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THE OPTIMAL COMPENSATION OF REACTIVE POWER IN ELECTRIC NETWORKS USING THE SENSITIVITY TECHNIQUE

M.Hamed
Faculty of Engineering
Suze Canal University
Port Said, EGYPT

H.Yasin
Faculty of Engineering
Menoufia University
Shebin El-Koum, EGYPT

ABSTRACT :

This paper presents mathematical analysis for optimal compensation of reactive power in electric networks. This optimization is needed to control the voltage level of busbars above the permissible minimum value. The fluctuation of consumed loads causes a deviation from the optimal operation so that a new optimal deviation of power flow must be deduced by sensitivity approach.

The sensitivity matrices for both voltage and phase shift dependency are derived. The elements of both sensitivity matrices are computed in per unit system. The diagonal element is taken as a reference. The proposed algorithm is programmed. It is applied to different networks for two voltage levels (250 and 230 KV). The number of controlled buses may be varied.

INTRODUCTION

Reactive power injection in power systems is an economical mean for maintaining voltage levels under different operating conditions within acceptable limits. The problem is where and how much VAR capacity are required to satisfy the imposed conditions.

Reactive power optimization is a nonlinear programming problem which has been solved by linear techniques. The trial and error approach depends on the engineering judgements for locating the reactive power sources. The dynamic programming approach is based on selecting the most sensitive bus for the reactive power injection at low voltage buses in a system [1]. A capacitor unit is added at different nodes. The voltage changes are calculated by a linearized model, and then, searching for the most sensitive bus in the new set of low voltage buses. This problem may be solved using linear approximation [2]. The sensitivity matrix is constructed and linear programming technique is for the optimization purpose [3].

PROBLEM FORMULATION

The reactive power optimization, in order to keep the load bus voltages within certain limits, is an over important subject which must be treated.

There are two methods to change the reactive power Q_i at the i-th bus : either voltage variation ΔV under constant active power P_i or phase shift displacement $\Delta\theta$ under a constant voltage. The relation between the reactive power variation ΔQ under constant voltage and phase shift change $\Delta\theta$, using the sensitivity matrix for phase shift dependency S_θ, may be formulated as

$$\Delta\theta = S_\theta \ \Delta Q \qquad (1)$$

This relation for voltage dependency S_V becomes

$$\Delta V = S_V \ \Delta Q \qquad (2)$$

THE SENSITIVITY MATRIX

Since the active power deviation depends on voltage vector (magnitude and angle), Jacob's matrix $(\partial W/\partial x)$ becomes an important instrument in this analysis. The matrix $(\partial W/\partial x)$ can be expressed in simple form as [4]:

$$\frac{\partial W}{\partial x} = \begin{bmatrix} \frac{\partial Q}{\partial V} & \frac{\partial Q}{\partial \theta} \\ \frac{\partial P}{\partial V} & \frac{\partial P}{\partial \theta} \end{bmatrix} \qquad (3)$$

Matrix $\frac{\partial W}{\partial x}$ may be devided into two parts : upper and lower. The upper part includes the relation of reactive power while the lower part contains that relation for active power. The elements of both parts will be computed using the given program (see Fig. 1). The maximum permissible number of nodes N is 26 and number of links should not exceed 27 for application on computer since dimension of the calculated matrix becomes large. The active and reactive powers at both ends of transmission line may be simplified in the form [5] :

$$P + jQ = \frac{T}{Z} \left[\frac{V_i^2}{B'} e^{j\phi} - \frac{V_i V_k}{C} e^{j(\theta_i - \theta_k + \phi - T.F)} \right] \qquad (4)$$

where i and k indicate the bus numbers of transmission line ends, Z is the branch impedance, ϕ is the angle between two voltages at both ends, F is impedance angle of transformer branch. The terminal conditions may be defined by coefficients B', C and T.

