

## Thermodynamic Optimization of Solar Collectors Operating Conditions

الأمتلة الترموديناميكية لظروف تشغيل المجمعات الشمسية

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الخلاصة

في هذا البحث تم دراسة أمتلة الاكسرجي المنتج بواسطة المجمعات الشمسية. بتجميع معادلات توزيع الحرارة و قوانين الديناميكا الحرارية الأول و الثاني ومفهوم الاكسرجي تم استنتاج ظروف التشغيل المثلى تحليليا. وكذلك تم الحصول على معادلات رياضية لدرجة حرارة الخروج المثلى المخرجة و كذلك لكمية المانع المناسبة بالمجموع. يمكن أن تحسن هذه النتائج من تصميم المجمعات الشمسية وبالتالي تعزيز أدائها.

### ABSTRACT

In this paper, optimization of exergy delivery by solar collectors is considered. Combining the temperature distribution, thermodynamic laws, and the exergy concept, the optimum operating conditions are analytically derived. Simple analytical expressions for the optimum outlet temperature and mass flow rate are obtained. The obtained results of this work may improve the solar collectors design and hence enhance their performance.

**keywords** Solar collectors, Operating conditions, Thermodynamic, Optimization, Exergy

### INTRODUCTION

Performance of solar energy systems is largely dependent on solar insolation that is transferred to the fluid. The working temperatures are determined by the application of useful energy production. The optimum operating conditions of solar collectors have been investigated on the basis of the collected thermal energy. This technique is generally used to maximize the difference between the collected thermal energy and the required pumping power, Kovarik et al. [1976] and Winn et al. [1981]. In fact, this criterion equalizes the value of mechanical energy required and the obtained thermal energy. The required pumping power is converted to thermal energy by friction. This process reduces the quality of energy, but not the quantity. With the use of energy balance only in the thermal analysis, it is impossible to determine the optimum design and operating conditions for an independent collector, Suzuki [1988]. Because, on one hand, according to the law of conservation of energy, high temperature heat collection is accompanied with a large heat leakage to the surroundings, and for

suppressing this, the thermal energy must be collected at low temperature. On the other hand, an energetic efficiency curve of the solar collector is decreasing monotonically with respect to outlet temperature. The quality as well as the quantity of thermal energy can be treated by means of exergy, Kotas [1985]. Exergy is an equivalent concept of availability or available work. There has been growing interest in the use of exergy principles for analyzing and evaluating energy demand as well as technologies available to meet it. Specifically, the second law of thermodynamics has been used for better understanding the irreversible nature of real processes and systems and thereby defines the upper bounds of available energy, Brodyansky et al. [1994]. Many researchers have activated the second law analysis of various energy conversion components and systems, see Bejan [1988]. For example, the exergy analysis of the collector performance and design can provide the answer for achieving maximum convertible energy output, and hence improving the performance. Derivation of optimum operating conditions (OOC) of a solar collector by the use of exergy concept or its equivalent has been carried out by some investigators. Bejan et al. [1981] have obtained the OOC on the basis of entropy generation for isothermal and nonisothermal collector models, where the inlet temperature assumed equal to the ambient temperature. Suzuki et al. [1987], have derived an equation for determining the collector outlet temperature that will optimize the system performance. However, because of the non linearity of this equation, its solution is very difficult. Fujiwara [1985] has showed that the criterion based on thermal efficiency and pumping power does not give the optimum operating conditions when the friction heat is included in the analysis. He has showed graphically that the optimum flow rate becomes infinite in some regions of the inlet temperature.

In this article, the exergy analysis of solar collectors is presented. A number of simple relations are developed analytically for OOC calculations. These relations can readily be used for computer solar collector programming.

### EXERGY ANALYSIS

In steady flow with no changes in velocity and altitude, the fluid flow between the collector inlet and outlet gains an amount of energy equal to  $Q_c$  according to the first law of thermodynamics.

$$Q_s = Q_c - Q_l \quad (1)$$

By combining this equation with the following second law form;

$$-\frac{Q_c}{T_a} + \frac{Q_s}{T_s} + mT(s_1 - s_2) + S_{irr} = 0 \quad (2)$$

The symbols has the meaning given in the nomenclature.

