# THE APPLICATION OF MATHEMATICAL PROGRAMMING IN THE DESIGN OF AXIALLY LOADED MEMBERS (I)

BY

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ABSTRACT:

This paper outlines the application of mathematical programming in the optimum design of machine elements. We adopt the technique of geometric programming in this study for the design of axially loaded members subjected to structural constraints with various degrees of complexity. A following paper will include a comparative study between the proposed design procedure and the traditional optimal design procedures.

## 1- INTRODUCTION:

The optimal design of machine member has always been of great interest. As the machine member constitute a structural compenent in the integrated design, the design paramaters are normally subjected to limitations or structural constraints, the design criterion, on the other hand strongly depend on the application.

Thus, the optimal design problem could finally be formulated as a mathematical programme with an objective function subject to constraints.

The difficulty of application of this concept emenates from the fact that both the objective function and/or the contraints are nonlinear in nature. The size of real design problem-with the above mentioned non-linearity feature was-in many cases-challenging.

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However the technique of geometric programming in the last decade offered an efficient tool to deal with many engineering problems. It is remerkable, that the application of this technique in the field of machine design is still in its preleminary stages.

This paper (I) investigates the application of geometric programming in the design of axially loaded members. A following paper (II) will report a comparative study with the traditional optimal design procedures and the efficiency of computational algorithm.

## 2- TENSILE BARS WITH STATIC AXIAL LOADS:

The design problem in Fig. (1) depict the structural contraint of an axially loaded machine member, the length is restricted by

$$L_{min} \leq L \leq L_{max} \qquad \text{..........(1)}$$

and the diameter is restricted by

$$d \leq d_{max} \qquad \text{..........(2)}$$

the weight of the machine elements is

$$W = w(\Pi d^2/4) \cdot L \qquad \text{..........(3)}$$

w = is the density.

The cost of the part C is estimated to be

$$C = \sum_i Co_i \; d^{\alpha_i} \; L^{\beta_i} \; S_Y^{\gamma_i} \qquad \text{..........(4)}$$

Co,a,b is known manufucturing constant, i the number of cost attributes inherent in the total manufucturing cost.

The allowable strength $\tau_{max}$ is given by

$$\tau_{max} \leq Sy/2N \qquad \text{..........(5)}$$

Sy = published tensile strength

N = Factor of safety

Since

$$\max = (\tfrac{1}{2})\ F/(\frac{\pi d2}{4}) \qquad \text{..........(6)}$$

The optimal design problem, for minimum cost criterion may be cited as:-

Minimize

$$C = \sum_{1}^{m} coi\ d^{\alpha i}\ L^{\beta i}\ sy^{\gamma i} \qquad \text{..........(4)}$$

subject to

$$\frac{2N}{s_y}\ \frac{4F}{\pi\ d2} \leq 1 \qquad \text{..........(5)}$$

$$d/d_{max} \leq 1$$
$$L/L_{max} \leq 1 \qquad \text{..........(6)}$$
$$L/L_{min} \geq 1$$

let $C_{oi} = coi\ sy^{\gamma i}$

$$\frac{8NF}{Sy\ \pi} = C11$$

$$\frac{1}{d_{max}} = C21 \qquad \text{..........(7)}$$

$$\frac{1}{L_{max}} = C31$$

$$L_{min} = C41$$

Then our problem is

Minimize

$$C = \sum_{i=1}^{m} C_{oi}\ d^{\alpha i}\ L^{\beta i} \qquad \text{.........(8)}$$

subject to

$$C11 \quad d^{-2} \leq 1$$
$$C21 \quad d \leq 1$$
$$C31 \quad L \leq 1 \qquad \text{..........(9)}$$
$$C41 \quad L^{-1} \leq 1$$

Consider the case of i=2
The dual problem is

Maximize

$$d(w) = \prod_{i=1}^{2} \left(\frac{coi}{woi}\right)^{woi} \left(\frac{C11}{w11}\right)^{w11} \left(\frac{C21}{w21}\right)^{w21} \left(\frac{C31}{w31}\right)^{w31} \left(\frac{C41}{w41}\right)^{w41} \qquad \text{...(10)}$$

Subject to:-

$$wo1 + wo2 = 1$$
$$\alpha 1wo1 + B1\ wo2 - 2w11 + w21 = 0 \qquad \text{..........(11)}$$
$$\alpha 2wo1 + B2\ wo2 + w31 - w41 = 0$$
$$woi,\ w11,\ w21,\ w31,\ w41 \geq 0$$

3- TENSILE BARS WITH VARYING AXIAL LOADS:

The design of coupling study with repeated energy loading is considered

3-1 Simple case

For thé simple case the subsidery design equations are:-

$$\Delta = \frac{(F_{max})\ L}{(\Pi\ d2/4)E} \qquad \text{..........(12)}$$

$$\sigma_{max} = \frac{F_{max}}{(\Pi\ d2/4)} \qquad \text{..........(13)}$$

Where $\sigma_1 max \leq \frac{2 \cdot se}{(1 + se/sy)N}$ ..........(14)

E = youngs Modulous

$\Delta$= Maximum axial Elongation

Se= Fatigue strength.