Using equation (4) the diagonal elements of submatrices of equation (3) may be determined as :

$$\frac{\partial P_i}{\partial V_i} + j \frac{\partial Q_i}{\partial V_i} = \sum_k \frac{2V_i}{Z_{ik}} e^{j\phi_{ik}} + \frac{V_k (e^j)}{C_{ik} Z_{ik}} \cos B$$

$$\frac{\partial P_i}{\partial \theta_i} + j \frac{\partial Q_i}{\partial \theta_i} = (1-j) \sum_k \left(\frac{V_i V_k}{C_{ik} Z_{ik}} \cos B \right) \qquad (5)$$

$$B = \theta_i - \theta_k + \phi_{ik} + F_{ik}$$

On the other hand, the off-diagonal elements can be derived by :

$$\frac{\partial P_i}{\partial V_k} + j \frac{\partial Q_i}{\partial V_k} = \frac{-V_i}{C_{ik} Z_{ik}} (\cos B + j \sin B)$$

$$\frac{\partial P_i}{\partial \theta_k} + j \frac{\partial Q_i}{\partial \theta_k} = \frac{-V_i}{C_{ik} Z_{ik}} (\sin B - j \cos B) \qquad (6)$$

The relation of active power deviation with respect to either reactive power of generators $\partial P / \partial Q_g$ or transmission lines $\partial P / \partial C$ and transformers $\partial P / \partial F$ should should be considered. This relation, but for reactive power variation ∂Q, must be inserted. Both relations may be included in matrix [$\partial W/\partial D$] as :

$$[\frac{\partial W}{\partial D}] = \begin{bmatrix} \frac{\partial Q}{\partial Q_g} & \frac{\partial Q}{\partial C} & \frac{\partial Q}{\partial F} \\ \frac{\partial P}{\partial Q_g} & \frac{\partial P}{\partial C} & \frac{\partial P}{\partial F} \end{bmatrix} \quad (7)$$

The diagonal elements of submatrix $\partial Q/\partial Q_g$ are equal to unity while all elements of submatrix $\partial P/\partial Q_g$ have zero value (6). Other elements of matrix $\partial W/\partial D$ can be derived by

$$\frac{\partial P_i}{\partial C_n} + j\frac{\partial Q_i}{\partial C_n} = \frac{V_i V_k}{C_{ik}^2 \, Z_{ik}} e^{jB}$$

$$\frac{\partial P_k}{\partial C_n} + j\frac{\partial Q_k}{\partial C_n} = \frac{-2V_k^2 \sin\phi_{ik}}{C_{ik} \quad Z_{ik}} \left(\frac{Z_{ik}}{C_{ik}^2} + j\right) +$$

$$+ \frac{V_k V_i \sin B}{C_{ik} \quad Z_{ik}} \left(\frac{1}{C_{ik}} + j\right) \quad (8)$$

$$\frac{\partial P_i}{\partial F_n} + j\frac{\partial Q_i}{\partial F_n} = \frac{V_i V_k}{C_{ik} Z_{ik}} (\sin B - j\cos B)$$

The given analysis is programmed and its flow chart is shown in Fig. 2. Finally, the sensitivity matrix S can be be formulated by the matrix equation :

$$[S] = [\frac{\partial W}{\partial x}]^{-1} [\frac{\partial W}{\partial D}] \quad (9)$$

This sensitivity matrix may be rewritten as a function of two sensitivity matrices for both voltage and phase shift dependency as

$$[S] = \begin{bmatrix} S_v \\ \hline S_\theta \end{bmatrix} = \begin{bmatrix} \partial V/\partial D \\ \partial\theta/\partial D \end{bmatrix}$$

The elements of sensitivity matrix S may be computed in per unit based on the diagonal element of each row, using a computer program (see Fig. 3).

APPLICATION

An electric network with 7 controlled buses as shown in Fig.4 is considered. Both sensitivity matrices are computed for two different base voltage such as 250 and 230 KV with reference bus number (0). The calculated sensitivity matrix for voltage dependency is given in Table 1.

Table 1.

The evaluated per unit sensitivity matrices for voltage dependency

Base voltage (KV)	BUS NUMBER						
	1	2	3	4	5	6	7
	1.00	0.91	1.03	1.05	1.22	1.31	1.19
	0.57	1.00	1.14	1.16	1.34	1.44	0.68
	0.43	0.76	1.00	1.02	1.18	1.27	0.51
250	0.35	0.60	0.78	1.00	0.92	0.99	0.42
	0.27	0.50	0.66	0.67	1.00	1.08	0.32
	0.20	0.37	0.50	0.51	0.76	1.00	0.24
	0.44	0.39	0.44	0.45	0.52	0.56	1.00
	1.00	1.06	0.80	0.80	1.03	1.08	0.90
	0.94	1.00	1.93	2.11	2.46	2.58	2.16
	0.49	0.52	1.00	1.32	1.54	1.61	1.35
230	0.35	0.40	0.76	1.00	1.17	1.23	1.28
	0.22	0.23	0.45	0.59	1.00	1.05	1.60
	0.14	0.15	0.29	0.39	0.66	1.00	0.39
	1.11	3.52	0.89	0.98	1.14	1.20	1.00

The sensitivity matrices (for phase shift dependency) are also obtained. The results are listed in Table 2.

