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NONLINEAR ANALYSIS OF RC BEAMS WITH OPENINGS SUBJECTED TO TORSION

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ABSTRACT

Transverse openings are sometimes very essential for accommodating utility services in reinforced concrete structures. Most current codes and specifications have no provisions for specific design of reinforced concrete beams with openings and subjected to torsion.

The object of this research is to develop a nonlinear 3-D finite elements constitutive model for R.C. beams with openings subjected to torsion by simulate the experimental behavior. A computer program based on finite element method is developed to include the nonlinear material properties of both concrete and steel reinforcement. The output results of the F.E. program are compared and verified with the experimental results done by other researchers. The effect of some important parameters such as tension stiffening, shear retention parameters, compressive strength of concrete, as well as the opening dimensions, and their locations are studied and investigated. Finally the main results are summarized.

إن الحاجة إلى تصميم منشأ اقتصادي جعلت المصممين يحاولون البحث عن بديل للمساحة أسفل الكمرات التي تستخدم لمرور شبكات الخدمات المختلفة، فكان عمل الفتحات في أعصاب الكمرات هو الحل الأمثل مما يوفر في الارتفاع الكلي للمنشأ وبالتالي تكلفة المنشأ.

وبالرغم من وجود عدد من الأبحاث السابقة التي تناولت تأثير عزوم اللي على الكمرات الخرسانية المصمتة إلا أن عدد الأبحاث التي تناولت تأثير هذه القوى على الكمرات الخرسانية ذات الفتحات يعتبر محدودا للغاية بالإضافة إلى أن الكودات المختلفة لا يوجد بها معادلات لتصميم هذا النوع من الكمرات.

ولقد تم في هذا البحث تطوير برنامج للحاسب الآلي لتمثيل هذه الكمرات وذلك باستخدام العناصر المحددة ثلاثية الأبعاد تم فيها تمثيل الحديد كأسياخ مدفونة داخل الخرسانة مع الأخذ في الاعتبار العوامل التي تمثل السلوك اللاخطي للمواد المستخدمة، وقد تم التأكد من صحة نتائج البرنامج بمقارنة نتائجه مع النتائج المعملية المعدة بواسطة باحثين آخرين ثم دراسة كيفية تمثيل عزم الإلتواء وطريقة التثبيت بطرقة العناصر المحددة كما تم دراسة تأثير بعض العوامل المؤثرة في تحليل السلوك اللاخطي لكل من الخرسانة وحديد التسليح وكذلك تأثير أبعاد الفتحات علي سلوك هذا النوع من الكمرات الخرسانية المسلحة وأخيرا تم توضيح أهم النتائج والاستنتاجات التي أمكن التوصل إليها.

Keywords: Finite Elements, Nonlinear Analysis, Tension Stiffening, Shear Retention, R.C. Beams, Openings, compressive strength.

1. INTRODUCTION

Transverse openings, in concrete beams represent, from a practical viewpoint, a means of accommodating utility services in a building structure. The ability to accommodate such services through a member instead of below or above the member results in a compact design and an overall saving in terms of total building height. Provision of openings through a beam, however, changes its simple mode of behavior to a more complex one [1]. Therefore, the design of such beams needs special treatment. Most current codes and specifications, however, have no provisions for specific design of such members.

The experimental works require, qualified laboratories with special equipments, financial supports, and time facilities

Most of the researchers depend on the analytical study of the problem by improving theoretical models to estimate torsional strength of members and the way they can resist torsional moments. To verify the results of the theoretical models, some important comparisons with the available experimental works should be essential. Three dimensional nonlinear finite element program is developed [2] and extended [3] to study and simulate many types of reinforced concrete structures. The torsional behavior of rectangular sections with or without openings is investigated.

The analysis based on the finite element method was extended to consider the material nonlinearities represented by stress-strain relationships for both concrete and reinforcing steel, concrete cracking, crushing, tension stiffening effect and shear retention factors. An isoparametric element with 20 nodes is used in the analysis. Each node has three degrees of freedom. The reinforcing steel was represented by the embedded bars in the concrete element [2].

One of the objectives of this paper is to verify and assure of the reliability of the developed three dimensional nonlinear finite element program on reinforced concrete structures under pure torsion. Another objective of this appraisal, in addition to testing the model reliability, is to identify the important material and solution parameters, which affect the torsional behavior of the considered beam.

2. EXPERIMENTAL WORK

2.1 Test Specimens

The results of the test program done by Mansur et al [4] was chosen in this analysis to be simulated and compared with the finite element analysis. The purpose of the test was to study the behavior of reinforced concrete beams with large rectangular openings under pure torsion. The test program consisted of ten reinforced concrete beams with rectangular cross section. The major variables of the study were the length, depth, and eccentricity of the openings.

Each beam was 3.3 m long with an overall cross section of 200 mm by 400 mm. The beams were supported on a span of 2.5 m. The dimensions and the reinforcing details of the test beam are shown in Fig. 1. The yield forces of reinforcing bars was 62.2 kN for longitudinal reinforcement and 10.6 kN for stirrups and the concrete cylinder strength f_c' was 44.4 N/m^2.

In this paper, one of these beams, B4, was chosen for the comparative study. Details of opening and concrete dimensions are shown in Fig. 1.

2.2 Description of Test Rig

The test beam was supported at each end on a cylindrical bearing. These bearings permitted free torsional rotation at the supports. A torsion arm was attached to the beam at each support. The load was applied to one of the torsion arms by means of a 200 kN hydraulic jack while the other torsion arm was held in position. Torsional rotations were measured within the central 1.6 m of the beam irrespective of opening length. Fig. 2 shows the test setup and instrumentation. The load on the torsion arm was applied in increments up to failure. The experimental cracking torque was 4.79 kN.m and the ultimate torque was 14.66 kN.m.