By adding the structural limitation and the same objective criterion, our primal problem is:-

$$\text{Min } z = \sum_{i=1}^{m} \text{coi } d^{\alpha i} L^{\beta i} \qquad \text{..........(15)}$$

subject to:-

$$\left(\frac{4\ F_{max}}{\Pi\ E}\right) L\ d^{-2} \leq 1$$

$$\left(\frac{2\ F_{max}\ (1+\ se/sy)\ N}{\Pi\ \ se}\right)\ d^{-2} \leq 1 \qquad \text{..........(16)}$$

$$\frac{1}{d_{max}} \quad d \quad \leq 1$$

$$\frac{1}{L_{max}} \quad L \quad \leq 1$$

$$L_{min} \quad L^{-1} \quad \leq 1$$

Let i=2, $\left(\frac{4\ F_{max}}{\Pi E}\right) = C11$, $\frac{2\ F_{max}\ (1+se/sy)N}{\Pi se} = C21$,

$\frac{1}{d_{max}} = C31$ , $\frac{1}{L_{max}} = C41$ , $L_{min} = C51$

Our dual problem is this:-

$$\text{max. } \prod_{i=1}^{2} \left(\frac{Coi}{woi}\right)^{woi} \left(\frac{C11}{w11}\right)^{w11} \left(\frac{C21}{w21}\right)^{w21} \left(\frac{C31}{w31}\right)^{w31} \left(\frac{C41}{w41}\right)^{w41} \left(\frac{C51}{w51}\right)^{w51} \qquad \text{..........(17)}$$

Subject to:-

$$wo1 + wo2 = 1$$

$$\alpha_1\ wo1 + B1\ wo2 - 2\ w11 - 2\ w21 + wo1 = o \qquad \text{......(18)}$$

$$\alpha_2\ wo1 + B2 + w11 + w41 - w51 = o$$

3-2 THE COMPLEX CASE:-

In the complex case depicted in Fig. (3), for a study The total absorbed energy (P.E) is given by

$$(P.E) = (P.E)1 + (P.E)2 = \frac{2\ (F_{max})\ L1}{\Pi\ d_1^2\ E} + \frac{2(F_{max})\ L2}{\Pi\ d_2^2 E}$$

$$= \frac{2\ (F_{max})}{\Pi\ E}\ \left(\frac{L1}{d1^2} + \frac{L2}{d2^2}\right) \quad ...(19)$$

Equation (19) Constitute the primary design equation. The subsidery design equations will be based on significant stress

$$(\sigma_A)\ max = KA\ \frac{F_{max}}{(\Pi\ d_2^2/4)}$$

$$(\sigma_B)\ max = KB\ \frac{F_{max}}{(\Pi\ d_1^2/4)} \quad ..........(20)$$

$$(\sigma_R)\ max = Kr\ \frac{F_{max}}{(\Pi\ dr^2/4)}$$

Where A,B,r designate shoulder and fillet regions shown in Fig. (3).
One of three equation in (20) will dominate the other two.
Limit equations of the problem are:-

$\sigma_1$ = Principle stress

$$(\sigma_1)\ max = \frac{2\ se}{(1 + sc/sy)}$$

$$d1 \leq d1\ max \quad ..........(21)$$

$$d2 \leq d2\ max$$

$$LT\ (Min) \leq LT = L1+L2 \leq L\ (max)$$

It is natural to assume that r (max) dominate as it is always possible to select r1,r2 to satisfy this condition, substituting we get

$$(P.E) = \frac{\Pi\ (\sigma_r)^2 \max\ d_4^4}{8\ E\ K^2 r} \left[ \frac{L1}{d1^2} + \frac{L2}{d2^2} \right]$$

Let $\sigma_r = \sigma 1_{max}$ then

$$(P.E) = \Pi/2 \left( \frac{Sc^2}{(1+p)^2} \right) \frac{dr^4}{E2\ Kr^2} \left[ \frac{L1}{d1^2} + \frac{L2}{d2^2} \right] \qquad \text{..........(22)}$$