The single line diagram of another network is shown in Fig.5 to check the above calculations. The voltage level of such a network is also 220 KV. The bus number is indicated on the figure. The bus number (0) is considered as the base bus with two base voltage (250 and 230 KV). If all buses of this network are under control, the sensentivity matrices for voltage and phase shift dependency will be computed. The results of calculations are written in Tables 3 and 4 in per unit values.

Table 2.

The calculated per unit sensitivity matrices for phase shift dependency

Base voltage (KV)	bus number						
	1	2	3	4	5	6	7
	1.00	0.82	0.93	0.95	1.10	1.19	1.19
	0.53	1.00	1.14	1.16	1.34	1.45	0.63
	0.40	0.75	1.00	1.02	1.18	1.27	0.47
250	0.32	0.61	0.81	1.00	0.96	1.03	0.38
	0.25	0.46	0.61	0.63	1.00	1.08	0.29
	0.17	0.31	0.41	0.42	0.67	1.00	0.20
	0.33	0.27	0.31	0.32	0.37	0.39	1.00
	1.00	1.06	0.90	0.99	1.15	1.21	1.01
	0.94	1.00	1.69	1.86	2.17	2.27	1.90
	0.54	0.58	1.00	1.23	1.44	1.51	1.26
230	0.43	0.45	0.80	1.00	1.17	1.23	1.17
	0.28	0.30	0.51	0.68	1.00	1.05	0.70
	0.22	0.24	0.43	0.55	0.81	1.00	0.56
	1.03	2.39	0.89	0.98	1.14	1.20	1.00

Table 3.

The per unit sensitivity matrix for voltage dependency

Base voltage (KV)	bus number				
	8	9	10	11	12
	1.00	1.09	1.54	1.66	1.56
	0.47	1.00	0.72	0.78	0.73
250	0.32	0.34	1.00	1.08	1.02
	0.24	0.26	0.80	1.00	0.73
	0.29	0.31	0.93	1.00	1.00
	1.00	1.08	1.70	1.86	1.16
	0.38	1.00	0.65	0.72	0.67
230	0.25	0.27	1.00	1.09	1.02
	0.18	0.20	0.72	1.00	1.74
	0.22	0.24	0.89	0.97	1.00

Table 4.

The calculated per unit sensitivity matrix for phase fhift dependency S_θ

Base voltage (KV)	BUS NUMBER				
	8	9	10	11	12
	1.00	1.09	1.54	1.66	1.56
	0.37	1.00	0.58	0.62	0.58
250	0.26	0.28	1.00	1.08	1.02
	0.18	0.20	0.71	1.00	0.72
	0.22	0.24	0.87	0.95	1.00
	1.00	1.08	1.70	1.86	1.73
	0.46	1.00	0.78	0.85	0.79
230	0.29	0.32	1.00	1.09	0.94
	0.23	0.25	0.81	1.00	0.83
	0.27	0.29	0.94	1.02	1.00

It is seen that, the diagonal element does not always represent the maximum value for sensitivity of voltage variations in both cases. Since it was concluded in reference [3] that, the diagonal element has always the optimal value, more calculations should be applied. Connecting the two given networks of Fig. 4 and Fig. 5 at the bus number (0), a third network will appeared as shown in Fig. 6. Hence, for this network, the sensitivity matrices (S_o) can be expressed as

$$[S_o] = \begin{bmatrix} S_1 & S_3 \\ S_3 & S_2 \end{bmatrix} \qquad (11)$$

In this case the matrix S_o may be either for phase shift dependency S_θ or for voltage dependency S_v. The submatrix S_1 is the sensitvity matrix for the network of Fig. 4, while sensitivity matrix of network of Fig.5 becomes S_2. Also the third submatrix S_3 will be zero matrix. This means that the above calculated matrices are the same for the third network [7]. The results of calculations for third network prove that equation (11) is real and true. It is shown from computations that any compensation at any busbar of any section (Fig. 4 or Fig. 5) will not affect on the voltage level of other part. This may be due to the selection of base bus (0).