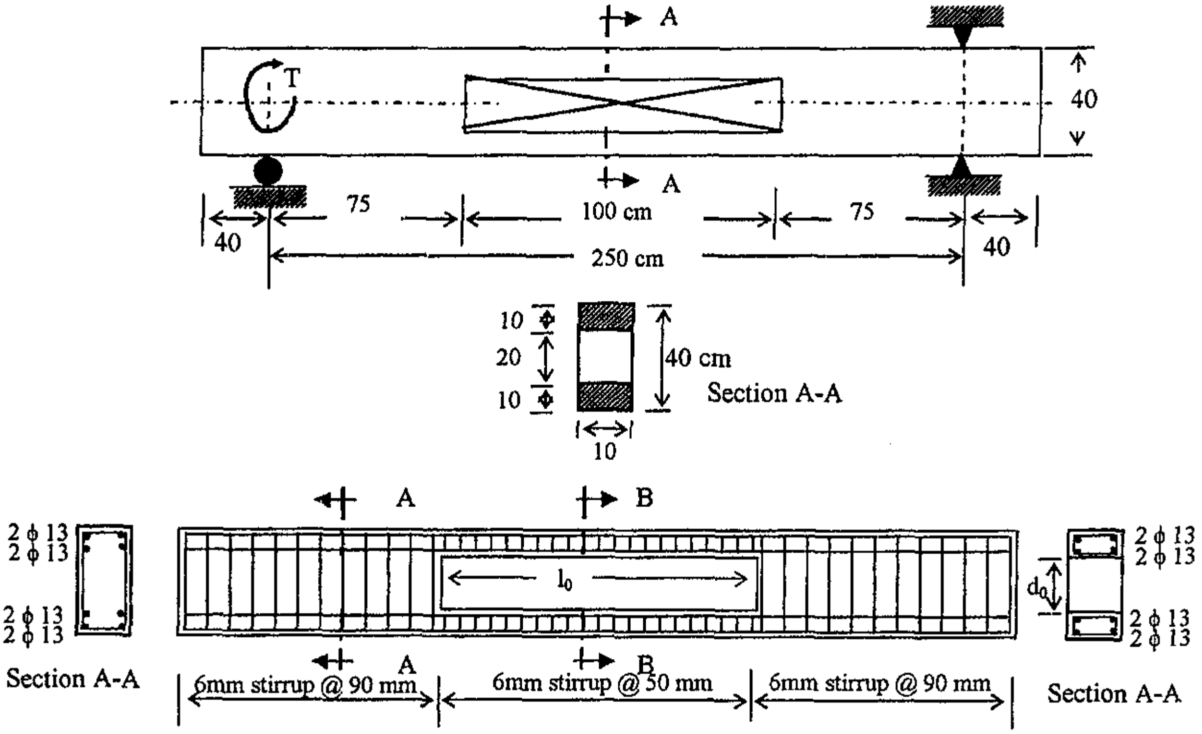

Fig. 1 Dimensions and reinforcement details of the tested beam [4]

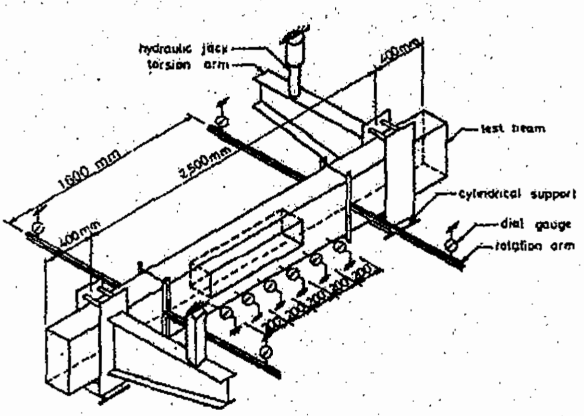

Fig. 2 Test Setup and instrumentation for experimental work [4]

3. MODELING OF THE TESTED BEAM BY FINITE ELEMENT METHOD

In the analysis, the beam is divided into 22 elements to simulate the reinforced concrete specimens as shown in Fig. 3. Reinforcements including stirrups and longit-udinal bars are represented for each element with its actual location, area, and direction as shown in Fig. 4. A Gauss rule with 3x3x3 was used for this study.

To check the reliability of the program for torsional analysis, load application, boundary conditions, shear retention parameters, and tension-stiffening parameters were considered to simulate the problem.

3.1 Loading Simulation

To properly simulate the experimentally applied torque, three sets of nodal loads are assumed in the F.E. analysis to simulate the exact loading as shown in Fig. 5.

