Mansoura University
Faculty of engineering
Department: Engineering Math. &Phys
Year: First year graduate

Course: Complex analysis
Code: 8504
Date: 22-9-2013 Time: 3 hrs
Full Mark: 60

Question1: [7 points] Answer true or false. If the statement is false, state why its false then write the correct statement.

a) There does not exist an analytic function $f(z) = u(x,y) + iv(x,y)$ for which $u(x,y) = y + 5x$.

b) If the function $f(z) = u(x,y) + iv(x,y)$ is analytic at point z. Then necessarily the function $g(z) = v(x,y) - iu(x,y)$ is analytic at z.

c) If $|e^z| = 2$, then z is a pure imaginary number.

d) The mapping $w = e^z$ takes vertical lines in the z −plane onto horizontal lines in the w-plane.

e) If $\int_C f(z)\, dz = 0$ for every simple closed contour C, then f is analytic within and on C.

f) If f is analytic within and on the simple closed contour C and z_0 is a point within C, then

$$\int_C \frac{f'(z)}{z - z_0} dz = \int_C \frac{f(z)}{(z - z_0)^2} dz.$$

g) If z_0 is a simple pole of a function f, then it is possible that $Res(f(z), z0) = 0$.

Question 2: [5 points] Fill in the blanks

a) The region in the complex plane consisting of the two disks $|z + i| < 1$ and $|z - i| < 1$ is......................................(connected/ not connected).

b) The statement "There exists a function $f(z)$ that is analytic for $Re(z) \geq 1$ and is not analytic anywhere else" is false because.............................

c) The complex exponential function e^z is periodic with period of....................................

d) If $\ln z$ is pure imaginary, then $|z| =$..................................

e) If n is a positive integer and C is the contour $|z| = 2$, then $\int_C z^{-n} e^z dz =$...

Question 3: [five points for each sub question]

a) Derive the trigonometric formula

$$\sin 3\theta = 3\cos^2\theta \sin\theta - \sin^3\theta.$$

b) Show that if $f(z) = x^3 + i(1-y)^3$, then $f'(z) = 3x^2$ only when $z = i$.

c) Show that if a function $f(z)$ and its conjugate are both analytic in a given domain D then $f(z)$ must be constant throughout D.

d) Show that if $v(x,y)$ and $V(x,y)$ are harmonic conjugates of $u(x,t)$ in a domain D, then $v(x,y)$ and $V(x,y)$ can differ at most by an additive constant.

e) Find all roots of the equation $\sin z = \cosh 4$.

f) Let C denote a contour of length L, and suppose that a function $f(z)$ is piecewise continuous on C. If M is a nonnegative constant such that

$$|f(z)| \leq M$$

for all points z on C at which $f(z)$ is defined, then prove that

$$\left| \int_C f(z)\, dz \right| \leq ML.$$

g) Evaluate the integrals

$$\int_{|z-i|=2} \frac{1}{(z^2+4)^2}\, dz, \qquad \int_{|z|=2} \tan z\, dz, \qquad \int_{|z|=2} \frac{dz}{\sinh 2z}$$

h) Give two Laurent series expansions in powers of z for the function

$$f(z) = \frac{1}{z^2(1-z)},$$

and specify the regions in which those expansions are valid.

i) Show that

$$\int_{-\infty}^{\infty} \frac{\cos 3x}{(x^2+1)^2}\, dx = \frac{2\pi}{e^3}.$$

j) Find the linear fractional transformation that maps the points $z_1 = -i, z_2 = 0,\ z_3 = i$ onto the points $w_1 = -1, w_2 = i, w_3 = 1$. Into what curve is the imaginary axis $x = 0$ transformed?