The change in number of controlled bus should be checked to prove the above conclusion. The sensitivity matrices for both voltage and phase shift dependency are evaluated for network of Fig. 6 with 5 controlled buses. The results are listed in Table 5.

On the other hand for network of Fig. 6 with three controlled buses, the sensitivity matrices for voltage and phase shift dependency are also obtained. The results are written in Table 6.

Table 5.

The per unit sensitivity matrices for 5-controlled bus network

Type of matrix	NUMBER OF CONTROLLED BUS				
	1	2	3	4	5
	1.00	0.84	0.96	0.98	1.14
for voltage	0.54	1.00	1.14	1.16	1.35
dependency	0.41	0.75	1.00	1.02	1.19
S_V	0.33	0.61	0.81	1.00	0.97
	0.52	0.47	0.62	0.64	1.00
	1.00	0.28	0.32	0.33	0.38
for phase	0.50	1.00	1.14	1.16	1.35
shift depend-	0.37	0.74	1.00	1.02	1.19
ency S_θ	0.32	0.62	0.83	1.00	0.98
	0.11	0.27	0.39	0.40	1.00

Table 6.

The computed sensitivity matrices for 3 controlled bus network

Type of matrix	number of controlled bus		
	1	3	5
for voltage	1.00	0.69	0.86
dependency S_V	3.04	1.00	2.60
	1.46	0.38	1.00
for phase shift	1.00	0.61	0.86
dependency S_θ	1.99	1.00	1.61
	1.47	0.45	1.00

From Tables 1 : 6 it is concluded that, the diagonal elements of the sensitivity matrix S_{ii} are not always, greater than the off-diagonal S_{ij} where $(i \neq j)$.

THE OPTIMAL COMPENSATION OF REACTIVE POWER

The objective function of reactive power distribution may be simplified in the form [3] :

$$f(\Delta Q) = \Delta Q_1 + \Delta Q_2 \tag{12}$$

The optimal solution for equation (12) may be deduced any where inside the area ABCD, i.e. one of the extereme points A, B, C or D (see Fig.7). The lines ab and cd (Fig. 7) can be determined by [3] :

$$\begin{aligned} \Delta Q_1 &= - \Delta Q_2 \cdot S_{12}/S_{11} + \Delta V_1'/S_{11} \\ \Delta Q_2 &= - \Delta Q_1 \cdot S_{21}/S_{22} + \Delta V_2'/S_{22} \end{aligned} \tag{13}$$

where

$$\Delta V_1' = V_{min} - V_1 \geq 0$$

$$\Delta V_2' = V_{min} - V_2 \geq 0$$

It was concluded in [3] that, the minimum point is always the point A for all cases (Fig. 7 a), since the diagenal elements S_{ii} of the sensitivity matrix are larger than the off-diagonal S_{ij} for the same row. The results of present paper show that, sometimes the element S_{ii} is less than S_{ij} so that optimal solution will not be always point A. This statement is proved also geometrically as given in Fig. 7. The values of angles θ_1 and θ_2 may be varied in wide range as concluded from tables 1 : 6. Thus all possible values of these angles are drawn in Fig. 7. The mathematical analysis for the same problem should be programmed. Its block scheme is shown in Fig. 8.

As an application, node voltage distribution in a given network of Fig. 6 is firstly calculated [7]. Then the required reactive power compensation to raise all voltage buses, which have voltages under 220 KV, at least to minimum permissible value of 220 KV is computed. The results are listed in Table 7.

From Table 7 it is concluded that, the compensation of reactive power may be achieved by all controlled buses to give the required optimal solution, so that a maximum reactive compensation of -0.2553 MVAR appears at the bus No. 8 where the voltage is 224.1 KV, i.e. larger than the minimum specified voltage.

If the diagonal element of the sensitivity matrix is the largest of row elements, the compensation of the reactive power can be realised by the same bus where also the voltage deviation is required.

On the other hand, if one of the off-diagonal elements is the largest, the reactive power compensation will include the line reactive power.

