Eng. Research Bull.
Faculty of Eng. & Tech.,
Menoufia University.
Vol. VIII Part II, 1986.

PP: 87-102.

Analysis of statically indeterminate slabs and beams applying the method of concentrated deformation

by

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Synopsis

The finite element and more recently the boundary element method,are widely applied.Many computer programs for solving several problems are available. The concentrated deformation method can have the same field of application with the advantage of large reduction of computer work due to the reduction of the number of elements,yielding the same accuracy. There is no need in many cases to add other type of elements to determine the reactions as for example the reactions for continuons slab and beam. Simple solution is obtained in case of existance of a real joint ,especially if the joint has a stiffness different from the monolithic body (as composite beams),being much complicated in other methods.

Notation

a = Length of element in the x direction.

b = Width of element in the y direction.

E_x = Modulus of elasticity of element in the x direction .

E_Y = Modulus of elasticity of element in the Y direction .

I_x = Moment of inertia of element in the x direction.

I_Y = Moment of inertia of element in the y direction.

G = Shear modulus.

t = Thickness of element

P = Potential energy or work done

Q = Shearing force.

q = Uniform load on the roof.

M = Bending mement.

S = Stiffness.

T = Torsional moment.

α = Angle of rotation in bending.

ϕ = Angle of rotation in torsion .

ω = Stiffness in bending

ψ = Stiffness in torsion .

ξ = Stiffness in shear .

η = Stiffness in compression or tension .

introduction:

In case of the method of concentrated deformation the structure is divided into elements as in other methods. To get the same accuracy of the other methods the number of elements is lesser. In case of pre-cast structures the pre-cast unit can be considerd as one element for easier approach and the joints are considered as the boundary of the element. The elements are considered absolutely stiff. All the deformations are concentrated at the boundary (joints between the elements). As a first approach the stresses at the boundary were considered uniform and equal(1). In such approach to get good accuracy, the elements must be small enough using a large number of elements . But by considering the stresses distributed nonuniformly a good accuracy can be achieved with a smaller number of elements. The approach to the solution is based on the equilibrium of elements which is considerably simple and easier, compared with other methods. It is very interesting to state that when the element boundary are real joints with stiffness other than that of the elements, no change in the approach to the solution will occur, as will be discussed later. Application of this method has proven to be successful for statically determinate composite beams (2).

Method of analysis:

As a simple approach the method will be explained on an example of beamless slab of 2 panels, as shown in fig. 1a.

The roof must be divided into suitable rectangular elements, of dimensions axb. In our example, it is divided into elements of 0.5 x 0.5m. The system will be considered as stiff plates connected by elastic joints which can resist bending moments, torsion and shear. The joints between the elements can be conditional joints (due to division). or can be real joints in case of precast structures. The elements can be of diffrent material (physical parameters) or geometry (dimensions). Because the elements are considered as absolutely stiff, all the deformations (due to bending, torsion and shear) are considered to be concentrated at the element edges and at the joints. The connection between the elements is considered to consist of three diffrent connections: i) bending conection ii) torsion connection iii) shear connection fig. 1b.

Each of the above connections is considered as a complex connection which consists of the part which takes the effect of own deformation of the connected elements itself and the effect of deformation of the real joint if exists. So the stiffness of the joint due to bending or torsion or shear is considered to consist generally of three parts of stiffness which can generally be determined as follows:

$$\frac{1}{S} = \frac{1}{S_i} + \frac{1}{S_j} + \frac{1}{S_r} \; .... \qquad I$$

Where : S is the joint stiffness
S_i & S_j the elements stiffness
S_r is the real joint stiffness which must be determined from experiments. In case there is no real joints, S_r is taken equal to infi...

The three values of S (in bending, torsion and shear can be determined as follows :

As stated before the slab is divided into elements of dimensions axb in the x and y directions, respectively, and of thickness t .

Case of bending:

When the edge b is acted upon by bending mement= M_x

Potential energy for element of volume a.b.t:

Knowing that : $\sigma_x = \frac{M_x . Z}{I_x}$, $E_x = \frac{\sigma_x}{\varepsilon_x}$, $I_x = bt^3/12$

$$P = \frac{1}{2} \int_v \sigma_x . \varepsilon_x \, d_v$$

$$= \frac{ab}{2E_x} \frac{M_x^2}{I_x^2} \int_{-t/2}^{t/2} z^2 \, dz$$

$$= \frac{abM_x^2}{2E_x I_x^2} . \frac{t^3}{12}$$

$$= \frac{a \, M_x^2}{2E_x I_x} \quad ....(1)$$

and knowing that $M_x = \omega_x . . \alpha_y$
Where ω is the stiffness in bending and α_y is the angle of rotation .