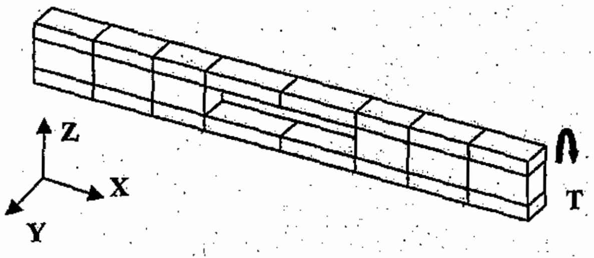

Fig. 3 Finite element mesh

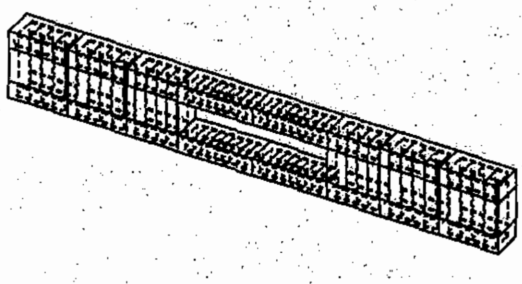

Fig. 4 FE Modeling of steel bars as imbedded reinforcement for beam B4

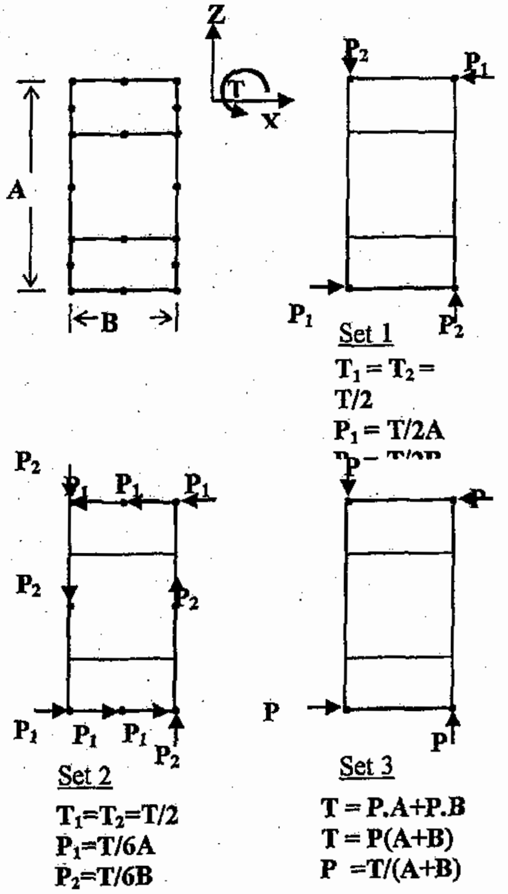

Fig. 5 Modeling of the applied load by three sets of loading

Figure 6 shows the effect of different simulation of load sets on the non-linear behavior and torque-angle of twist behavior of the tested beam. It can be seen from this figure that load application of Set 1 and 2, where the torque is applied as a set of two equal couples, gives an overall better results than the other load Set 3, therefore Set 1 is used for all subsequent analysis in this research.

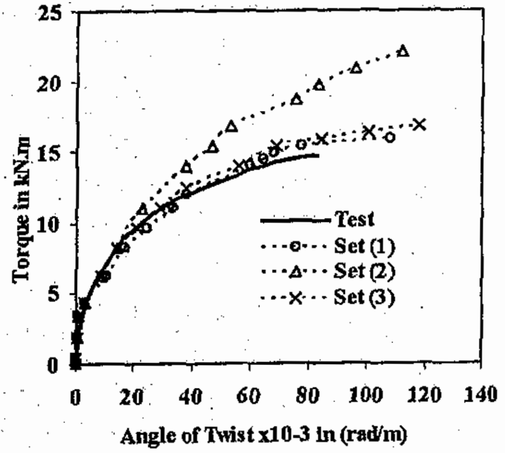

Fig. 6 Effect of load application on the non-linear behavior

3.2 Boundary Conditions

Six cases of boundary conditions were examined as shown in Fig. 7, where:

Case (1): all external nodes at one end are completely fixed to prevent any warping of the cross-section,

Case (2): the four corner nodes of the beam at one end are completely fixed to allow symmetrical warping to occur,

Case (3): is similar to case (2) but instead the four middle nodes were fixed,

Case (4): Three nodes only were completely fixed, this being the minimum to prevent unconstrained movement,

Case (5): all nodes at the end are fixed in the y- and z-directions while only three nodes fixed in the x-direction, this being an attempt to simulate the skew- symmetry nature of the problem, and finally

Case (6): is a combination of case (2) and case (3).

Fig. 8 shows the nonlinear torque-twist curves for different cases of boundary conditions and Fig. 9 compares ultimate torques for all boundary conditions with the test results. Boundary condition, Case (5) gives a too stiff post-cracking response and a large overestimate of the ultimate torque. Case (3) produced the closest fit to the experimental results; therefore, it is applied for all subsequent analysis