CONCLUSIONS

The diagonal elements of the sensitivity matrices for either voltage or phase shift dependency may be smaller than the off-diagonal elements.

The optimal compensation of power flow depends completely on the calculations of sensitivity matrices for both phase shift and voltage dependency.

Optimization of reactive power compensation can be achieved by re-distributing the reactive power economically between all controlled buses using the sensitivity technique.

Table 7.

The calculated reactive power compensation

Node No.	Bus voltage (KV)	Phase shift (rad.)	ΔQ (MVAR)
0	230.0	0.0	0.0
1	220.0	-0.105	0.0031
2	221.3	-0.157	-0.0413
3	220.0	-0.195	0.0291
4	220.1	-0.199	-0.0013
5	220.0	-0.286	0.0018
6	220.5	-0.300	-0.0018
7	218.6	-0.127	0.0048
8	224.1	-0.042	-0.2553
9	217.0	-0.087	0.0581
10	201.4	-0.215	0.0396
11	195.8	-0.260	0.0205
12	199.8	-0.225	0.0196

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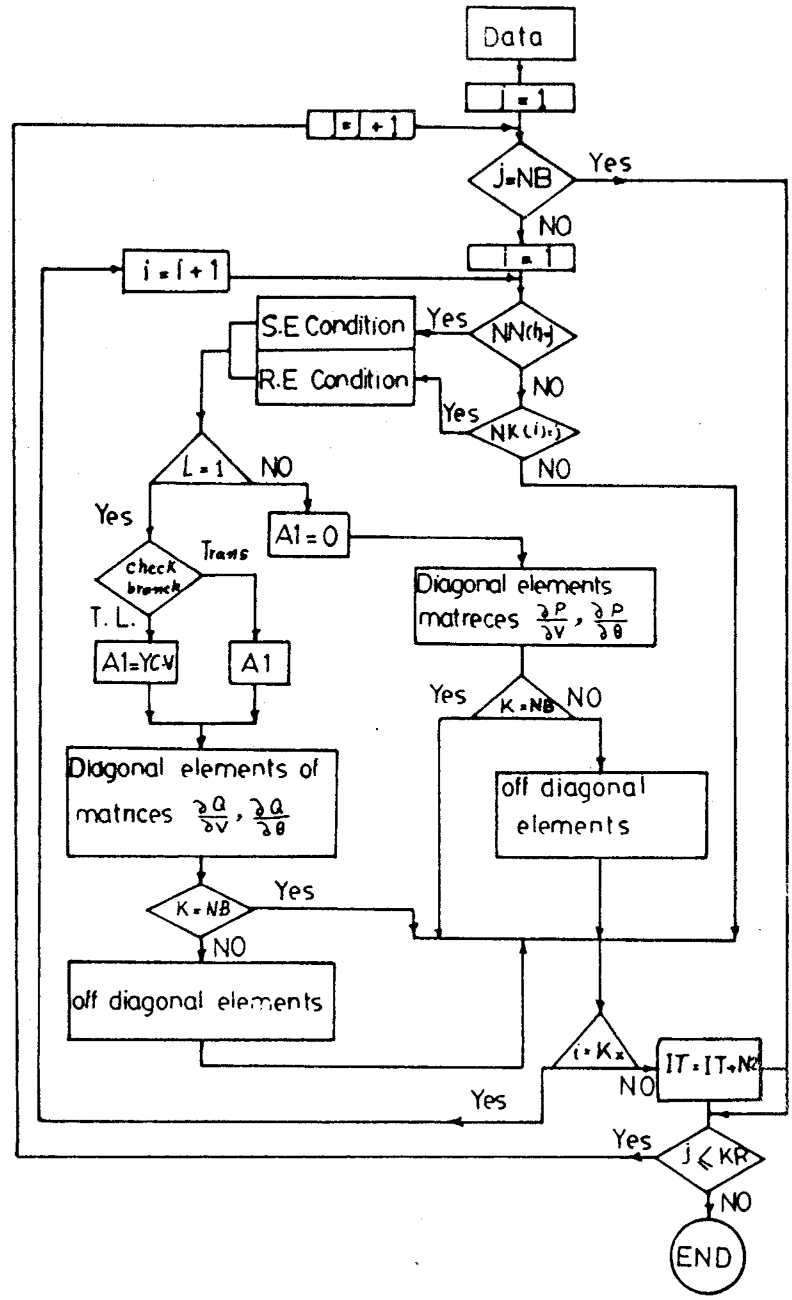