The exergy loss by the collector per unit time can be obtained from the following equation, Bejan [1988];

$$\dot{E}x_{loss} = T_a \cdot S_{irr} \quad (3)$$

From equations (1), (2), and (3) the following relations are obtained;

$$Ex_{loss} = Q_s \left(1 - \frac{T_a}{T_s}\right) - \dot{m}(h_o - h_i) + T_o(s_o - s_i) \quad (4)$$

or

$$= Q_s \left(1 - \frac{T_a}{T_s}\right) - \dot{m} C_p (T_o - T_i) + T_o \dot{m} C_p \ln \left(\frac{T_o}{T_i}\right) \quad (5)$$

where the first term  $Q_s \left(1 - \frac{T_a}{T_s}\right)$  in the above equations represents the exergy flux from the sun. As any exergy quantity, it is expressed by multiplying energy difference from its dead state by energy availability ratio (EAR). The EAR is the maximum convertible fraction of an energy quantity into work when using the reversible processes, Suzuki [1988].  $T_s$  is the apparent sun temperature as an exergy source ( $\approx 5800K$ ). Also, the destroyed exergy can be represented by the following equation ;

$$\dot{Ex}_{loss} = \dot{Ex}_i - \dot{Ex}_o \quad (6)$$

where  $Ex_o$  presents the exergy gained by the working fluid due to increasing the fluid temperature by the insolation and  $Ex_i$  is the exergy associated with the solar radiation .

In equation (6),  $Ex_o$  takes the following form;

$$Ex_o = \dot{m} C_p (T_o - T_i) - T_o \dot{m} C_p \ln \left(\frac{T_o}{T_i}\right) \quad (7)$$

In steady operating conditions , the mass flow rate per unit area  $\dot{M}$ , and the collector inlet and outlet temperatures  $T_i, T_o$  , are linked by the following equation, Suzuki et al. [1987]:

$$T_i = \theta - (\theta - T_o) e^{\beta} \quad (8)$$

where;

$$\beta = U / \dot{M} C_p$$

$$\theta = T_a + \frac{\tau \alpha Q_s}{U}$$

$\theta$  : The stagnation temperature . It is the highest temperature of the collector which would be encountered under the condition of no fluid flow through the collector , Fujiwara [1983].

### EXERGY OPTIMIZATION

Now, what is the optimal operating conditions in order to achieve the maximum net exergy flow?. In other words, what are the optimal values of  $T_i, T_o$  and consequently the optimal value of  $\dot{m}$  that maximizing the net exergy flow. The answers of these questions are presented in this section. Eliminating  $T_i$  in equation ( 7) by equation (8) ,  $\dot{Ex}_o$  can be expressed as a function of  $T_o$  and  $\dot{m}$  as follows ;

$$\dot{E}x_o = mC_p e^\beta (\theta - T_o) - mC_p T_o \ln \left( \frac{T_o}{e^\beta (T_o - \theta) + \theta} \right) + mC_p (T_o - \theta) \quad (9)$$

By differentiating the above equation with respect to  $T_o$  and setting  $\frac{\partial E_o}{\partial T_o}$  equals zero, the following equation is obtained

$$\frac{mC_p \left[ T_o e^{2\beta} (T_o - \theta) - e^\beta (T_o \theta + T_o (T_o - 2\theta)) + \theta (T_o - T_o) \right]}{T_o (e^\beta (T_o - \theta) + \theta)} = 0 \quad (10)$$

After some mathematical manipulation the optimum outlet temperature  $T_o^{opt}$  of the fluid can be obtained as follows ;

$$T_o^{opt} = \pm \frac{\sqrt{\theta e^{-\beta}} \sqrt{\theta e^{2\beta} + 2e^\beta (2T_a - \theta)} + \theta}{2} - \frac{\theta e^{-\beta}}{2} + \frac{\theta}{2} \quad (11)$$