The work down by the moment:

$$P = 2.\tfrac{1}{2}. M_x . \alpha_y$$

$$= M_x^2 / \omega_x \quad ....(2)$$

From (1) and (2) :

$\omega_x = 2E_x I_x / a$ where $I_x = bt^3/12$

Similarly: $\omega_y = 2E_y I_y / b$, $I_y = at^3/12$

And in case of isotropic body = $E_x = E_y = E$.

Case of Shear:

When the edge b of the element is acted upon by a shearing load Q :

Knowing that $\gamma_{zx} = \tau_{zx} / G$, and

$$\tau_{zx} = \frac{Q.S}{I_x . b} = \frac{Q \, b(\frac{t}{2} - Z)\,[Z + (\frac{t}{2} - Z)/2]}{I_y . b} , \quad I_x = bt^3/12$$

$$= \frac{Q\,(\frac{t}{2} - Z)\,(\frac{t}{2} + Z)/2}{I_x} = \frac{Q}{2.I_x} \left(\frac{t^2}{4} - Z^2\right)$$

Potential energy :

$$P = \tfrac{1}{2}. \int_v \tau_{zx} . \gamma_{zx} . d_v$$

$$= \frac{1}{2} \cdot \frac{Q^2}{4 \cdot I_x^2} \cdot \frac{1}{G} \cdot a.b \int_{-t/2}^{t/2} \left(\frac{t^2}{4} - Z^2 \right)^2 dz$$

$$= \frac{Q^2}{8 \cdot I_x^2 \cdot G} \cdot ab \int_{-t/2}^{t/2} \left[\frac{t^4}{16} - \frac{t^2 . Z^2}{2} + Z^4 \right] dz$$

$$= \frac{Q^2}{8 \cdot I_x^2 \cdot G} \; ab \left[\frac{t^5}{16} - \frac{t^2}{2} \cdot \frac{t^3}{3} \cdot \frac{1}{4} + \frac{t^5}{5} \cdot \frac{1}{16} \right]$$

$$= \frac{Q^2 \; ab}{8 I_x^2 \; G} \cdot \frac{t^5}{30} = \frac{3Q^2 a}{5Gtb} \quad \ldots\ldots \quad (1)$$

And knowing that $Q = \xi_x . \Delta$,

Where ξ =Stiffness in shear

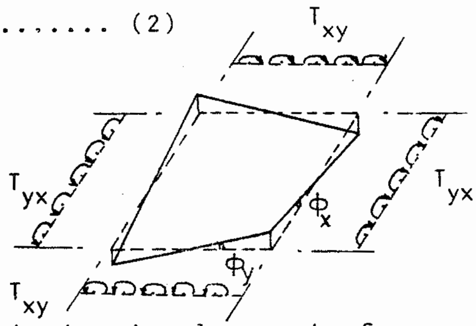
Work done by shear Q

$$P = 2 . \tfrac{1}{2} . Q . \Delta \quad = \frac{Q^2}{\xi_x} \quad \ldots\ldots\ldots \quad (2)$$

From (1) and (2)

$$\xi_x = \frac{5}{3} \; \frac{G. \; tb}{a}$$

Similarly $\quad \xi_y = \frac{5}{3} \cdot G.t. \frac{a}{b}$



Case of torsion:

The element is acted upon by torsional moment of equal intensity at all the edges

For the shown case of deformation potential energy of the element :

$$P = \tfrac{1}{2} \int_v \tau_{yx} \cdot \gamma_{yx} \; dv$$

Where: $\gamma_{yx} = \tau_{yx}/G.$

Work done by torsion :

$$P = \tfrac{1}{2} \left(2 \cdot T_{yx} . b . \; \Phi_x + 2T_{xy} \cdot a . \; \Phi_y \right)$$

$$= T_{xy} \left(b . \Phi_x + a . \Phi_y \right) \quad \ldots\ldots\ldots \quad (2)$$

Following the same analysis as before, the stiffness due to torsion can be expressed as follows:

$$\psi_x = \frac{Gt^3}{3} \cdot \frac{b}{a} \quad \& \quad \psi_y = \frac{Gt^3}{3} \; \frac{a}{b}$$

After determining the stiffness of the elements and if a real joint exists, its stiffness must be given from the standards or experiments, the general stiffness of the joint will be determined from equation(1) for bending, torsion and shear .