Fig. 1. Flow chart computer program for calculating the elements matrix $\frac{\partial W}{\partial x}$

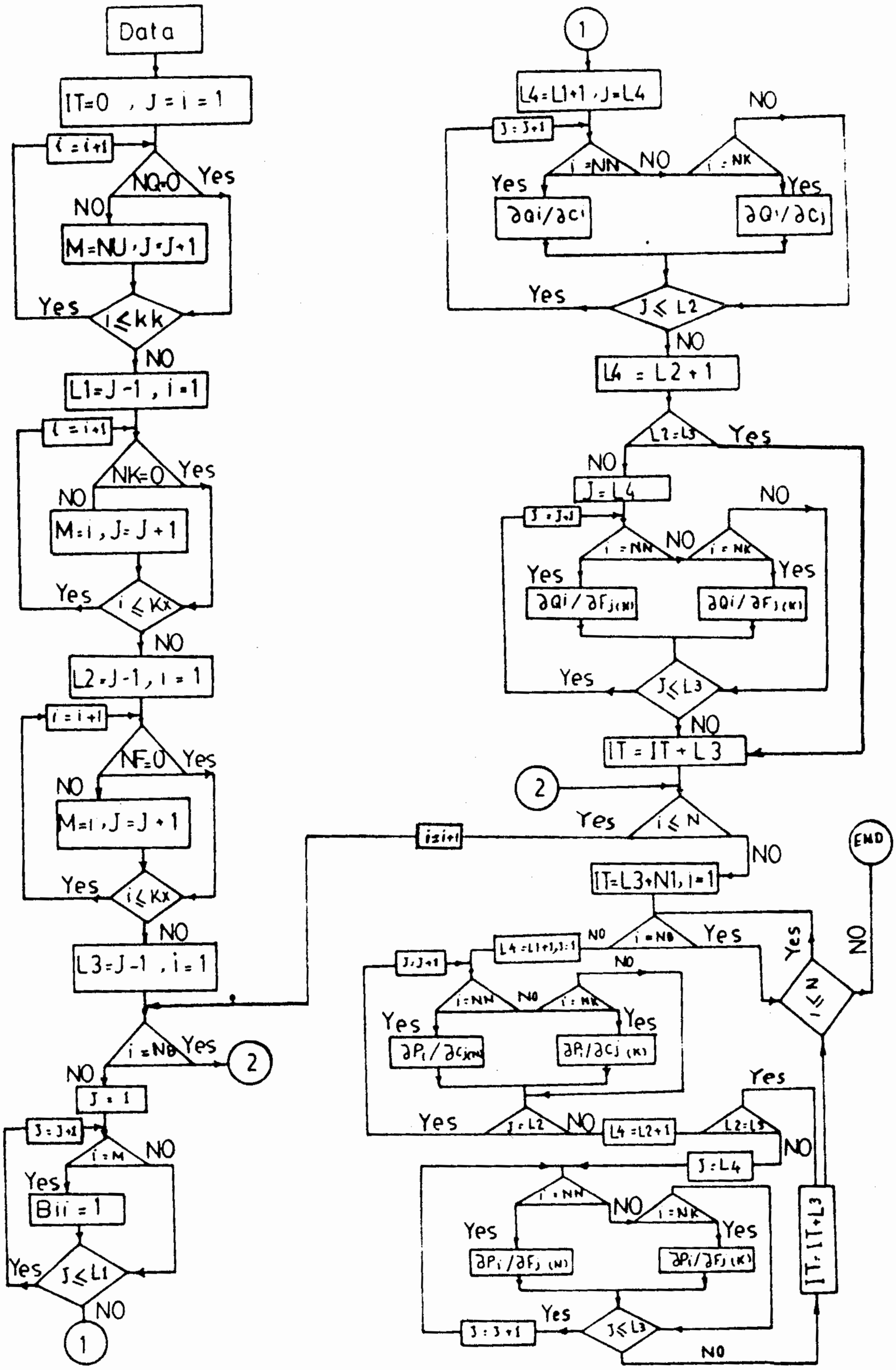

Fig.2. The flow chart of a program for calculating the elements of matrix [∂W/∂D]