The above equation represents  $T_o^{opt}$  as a function of  $\beta$  and stagnation temperature  $\theta$ . From equations (8) and (11),  $T_o^{opt}$  can be calculated as a function of  $T_i$  and  $\theta$  from the following implicit equation:

$$\frac{2T_o^{opt}}{\theta} = \pm \sqrt{\left( \frac{T_o - \theta}{T_i - \theta} \right)^2 + 2 \left( \frac{T_o - \theta}{T_i - \theta} \right) \left( \frac{2T_a}{\theta} - 1 \right) + 1} - \left( \frac{T_o - \theta}{T_i - \theta} \right) + 1 \quad (12)$$

At fixed values of  $T_i$ ,  $T_a$  and  $\theta$  the above equation may be solved iteratively. Starting with a reasonable approximation for  $T_o$  to the solution for  $T_o^{opt}$ , and generates sequence of values  $\{T_o^k\}_{k=0}^{\infty}$  that convergence to  $T_o^{opt}$ .

In other words, the converged solution may be obtained if the condition  $T_o^{opt} - T_a \cong 0.0$  is satisfied, under this condition (i.e. at  $T_o^{opt} \cong T_a$ ) equation (12) can be simplified to the following simple form:

$$\frac{T_o^{opt}}{\theta} = \frac{T_a}{T_i} \quad (13)$$

Using this obtained simple equation, the optimal outlet temperature of the solar collector can be calculated by knowing  $\theta$ ,  $T_a$  and  $T_i$ . From equations (7 and 13), the optimal net exergy flow can be calculated as follows:

$$\dot{E}x_o^{opt} = mC_p \left[ \left( \frac{T_a \theta - T_i^2}{T_i} \right) - T_a \ln \left( \frac{T_a \theta}{T_i^2} \right) \right] \quad (14)$$

Similarly, from equations (7), and (8) the optimum inlet temperature  $T_i^{opt}$  is expressed as ;

$$T_i^{opt} = \pm \frac{\sqrt{\theta} \sqrt{\theta e^{2\beta} + 2(2T_a - \theta)e^\beta} + \theta}{2} - \frac{\theta e^\beta}{2} + \frac{\theta}{2} \quad (15)$$

Also, a simple relation for  $T_i^{opt}$  as a function of  $T_o$ ,  $T_a$  and  $\theta$  is developed as ;

$$\frac{T_o^{opt}}{\theta} = \frac{T_a}{T_c} \quad (16)$$

Knowing the value of  $T_o^{opt}$ , the optimal flow rate  $\dot{M}_{opt}$  to be circulated through the collector can be obtained from equation (8) and (13) as follows ;

$$\dot{M}_{opt} = \frac{U}{C_p} \frac{1}{Ln \left( \frac{\frac{T_c}{T_o^{opt}} - 1}{\frac{T_c}{T_a} - 1} \right)} \quad (17)$$

### RESULTS AND DISCUSSION

Figures (1 and 2) show the dependence of optimum outlet temperature  $\left(\frac{T_o^{opt}}{\theta}\right)$  and the mass flow factor  $(\beta = U/MC_p)$  on the inlet temperature ratio  $\left(\frac{T_c}{\theta}\right)$  at different values of  $(T_a/\theta)$ . It can be seen from figure (1) that in the range of  $\frac{T_c}{\theta} \geq \left(\frac{T_a}{\theta}\right)^{1/2}$ , the flow rate  $\dot{M}$  becomes infinite ( $\beta = 0$ ). In the range  $\frac{T_c}{\theta} \leq \frac{T_a}{\theta}$  however,  $\beta$  becomes infinite ( $\dot{M} = 0$ ). The former range corresponds to the case that the net exergy has a higher quality (viz. has a high inlet temperature). However, an infinite pumping power is required for the infinite  $\dot{M}$ . The latter range corresponds to the case that the net exergy has no quantity because  $\dot{M}$  becomes zero at this region. Therefore, it can be deduced that the above two regions are practically unsuitable for an optimum operating condition.

