Answer all the following questions:

Q1 (40 Marks)

(a) Use the simple iteration method to find the smaller positive root of the equation $x^3 - 7x + 1 = 0$, correct to three decimal places and the larger positive root to two decimal places, hence find the third root.

(b) Use Newton's method to find the positive root, correct to three decimal places, of the equation $f(x) = x - 2\sin x = 0$.

(c) Find the real root of the equation $5x^3 + x^2 - 2 = 0$. Correct to four decimal places by the bisection method.

(d) Use Newton-Raphson method to solve the following system of equations:
$f(x,y) = x^2 + y - 3 = 0,\quad g(x,y) = x + y^2 - 5 = 0$.Close to the point $(2, -2)$.

Q2 (35 Marks)

(a) Use the following readings to fit the equation $y = a\, e^{bx}$.

x	1	2	3	4	5	6	7	8
y	15.3	20.5	27.4	36.6	49.1	65.6	87.8	117.6

(b) Evaluate $f(1.03)$ and $f(1.27)$ from the following table:

x	1.0	1.05	1.10	1.15	1.20	1.25
y	1.0	1.257625	1.531	1.820875	2.128	2.463125

Q3

(25 Marks)

(a) Define the Expressions: Basic solution, Extreme point, Convex set, Hyper-plane .

(b) Prove that the intersection of two convex sets is a convex set.

(c) Using Simplex method, solve the following linear programming problem:

$$\text{Maximize } Z = 2x_1 + x_2$$
$$\text{Subject to}$$
$$x_1 + 2x_2 \leq 10$$
$$2x_1 + x_2 \leq 11$$
$$x_1 - x_2 \leq 3$$
$$x_1, x_2 \geq 0$$

Good Luck