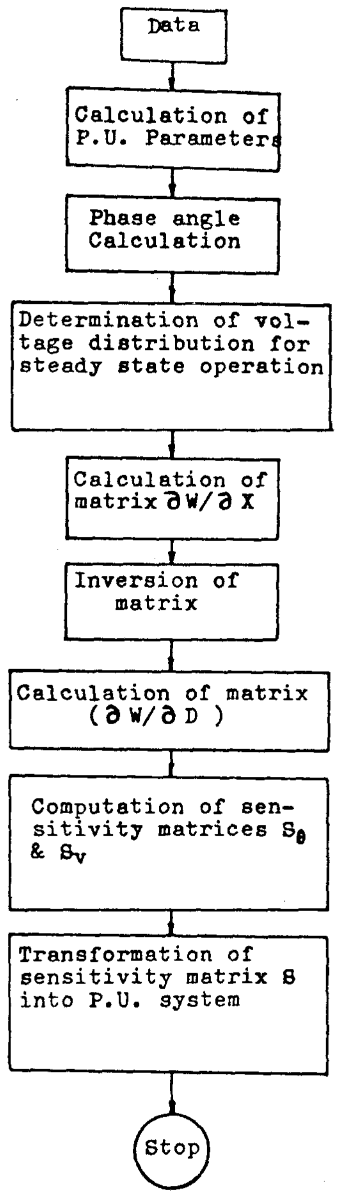

Fig.3. Block scheme of a computer program for calculation of sensitivity matrix.

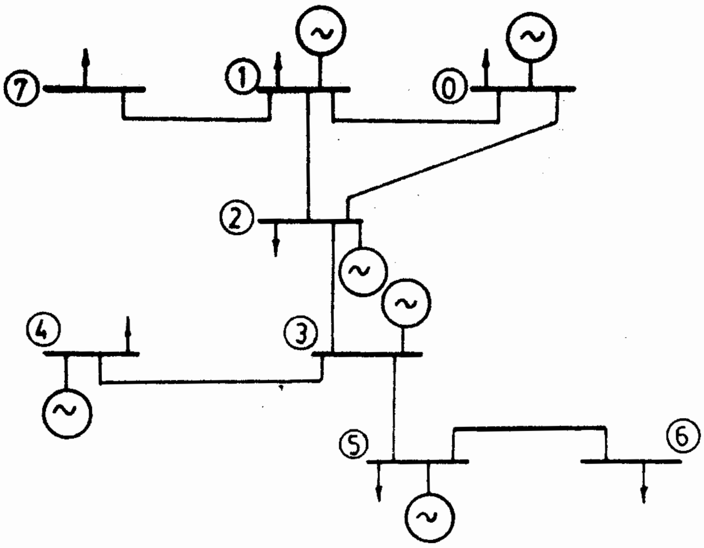

Fig. 4. Single line diagram of the proposed first network

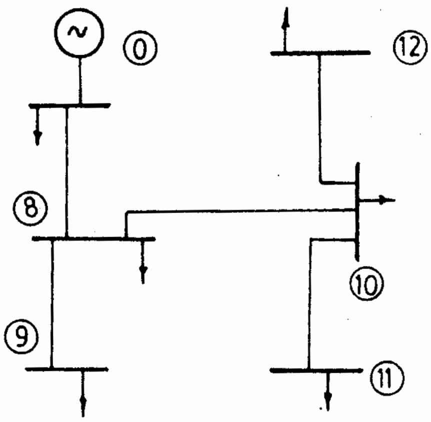

Fig. 5. Single line diagram of the second network
(220 KV)