The optimal design problem can be cited as follows:-

$$\text{Max. P.E} \qquad \text{..........(23)}$$

Subject to:-

$$d1 \leq d1_{max}$$
$$d2 \leq d2_{max}$$
$$L1 + L2 \leq LT_{max} \qquad \text{..........(24)}$$
$$L1 + L2 \leq Lt_{min}$$

$$\sum_i Coi\ d1^{\alpha 1i} d2^{\alpha 2i}\ L1^{\beta 1i} L2^{\beta 2i} \leq C \qquad \text{..........(25)}$$

Equation (25) is the "design" cost constraint
Some difficulties are faced in this design problem. Firstly the factor Kr which depend on dr This can be overcomed by assuming

$$Kr = Ks\ (dr)^{Qs} \qquad \text{..........(26)}$$

Also it is feasible to assume that:-

$$\frac{dr}{d1} \leq As \qquad \text{..........(27)}$$

Where Ks,Qs,As depend on the particular theading system.
The Problem thus will be:-

$$Max = \frac{\Pi}{2EN} \left(\frac{se}{1+se/sy}\right) d_r^{(4-2Qs)} \left[\frac{L1}{d2_1} + \frac{L2}{d2_2}\right]$$

Subject:-

$$d1 \leq d1_{max}$$

$$d2 \leq d2_{max}$$

$$L1+L2 \leq LT_{max}$$

$$L1+L2 \geq LT_{min}$$

$$\frac{1}{As} \; drd^{-1} \leq 1$$

$$\sum_i Coi \; d1^{\alpha 1i} \; d2^{\alpha 2i} \; L1^{\beta ic} \; L2^{\beta 2i} \leq c$$

4- SOLUTION:

The above models are solved by mathematical programming techniques, for various examples on computers. The compartive study will be reported and discussed in following paper (II).

5- REFERENCES:


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## APPENDIX

Geometric programming (G.P.) is a relatively new topic developed for solving algebric non-linear programming problems subject to non linear constraints. In mathematical programming, two problems can be constructed, one is called the primal and a corresponding one called the dual.

The G.P technique relies heavily on the dual.

We state the generalized (G.P) Problem.

$$\text{Minimize } yo(x) = \sum_{t=1}^{To} hot\ cot \prod_{n=1}^{N} xn\ \ aotn \quad \text{(A1)}$$

Subject to:-

$$ym(x) = \sum_{t=1}^{T_m} hmt\ \ cmt \prod_{h=1}^{N} xn\ \ aotn$$

$$m = 1, 2, \ldots, M \quad \text{(A2)}$$

hot, hmt = $\pm$ 1, cot, cmt $>$ 0

amtn, aotn urestricted in sign

tm = No. of terms in the m th constraint

to = No. of terms in the objective function

For the case when all hmt, hot = $\pm 1$, we have a posynomial G.P problem.

The primal Problem is

$$\text{Minimize } yo(x) = \sum_{t=1}^{T_o} cot \prod_{n=1}^{N} xn^{aotn} \quad \text{(A3)}$$

Subject to

$$ym\ (x) = \sum_{t=1}^{Tm} cmt \prod_{n=1}^{N} xn^{\ aotn}$$

$$cot, cmt > o$$

The dual problem is

$$\text{Maximize } d(w) = \prod_{m=0}^{M} \prod_{t=1}^{T} \left( \left( \frac{C_{mt} \cdot wmo}{wmt} \right) \right)^{wmt} \quad \text{(A5)}$$

Subject to

$$\sum_{t=1}^{t_o} wot = 1$$

$$\sum_{m=1}^{m} \sum_{t=1}^{tm} amtn\ \ wmt = o \quad n=1,2,.\ N \quad \text{(A6)}$$

$$Wmo = \sum_{t=1}^{Tm} Wmt \qquad m=1,\ldots,M \tag{A7}$$

Provided that we define woo = 1

$$T = \sum_{m=o}^{M} Tm \tag{A8}$$

From the duality theorem if X* is the optimal primal solution and W* is the optimal dual solution then

$$d\ (W)^* = (X)^* \tag{A9}$$

Hence for the terms is the objective function the following relationship at optimality holds:-

$$Wot\ (yo\ (X)^*) = cot \prod_{n=1}^{N} xn^{aotm} \qquad t=1,\ldots,To \tag{A10}$$

At last we like to mention that for the ith term of the mth consraint the following relation-ship holds:-

$$S_{mt} = \frac{W_{mt}}{\sum W_{mt}} \qquad Cmt \prod^{N} xn^{amtn} \tag{A11}$$

Where wmt is the generalized weight for the t th term in the mth constraint.

