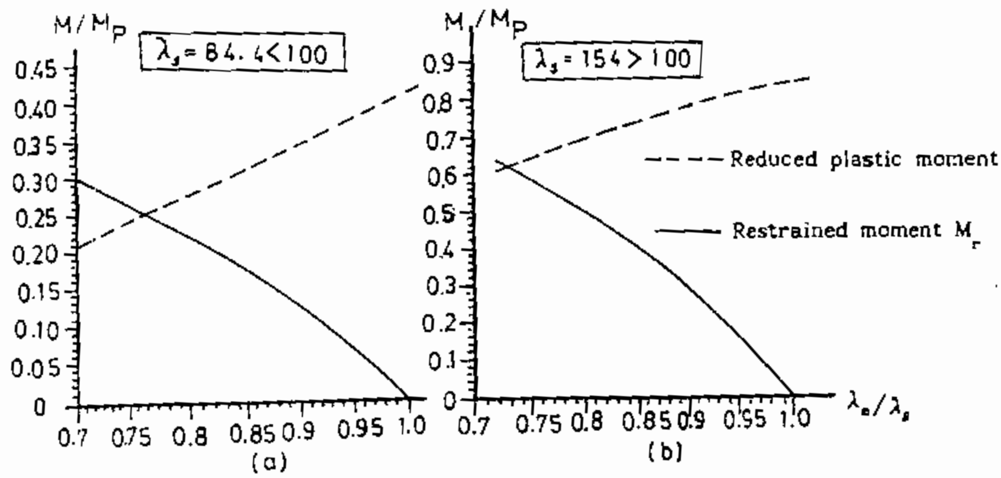


FIG. (11) **Results for two characteristic system slendernesses (the second method)**

According to the second method, P_A can be supported elastically for a large range of elastic effective slendernesses. For those parts of Fig.(11) to the left of the intersection points, the restraining moments cannot be achieved at the top of the column. Therefore, checks on column stability cannot be made using $l_b = l_e$. To the right of the intersection point, the yield moment M_Y exceeds the reduced plastic moment for small system slendernesses. However, this violation results in a reduction of the load carrying capacity (see 2.2.1) by less than 1% . If this violation is ignored, the check on column stability to the right of the intersection Points in Fig. (11) can be carried out with $l_b = l_e$, if the beam can provide the necessary restraining moment

All the results for the second method are summarized in Fig. (12). In the shaded area, the use of $l_b = l_e$ is allowed and the required restraining moment can be read off. For example, for the slenderness $\lambda_e = 100$ and $\lambda_s = 120$, the restraining moment $M_r = 0.32\ M_p$. To avoid the use of graphs to determine the restraining moment, a formula has been derived. On Linearization of the curves in Fig.(12), the following expression can be obtained for M_r.

$$\frac{M_r}{M_p} = f(\lambda_s)\,(\lambda_s - \lambda_e) \tag{11}$$

$$\text{with}\quad \left.\begin{array}{lll} \lambda_s \le 110 & : & f(\lambda_s) = 1.3 \cdot 10^{-4}\lambda_s + 2.6 \times 10^{-3} \\ \lambda_s > 110 & : & f(\lambda_s) = 1.7 \cdot 10^{-2} \end{array}\right\} \tag{12}$$

When the column is not completely loaded by its carrying capacity P_A, based upon $l_b = l_e$, the beam does not have to provide the full restraining moment according to Eqn.(11). For practical checks, the following equation should be used. This relationship accounts for the reduced load on the column.

$$\frac{M_r}{M_p} = f(\lambda_s)(\lambda_s - \lambda_e)\,\frac{(P_E - P_p)}{P_E - N}\cdot\frac{N}{P_A} \tag{13}$$

Using Eqn. (12) and P_E on the basis of $l_b = l_e$.

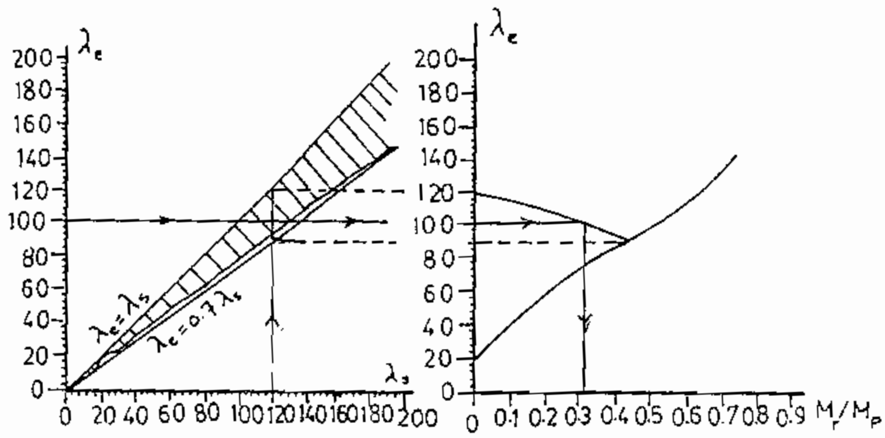


FIG. (12) Summary of the results of the second method

CONCLUSIONS

- Design formulae, derived for checking the stability of the individual column where $l_b = l_s$ are commonly used with $l_b = l_e$ for braced frames.

- On the basis of a parameteric study, it is concluded that the design formulae can be used with the elastic effective length ($l_b = l_e$) if the beams can provide the necessary restraining moment.

- So far, there are no design formula available for the necessary restraining moment.

- In this study, an approximate formula for the necessary restraining moment has been derived, Eqn.(13) for a single storey, single bay braced frame, with concentrated loads on the columns.

- Two methods for determining the necessary restraining moment have been used. A method based on the individual column approach, is not effective for geometrically non-linear problems, therefore, a second method has been developed, using a rotational spring at the top of the column. This rotational spring has two functions:it generates the restraining moment M_r and accounts for the elastic boundary condition. The geometrical non-linearity is therefore correctly taken into account.

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