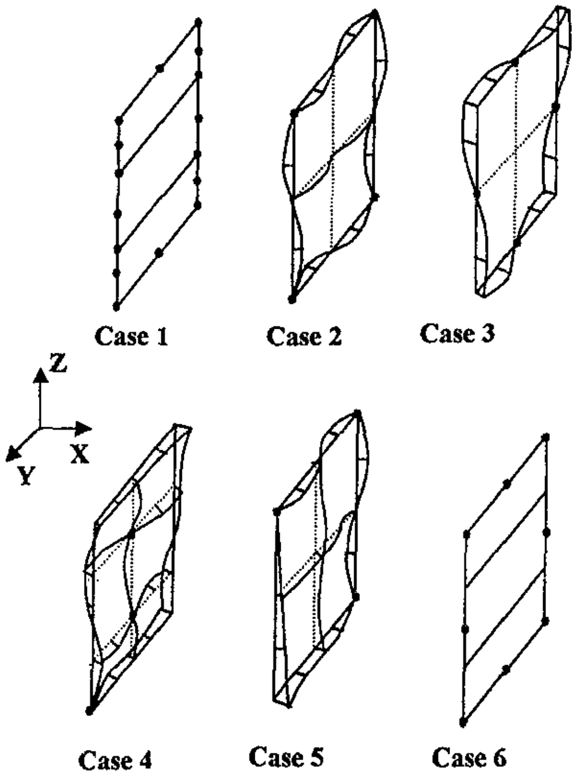

Fig. 7 Different boundary conditions

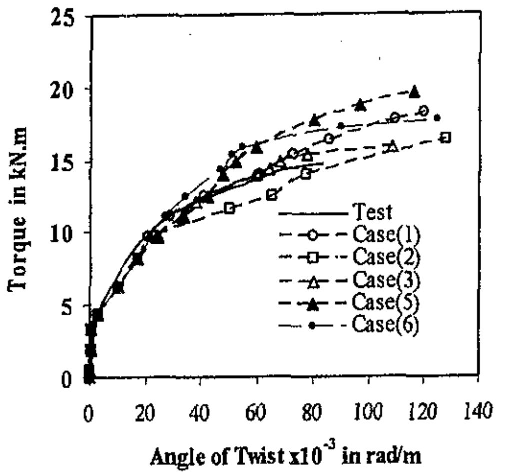

Fig. 8 Torque-angle of twist curves for different cases of boundary conditions

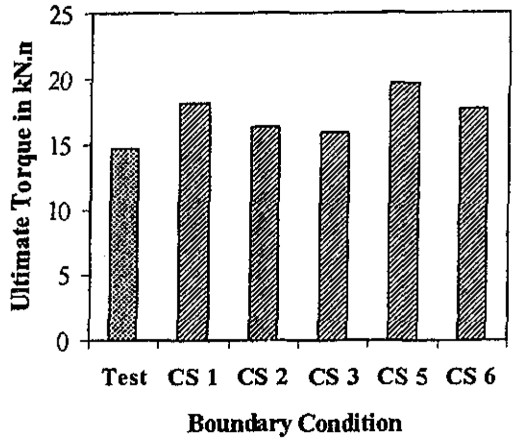

Fig. 9 Ultimate torques for different cases of boundary conditions

4. PARAMETERIC STUDIES

The main parameters affecting the results of torsion by the proposed nonlinear finite element model for reinforced concrete structures can be classified into two groups; the first contains material property parameters of both concrete and steel such as tension stiffening, shear retention factor, compressive and tensile strengths, as well as stress-strain relationships, and the second contains "numerical" factors such as the solution techniques used to solve non-linear equations, mesh size, number and size of increments; maximum number of iterations in each increment, convergence tolerance, the type of element used in the analysis, the number of Gauss points per element and other parameters such as simulation of supports and applied loads.

The aim of this section is to study the effects of some of the above parameters on the behavior of reinforced concrete members subjected to pure torsion.

In addition to the pervious factors, the effect of opening dimensions on the behavior of beam B4 will be investigated.

5. EFFECT OF NONLINEAR PARAMETERS

The following parameters that influence the nonlinear analysis were studied in this part:

- Shear retention factor
- Tension stiffening factor
- Compressive strength of concrete
- Opening dimensions

5.1 Effect of Shear Retention Parameters

The shear retention factor can strongly influence a nonlinear solution, especially if shear is prominent. The torsion problem is primarily a shear dominant problem.

To achieve an aim of incorporating a realistic shear retention factor to model shear transfer across cracked concrete, a quadratic function is used [1] based on the assumption of direct strain normal to the crack as shown in Fig. (10).

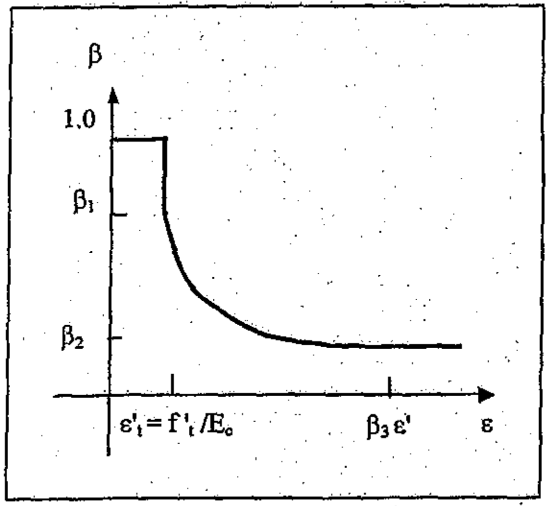

Fig. 10 Variable shear retension factor

This is given by

$$\text{For } \frac{\varepsilon}{\varepsilon_t} \le 1 \qquad \beta = 1 \tag{1}$$

$$\text{For } 1 < \frac{\varepsilon}{\varepsilon_t} \le \beta_3$$

$$\beta = \beta_1 - \beta_4[(1-2\beta_3) + 2\beta_3\left(\frac{\varepsilon}{\varepsilon_t}\right) - \left(\frac{\varepsilon}{\varepsilon_t}\right)^2] \tag{2}$$

$$\text{Where} \qquad \beta_4 = \frac{(\beta_1 - \beta_2)}{(1-\beta_3)^2} \tag{3}$$

$$\text{and } \frac{d\beta}{d\varepsilon} = 0 \quad \text{at} \quad \frac{\varepsilon}{\varepsilon_t} = \beta_3 \tag{4}$$

$$\text{For } \frac{\varepsilon}{\varepsilon_t} > \beta_3 \quad \beta = \beta_2 \tag{5}$$

Where ε' is the uniaxial cracking strain of concrete.

β_1 = the value to which the shear retention factor drops immediately upon cracking to represent the sudden loss of stiffness of cracking formation.

β_2 = the value to which the shear retention factor finally settles and continues as a constant to represent the residual shear stiffness due to dowel action once a crack has opened sufficiently for aggregate interlocking to cease.

β_3 = the ratio of final normal tensile strain, after which the shear retention factor remains constant, to the cracking strain to represent the rate of decay of stiffness as the crack widens.

Shear retention factors can strongly influence a nonlinear solution, especially if shear is prominent. The torsion problem is primarily a shear dominant problem and because of this dominance, and in order to isolate the effects of shear retention from those of tension stiffening, tension stiffening was assumed inactive for the study reported in this section.

The effects of the shear retention parameters (β_1, β_2, and β_3) on the nonlinear behavior of the investigated beam are considered with the following limits:

$\beta_1 = (0.0 - 1.0)$

$\beta_2 = (0.0 - \beta_1)$

$\beta_3 = 0.003 * E_c / f_t'$

Where E_c is the modulus of elasticity of concrete, and f_t is the tensile strength of concrete.