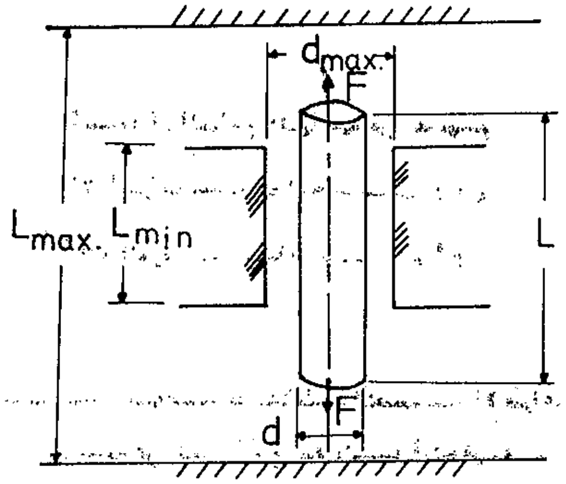


Fig. (1) Tensile bar with static axial lood

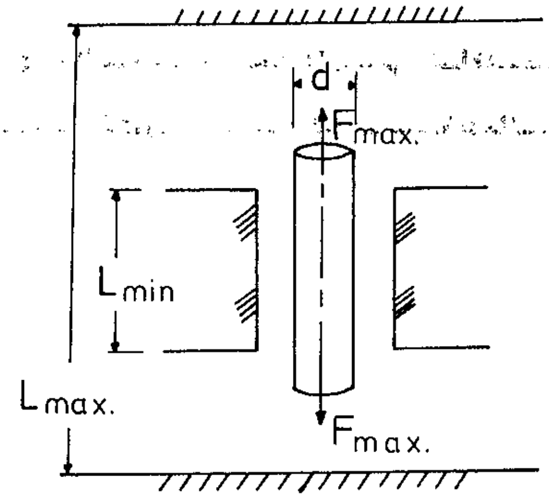


Fig. (2) Tensil bar with varying axil lood

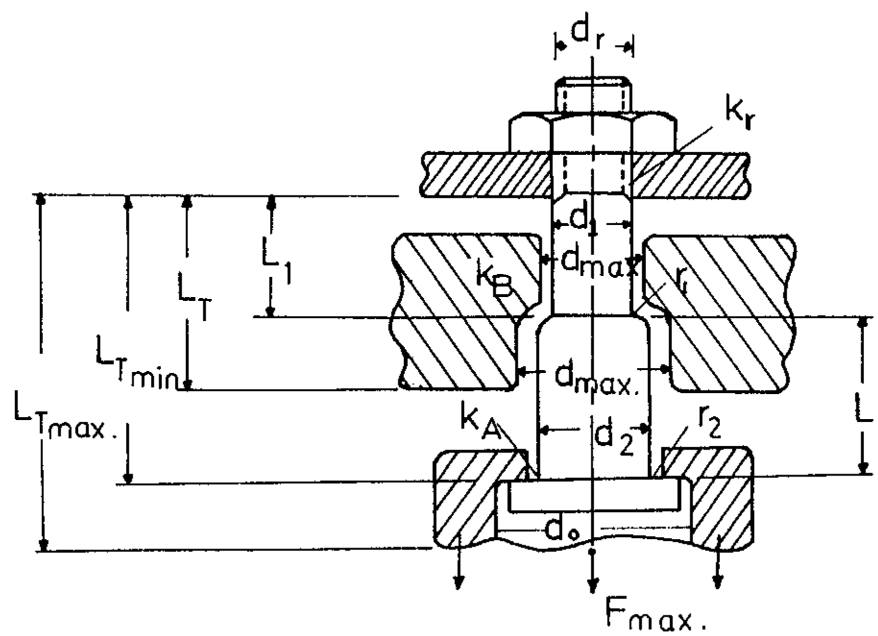


Fig (3) Complex Case

## استخدام البرامج الرياضية فى تصميم
## الاطراف محوريه الحمل (١)

الدكتورة :- سعاد سراج

فى هذا البحث نقدم صياغة متكاملة لتصميم الاطراف المحورية الحمل باستخدام البرمجة الهندسية فى نماذج متفاوتة التعقيد - واضافة قيود جديدة للتصميم تتيح للمصمم التعبير عن المسألة قيد الدراسة فى صوره أكثر واقعية.

وسوف يتبع هذا البحث بحث تكميلى لمناقشة طرق الحل باستخدام الحاسبات الالكترونية وبد الدراسة للحالة العامة للتصميم.