On the other hand, as shown in Figure (2), in the range of  $\frac{T_c}{\theta} \geq \frac{T_a}{\theta} \left(\frac{T_a}{\theta}\right)^{1/2}$ , the optimal outlet temperature decreases at a nearly constant rate by increasing the input temperature. Also the heat capacitance rate  $MC_p$  decreases exponentially with increasing the optimum outlet temperature. Once the fluid inlet temperature is specified with the other operating parameters in terms of the instantaneous values of  $Q_s$ ,  $T_a$ ,  $\tau\alpha$  and  $U$ , the optimum outlet temperature  $T_o^{opt}$  can be obtained from figure (2) or equation (13), then the optimal mass flow rate is calculated by equation (17) or determined from figure (1). Under the condition of uniform temperature distribution along the collector (i.e. for isothermal collector, where  $T_i = T_o$ ), it can be seen from equations (13,16, and 17) and figures (1), (2)

that  $M \Rightarrow \infty$  and  $\frac{T_o^{opt}}{\theta} = \frac{T_i^{opt}}{\theta} = \left(\frac{T_a}{\theta}\right)^{1/2}$ . This result can be confirmed by

putting  $\beta = 0$  (i.e.  $M \Rightarrow \infty$ ) in equations (8 and 11) which yields,

In fact these results agree with those obtained by Bejan et al. (1981), and Suzuki et al. (1987) for the isothermal model. Hence the optimum operating condition for the

isothermal collector can be specified by constant average temperature at  $\left(\frac{T_a}{\theta}\right)^{1/2}$

By considering the case of  $\frac{T_i}{\theta} = \frac{T_a}{\theta}$ , it can be seen from figure (2) or equation

(13) that  $T_o^{opt} = \theta$ . In other words, the optimum outlet temperature increases linearly with the stagnation temperature. It is a matter of interest to note that this analytical result does not agree with that numerically obtained by Bejan et al. (1984) where they suggested that  $T_o^{opt} = \theta^{0.7}$ . However, according to the definition of the stagnation temperature, mentioned before, the result which is reported in the present work seems more acceptable than that reported by Bejan et al. (1984)

### CONCLUSION

In summary, a number of simple equations representing the optimum operating conditions for solar collectors are derived analytically by the application of exergy concept. Once the collector has been designed or chosen, and the operating parameters in terms of  $T_a$ ,  $\theta$ , and  $T_i$  are specified, the following equations,

$$\frac{T_o^{opt}}{\theta} = \frac{T_a}{T_i}, \text{ and } \dot{M}_{opt} = \frac{\frac{U}{Cp}}{\ln\left(\frac{\frac{T_i}{\theta} - 1}{\frac{T_a}{T_i} - 1}\right)}$$

can be used to determine the values of

$T_o^{opt}$  and the mass flow rate  $\dot{M}_{opt}$  for that particular collector. The most satisfactory operating range for nonisothermal collectors is defined in the present work as

$\frac{T_i}{\theta} < \frac{T_i}{\theta} < \left(\frac{T_a}{\theta}\right)^{1/2}$ . However, for isothermal models, the optimum fluid temperature is

$(\theta T_a)^{1/2}$ . In the case of  $T_i = T_a$ , in other words, when the inlet temperature is equal to that of the ambient, it is found that; the optimum outlet temperature is linearly proportional with the stagnation temperature,  $T_o^{opt} = \theta$

### NOMENCLATURE

- A : collector surface area, (m<sup>2</sup>)
- C<sub>p</sub> : specific heat at constant pressure, (J/kg.K)
- h : specific enthalpy, (J/kg)
- Ex : rate of exergy, (W/m<sup>2</sup>)