Figure 11, shows torque-twist curves for the beam for different values of β_1, and Fig. 12 shows the variation of the behavior for different values of β_2 and β_3. It is clear from the figures that variation of shear retention parameters produced significant effects on ultimate torque and post-cracking stiffness.

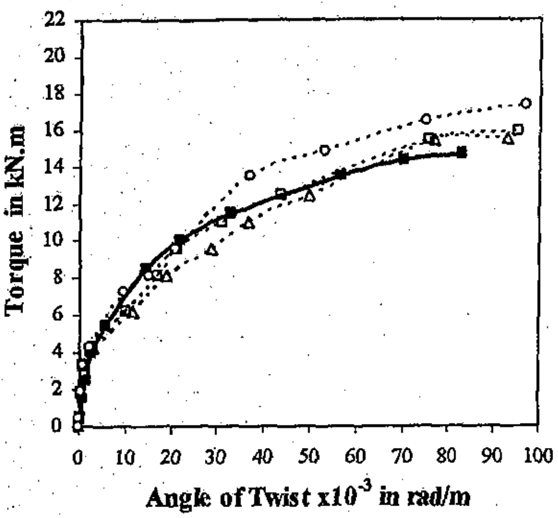

Fig. 11 Effect of shear retention parameter (β_1) on torque-twist behavior

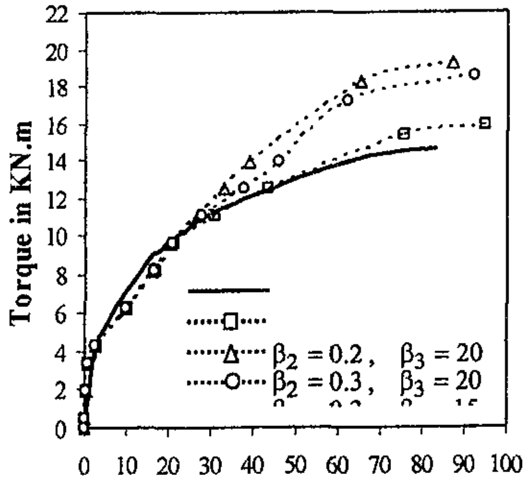

Fig. 12 Effect of shear retention parameters (β_2 & β_3) on torque-twist behavior

5.2 Effect of Tension Stiffening Parameters

When concrete reaches the ultimate tensile strength, primary cracks form at finite intervals along the length. The load is transferred across these cracks by reinforcement, but the concrete between cracks is still capable of carrying part of the stresses because of the bond between steel and concrete. This phenomenon is known as tension stiffening effect. The concrete stress is zero at the cracks, but is not zero if averaged over the length. Figure 13 shows the stress distribution of a cracked reinforced concrete element. As the load increases, more cracks develop and the amount of the tension carried by the concrete is gradually decreased. Several approaches based on experimental results have been employed to simulate this behavior. A gradual release of the concrete stress component normal to the cracked plane is considered in the present work as shown in Fig. 14.

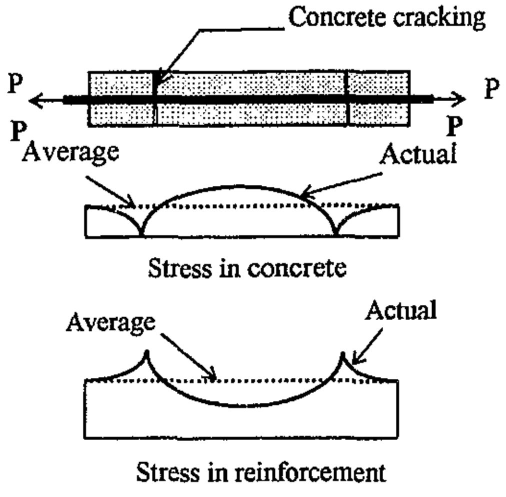

Fig. 13 Stresses in cracked reinforced concrete element

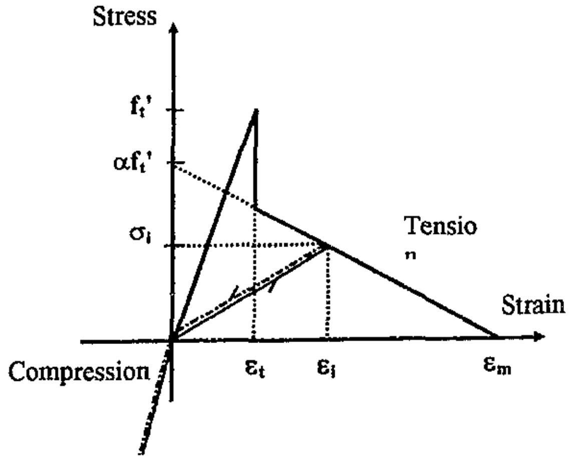

Fig. 14 Tension stiffening model [7]

The modulus of elasticity is assumed to decrease due to cracking as the strain increases [7] by the following formula:

$$E_i = \frac{\alpha f_t'}{\varepsilon_i}(1 - \frac{\varepsilon_i}{\varepsilon_m}), \qquad \text{and} \qquad \varepsilon_t \le \varepsilon_i \le \varepsilon_m . \qquad (6)$$

Where f_t' is the modulus of rupture of the concrete, ε_t is the uniaxial cracking strain, and α, εm are tension stiffening parameters which depend on concrete strength.

The normal stresses are obtained by:

$$\sigma_i = \alpha f_t' (1 - \frac{\varepsilon_i}{\varepsilon_m}), \qquad \text{and} \qquad \varepsilon_t \le \varepsilon_i \le \varepsilon_m \qquad (7)$$

α_1 = the ratio of the value to which the normal stress across the crack drops immediately at cracking to the tensile strength uncracked concrete.

