



Allowed Tables and Charts: *None*

Answer all the following questions (with neat sketches)

Question (1)

(50 Marks)

- A. For 3-dimensional incompressible flow develop the continuity and Navier-Stokes equations in Cartesian coordinates. Reduce the final forms to steady incompressible flow. (20 Marks)
- B. Explain how to develop the turbulent Reynolds-stresses tensor matrix in three-dimensional turbulent flow. (15 Marks)
- C. Give a brief outline of the Blasius solution of laminar boundary layer for flow over a flat plate in the form: $f f'' + 2 f''' = 0$. What are the boundary conditions from which the analytical solution can be developed. Write the function of the boundary layer and displacement thicknesses developing in streamwise direction. (15 Marks)

Question (2)

(50 Marks)

- A. Explain with neat sketches the following terms:
Length scale, Energy cascade, Energy backscatter, Two-point correlation. (8 Marks)
- B. For a boundary layer flow, derive the momentum integral equation of von Kármán, in which the momentum thickness Θ and displacement thickness δ_1 are related to the wall shear stress τ_w with the presence of pressure gradient and free-stream velocity U_∞ as the following relation:
$$\frac{d\Theta}{dx} + \frac{(2\Theta + \delta_1)}{U_\infty} \frac{\partial U_\infty}{\partial x} = \frac{\tau_{wall}}{\rho U_\infty^2}$$
 (18 Marks)
- C. Derive the growth of the turbulent boundary layer thickness, the displacement thickness, the momentum thickness and the wall skin friction coefficient of a turbulent flow over a flat plate (without pressure gradient). How do you compute the total drag force of the plate? (14 Marks)
- D. Discuss the different regions in the boundary layer. Write the law of the wall, from which how do you derive the Clauser's plot relation? (10 Marks)

Best wishes
Professor Wageeh El-Askary