$m$	: mass flow rate, ( kg/s )
$M$	: mass flow rate per unit area, ( kg/s.m <sup>2</sup> )
$Q_g$	: gained energy by the collector, ( W/m <sup>2</sup> )
$Q_s$	: solar radiation incident on the collector, ( W/m <sup>2</sup> )
$Q_l$	: heat loss to the ambient, ( W/m <sup>2</sup> )
$s$	: specific entropy, ( J/kg.K )
$S$	: entropy generation rate, ( W/m <sup>2</sup> .k )
$T$	: absolute temperature, ( K )
$T_s$	: apparent sun temperature as an exergy source, ( K )
$U$	: collector loss heat transfer coefficient, ( W/m <sup>2</sup> .k )
$\theta$	: stagnation temperature, ( K )
$\tau\alpha$	: collector transitivity absorptivity product

**Subscript**

a	: ambient
i	: inlet
irr	: irreversibility
o	: outlet

**Superscript**

opt	: corresponding to maximum rate of exergy collection
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**REFERENCES**

- Bejan, A .Advanced Engineering Thermodynamics. Wiley. 1988.
- Bejan, A.,Kearney, D W., and Kreith, K, .Second Law Analysis and Synthesis of Solar Collector Systems ASME Journal of Solar Energy Engineering, Vol 103, 1981, P 23
- Brodyansky, V.M, Sorin, M.V, and Goff P The Efficiency of Industrial Processes Exergy Analysis And Optimization. Elsevier, 1994
- Fujiwara, M Exergy Analysis for the Performance of Solar Collectors. ASME Journal of Solar Energy Engineering, Vol.105, 1983, P 163
- Kotas, T J The Exergy Methods of Thermal Plant Analysis Butterworths, London, 1985
- Kovarik, M, and Lesse, P.F. Optimal control of flow in low temperature solar heat collectors Solar Energy, Vol. 18, 1976, p. 431.
- Suzuki,A. A Fundamental Equations For Exergy Balance on Solar Energy Engineering. Vol 110, 1988, P.102
- Suzuki, A., Okamura, H, and Oshida, I. Application of Exergy Concept to the analysis of Optimum Operating Conditions of Solar Heat Collectors ASME Journal of Solar Energy Engineering, Vol.109, 1987, P.337
- Winn, C B, and Hull, D Optimal controllers of the second kind Solar energy, Vol. 23, 1979, p.529
- Winn, R. C, and Winn, C.B Optimal control of mass flow rates in flat plate solar collectors ASME Journal of Solar Energy Engineering , Vol. 103, 1981, p 113

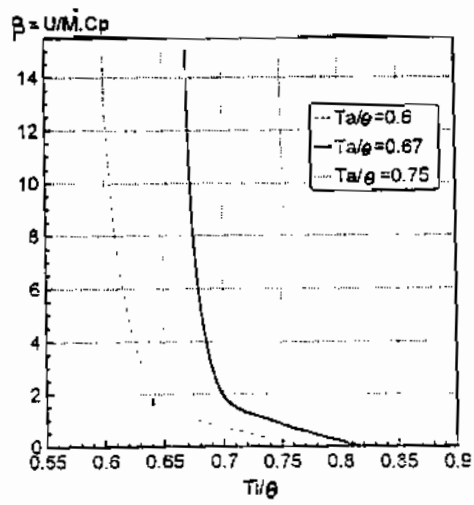


Fig. 1. Optimum mass flow rate as a function of inlet temperature for different  $T_a/\theta$

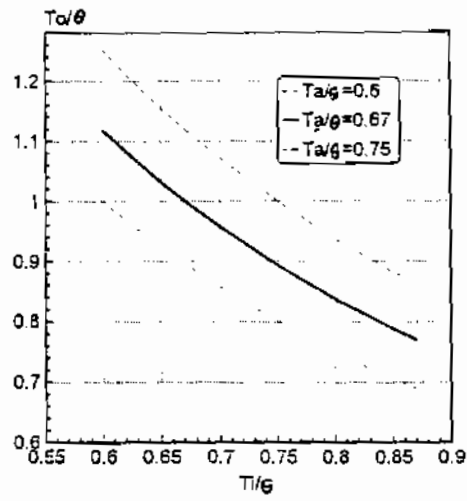


Fig.2. Optimum outlet temperature as a function of inlet temperature for different  $T_a/\theta$