α_2 = the ratio of the final tensile strain at which the tension stiffening effect becomes zero to the cracking strain of concrete.

Several values for the tension stiffening parameters α_1 and α_2 were examined, as shown in Figs. 15 and 16 with the shear retention parameters adopted from the previous section as:

$\beta_1 = 0.4$, $\beta_2 = 0.20$, $\beta_3 = 20.0$

It is evidently clear from the figures that all values of tension stiffening parameters produced slightly stiff initial post-cracking response. However, neglecting the tension stiffening parameters predicts reasonable post cracking response. Furthermore, the ultimate torques predicted with the inclusion of tension stiffening was only slightly higher than those predicted with no-tension stiffening. This can probably be attributed to the nature of the torsional cracking around the external perimeter of the cross section as distinctly different from flexural cracking. The maximum number of iterations was taken as 20 with a convergence tolerance equal to 1%. The element stiffness was recomputed at the beginning of each load increment. The load was applied in small equal increments of 0.48 kN.m until failure.

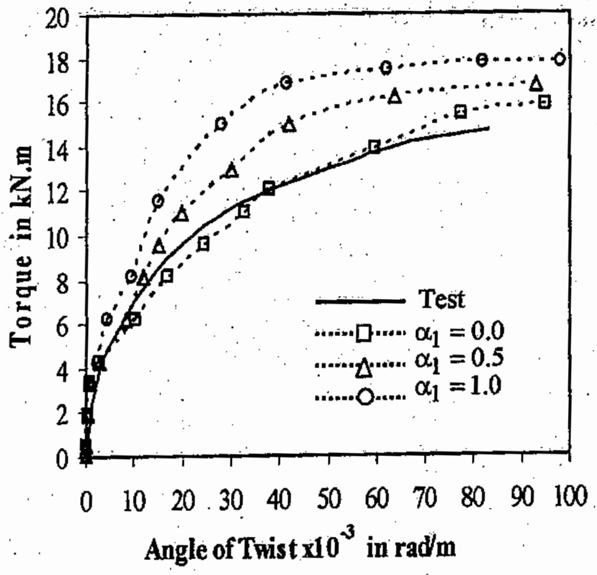

Fig. 15 Effect of tension stiffening (α_1) parameters on the non-linear behavior

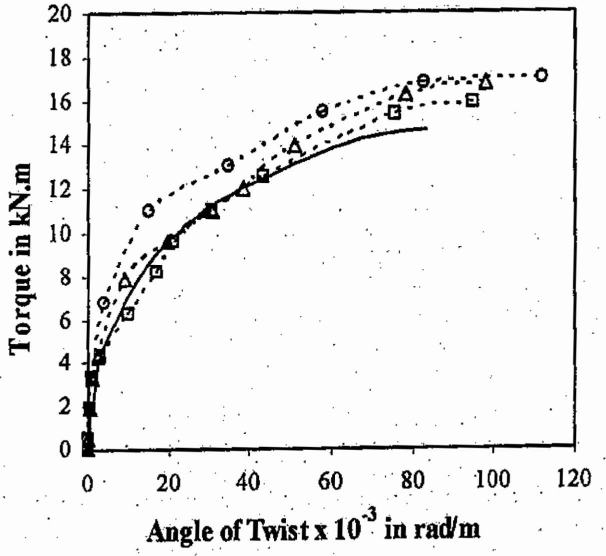

Fig. 16 Effect of tension stiffening (α_2) parameters on the non-linear behavior

Comparison between experimental and analytical results

The analytical results based on FEM model considering the previous factors are compared with the experimental results for the test beam. Figure 17 compare the torque–angle of twist curve and Fig. 18. compare the cracking ultimate torque. It is shown from these figures that a very good agreement between the experimental and analytical results was obtained.

Crack propagation of the test beam at different loading values is shown in Fig. 19. The experimental crack pattern of the test beam is shown in Fig. 20. It is clear from the figure that the predicted pattern of cracks using the FEM model was almost similar to the experimental results. It is shown from the crack patterns that stress concentration occurs at the corners of the opening.

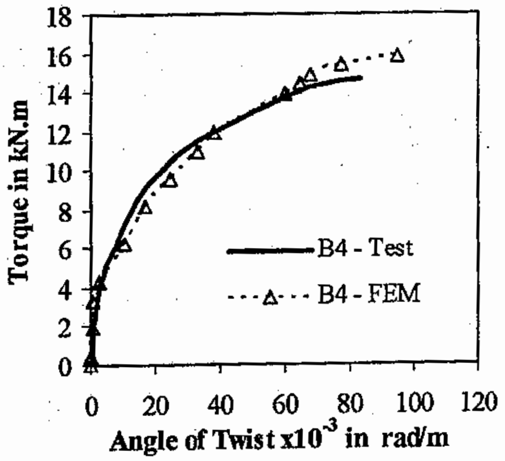

Fig. 17 Comparison of torque-twist curves between experimental and analytical results