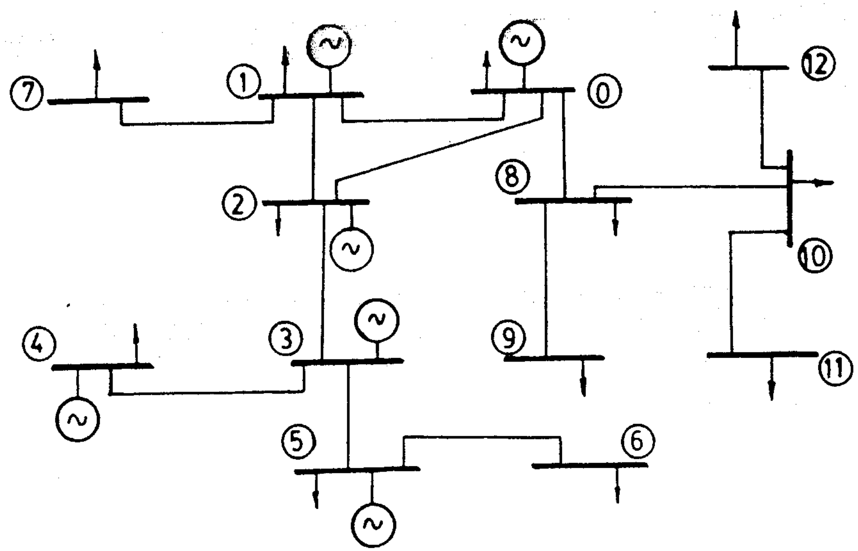

Fig. 6. Single line diagram of third electric network (220 KV)

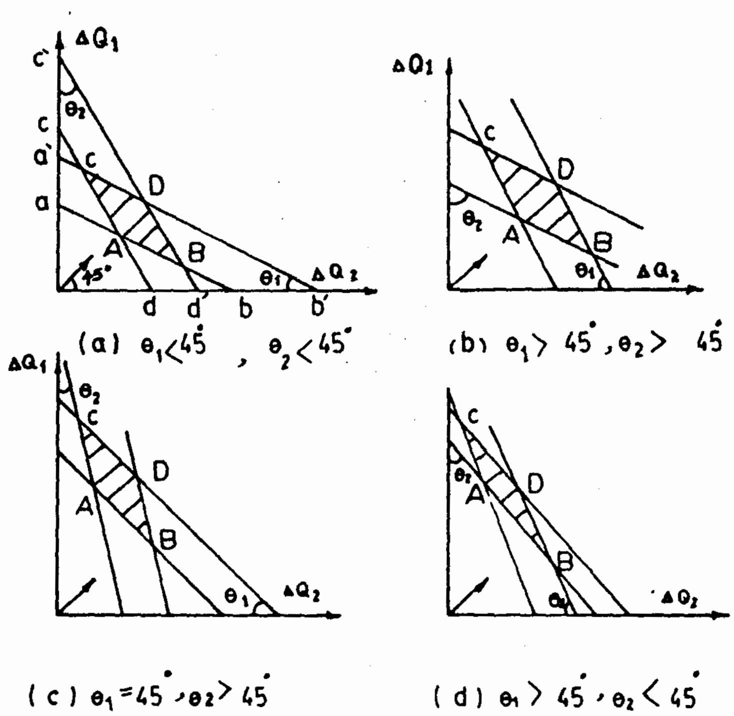

Fig. 7. Geometric Interpretation of linear programming for two constrained buses.

$\theta_1 = \tan^{-1} (S_{12} / S_{11})$.

$\theta_2 = \tan^{-1} (S_{21} / S_{22})$.

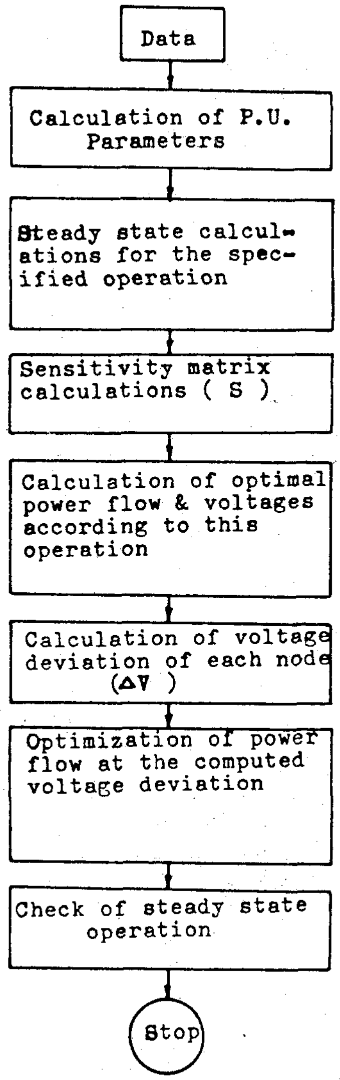

Fig.8. Block scheme of a computer program for optimal compensating of reactive power.