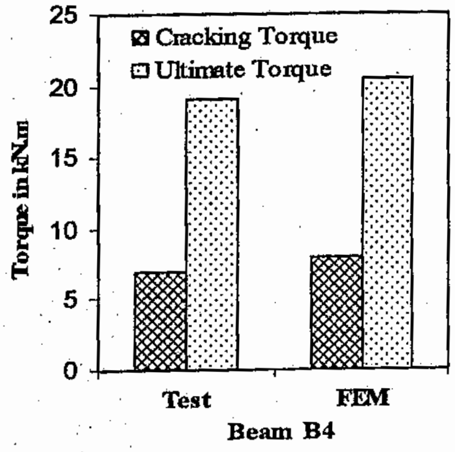

Fig. 18 Cracking and ultimate torques

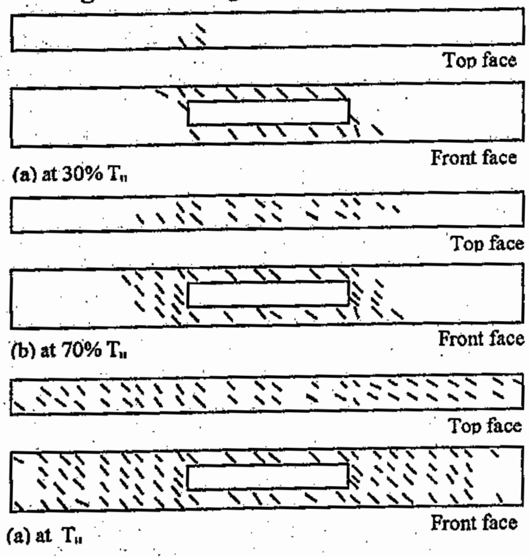

Fig. 19 Analytical crack patterns of beam B4

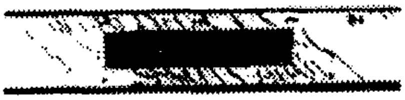

Fig. 20 Experimental crack pattern of beam B4

5.3 Effect of Compressive Strength

Many researchers investigated the effect of torsion on the behavior of normal and high strength concrete [6] with different compressive strengths. In this section three values of compressive strengths of concrete were applied to identify the concrete in the finite element model proposed for the test beams. The objective of this section is to show the effect of increasing the compressive strength on the torsion behavior. The following relations between the compressive strength, tensile strength and young's modulus [7] are considered in the analysis:

$$f_t' = 6.7\sqrt{f_c'} \quad \text{psi} \qquad (8)$$

$$E_c = 57000\sqrt{f_c'} \quad \text{psi} \qquad (9)$$

Three different values of compressive strength of concrete, f_c', and the corresponding tensile strength, f_t', and the modulus of elasticity E_c, are applied in the analysis. The given data were as follows:

Case (1):
f_c' = 3.55 kN/cm^2
f_t' = 0.33 kN/cm^2
E_c = 2821 kN/cm^2

Case (2):
f_c' = 4.44 kN/cm^2
f_t' = 0.37 kN/cm^2
E_c = 3155 kN/cm^2

Case (3):
f_c' = 5.55 kN/cm^2
f_t' = 0.41 kN/cm^2
Modulus of elasticity of concrete,
E_c = 3527 kN/cm^2

The previous cases are analyzed by the finite elements and the results are compared.

The effects of the compressive strength are shown in Fig. 21 and 22. It is noticed that, when compressive strength was increased by 25%, cracking and ultimate torques of the considered beam were increased by about 14% and 12.5%, respectively.

5.4 Effect of Openings Dimensions

The opening dimensions in the tested beam are assumed with different values in order to investigate their effects on the behavior of beam B4. The range of values for these parameters is chosen as to be within the practical range. The following parameters are considered:

- The opening depth, d_o
- The opening length, l_o

While investigating each one of the parameters, the other parameter is kept constant.

To investigate the effect of variation of opening dimensions on the behavior of reinforced concrete beams under torsion, test beam was investigated under the following conditions:

- Opening depth with values of 160, 180, and 200 mm and with a constant opening length of 800 mm.
- Opening length with values of 600, 800, 1000 mm with a constant opening depth of 200 mm.

Results of the analysis are illustrated in figures from 23 through 26.

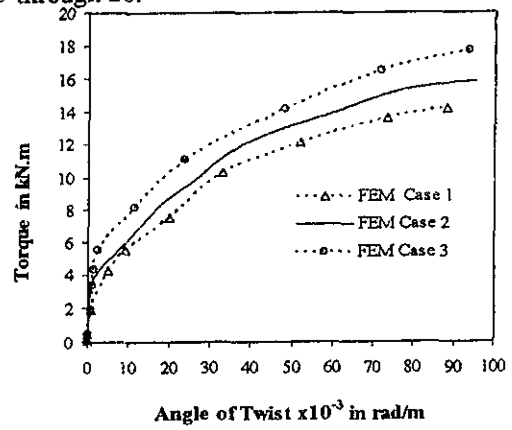

Fig. 21 Effect of compressive strength of concrete on the torque-twist behavior

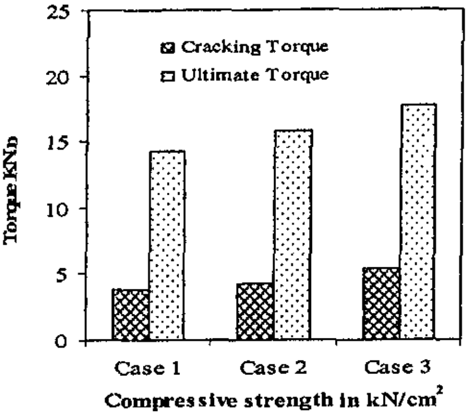

Fig. 22 Effect of compressive strength of concrete on the cracking and ultimate torques

It can be noticed that torsional capacity of a beam decreases with increasing length or depth of opening. The pre-cracking torsional stiffness of beams decreases with increasing length or depth of opening. After cracking, the slope of the torque-twist curves appears to be independent of opening dimensions. In other word, the slopes of the torque-twist curves in the post-cracking range were nearly the same for all the specimens regardless of the length or the depth of the openings.

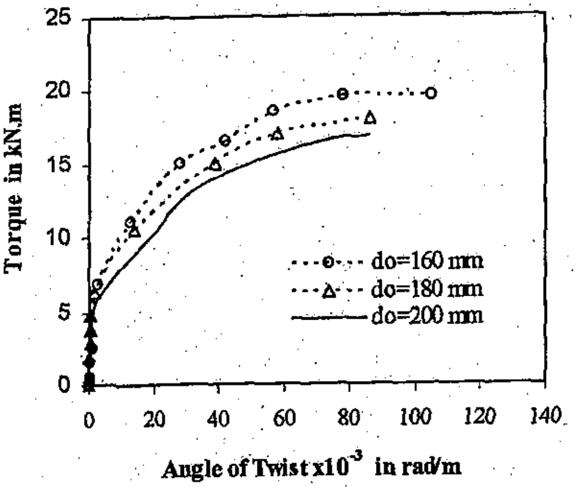

Fig. 23 Effect of opening depth on the torque-angle of twist behavior

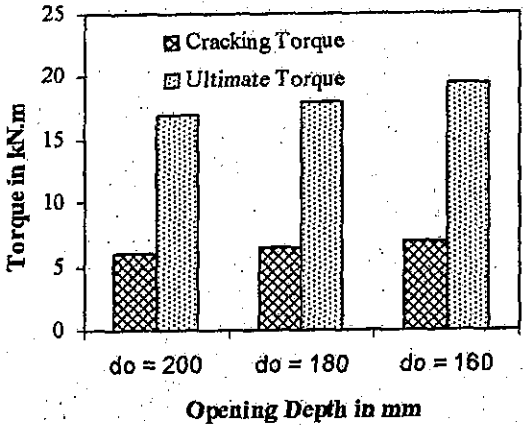

Fig. 24 Effect of opening depth on the cracking and ultimate torques

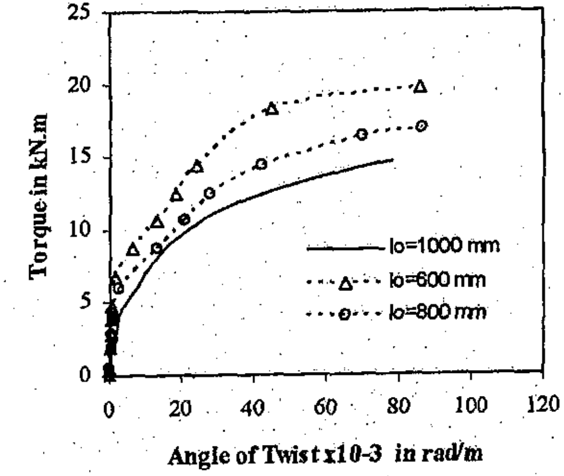

Fig. 25 Effect of opening length on the torque-angle of twist curve

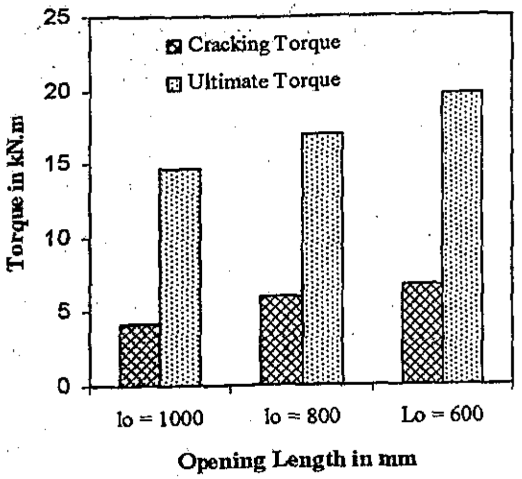

Fig. 26 Effect of opening length on the cracking and ultimate torques

6. CONCLUSION

1. The chosen isoparametric element with 20 nodes and embedded steel bars can represent the concrete and simulate the reinforcement with real dimensions and positions.
2. The developed three dimensional nonlinear finite element model simulates the pure torsional behavior of reinforced concrete rectangular beams within acceptable accuracy. In particular, torque-twist curves and ultimate strengths were all predicted well.
3. The finite element model predicts the behavior of reinforced concrete beams under combined loading reasonably well. Proper attention must be paid for selecting the nonlinear material parameters.
4. Suitable modeling of boundary conditions for torsion is important, and must be taken care of to allow for proper warping effects.
   Application of torque in the finite element program can be simulated by a set of two equal couples, since it gives acceptable simulation for the warping of the cross section.
5. Care and proper judgment must be used to identify what parameters affect the particular situation in question. Generally, when shear dominates the shear retention factor is more important and tension stiffening may be neglected or used with very small values of its parameters. On the other hand, when flexure dominates the tension stiffening may be more important and should be considered.
6. The results from the 3-D finite element program were harmonic with the experimental results and therefore, the proposed analysis with FEM proved to be a good tool for predicting the overall behavior of reinforced concrete beams with openings subjected to torsion and/or shear and bending moment.

7. Concrete grade affect the values of cracking and ultimate torques. It is noticed that increasing of *compressive strength value by 25% results in* increasing of cracking and ultimate torques of beam by about 14% and 12.5%, respectively.
8. Torsional capacity of a beam decreases with increasing length or depth of opening. It can be concluded from this study that increasing of opening length by a ratio of 50% of beam depth led to decreasing of ultimate torque by about 15.5%. In addition to that, a reduction in opening depth by a ratio of 0.05% of beam cross section results in an increasing in ultimate torque by about 7%.

7. LIST OF SYMBOLES

E_c	Modulus of elasticity of concrete
f'_c	Compressive strength of standard concrete cylinder
f_t	Tensile strength of concrete
α_1	the ratio of the value to which the normal stress across the crack drops immediately at cracking to the tensile strength uncracked concrete.
α_2	The ratio of the final tensile strain at which the tension stiffening effect becomes zero to the cracking strain of concrete.
β_1	The value to which the shear retention factor drops immediately upon cracking
β_2	The value to which the shear retention factor finally settles and continues as a constant
β_3	The ratio of final normal tensile strain, after which the shear retention factor remains constant, to the cracking strain.
ε'	Uniaxial cracking strain of concrete

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