As stated before the first step in the solution is to divide the roof into a suitable number of elements . As an example the flat slab of two panels 3x3m and thickness 10cm is devided to elements of 0,5x0.5m in the x and y directions as shown in figure 1a. The elements can carry any type of loading, but for simplicity in our example, the roof is assumed to carry a uniform load $q = 600\ kg/m^2$.

The next step is simply to study the equilibrium of the elements in the vertical direction and around the two axes x and y . simply $\Sigma Q = 0$, $\Sigma Mx = 0$, $\Sigma My = 0$. The positive direction of Q, Mx, My can be chosen at any direction, we will assume the possitive directions as shown in figure 1a.

To reduce the amount of work in the solution, the elements which have the same forces are given the same type and their equilibrium is shown in figure 2.. The Arabic figures used in figure 1a give the element number. The roman numbers give the type number. So, we have 72 elements and thirteen types.

The next step after determining the complex stiffness of the joint, and the equilibrium of the elements, is to apply the stiffness method. The vector of displacement of the system U (two angles of rotation around the x and y axis α_x and α_y and vertical displacement z for each element) is determined by solving the following matrix equations of the stiffness method:

$$[A] \cdot [S] \cdot [A^T]\ \bar{u} = \bar{P} \quad \ldots\ldots \text{II}$$

Where

[A] is the matrix of the equations of equilibrium

$[A^T]$ is the transposed matrix of [A].

[S] stiffness metrix

$\bar{P}$ vector of the external forces.

The matrix [S] is a diagonal matrix.

The matrix [A] [S] $[A^T]$ is the matrix of internal stiffness of the whole system.
Solving equation II we get the displacement for every element (α_x , α_y and Z)

Applying what was stated before for the example, since matrix [S] is a diagonal matrix then the matrix [S] written as one raw matrix and the matrix [A] will have the view shown in table 1 .

To determine the stiffness values in the example, the material is considered isotropic with the following characteristics: $E = 2{,}0.10^6\ t/m^2$, $\mu = 0{,}25$, $G = E/2(1+\mu) = 0{,}4E$ and the roof is assumed to carry a load of $0{,}600 t/m^2$ (0,150 t/element since the element is 0,5x0,5m)

For simplicity all the elements are of the same material, having the same dimensions and thickness. No real joint exists, then the stiffnesses of all elements are equal, and are equal in both directions (a=b). Then:

$$S = \frac{1}{\frac{1}{S_i} + \frac{1}{S_j}} = \frac{Si}{2} = \frac{Sj}{2}$$

$$\omega^m = \omega^n = \frac{\omega_{element}}{2} = \frac{2EI}{2a} = \frac{50.10^3}{12.50} E = \frac{250}{3} E$$

$$\psi^m = \psi^n = \frac{\psi_{element}}{2} = \frac{1}{3} Gt^3 . \frac{a}{b}/2 = \frac{1}{6} .0{,}4E.10^3 = \frac{200}{3} E$$

$$\xi^m = \xi^n = \frac{\xi_{element}}{2} = \frac{5}{3} Gt \ \frac{a}{b}/2 = \frac{5}{6} .0{,}4E.10 = \frac{10E}{3}$$

To determine the reaction at the supports a reasonable value of the stiffness of the supports is assumed equal to 50 ξ at the external corners. For the reaction at mid span in order to keep the symmetry of the global stiffness matrix $[A]\ [S]\ [A^T]$ it is assumed to have two equal reactions at the same point for the left and right elements having the same stiffness of the external corner reactions:

$$\eta_1 = \eta_2 = 50\,\xi$$

To reduce the volume of work, since the matrix of internal stiffness $[A][S][A^T]$ is a symmetrical matrix, it is enough to have the values only above the diagonal (in the band width)

The band width =3xMax.difference in jointed element number + 2.

For our example: band width is 3,0.6,0 + 2= 20. Since according to the stiffness method for our case the strains in the joint will be affected only by the displacement of the elements directly adjacent to it(this is clear from table 1), and since the complex stiffness matrix of the joint is a diagonal matrix, the raws of the internal stiffness matrix can be determined mathematically for every element taking into consideration the effect of the elements jointed to it as shown in table 2.

The solution of the matrix equations was done by the computer.

As stated before the solution of the matrix equation gives the displacements (α_x, α_y and Z) at the center of each element. To get the vertical displacement (in the Z direction) for any other point in the element except at the center the two angles of rotation α_x and α_y must be taken into consideration .

The bending moment at any joint between two elements i and j :

$$M = \omega(\alpha_i - \alpha_j)$$

Where α is considered in the direction of the bending moment.

The torsional moment at any joint between two elements i and j :

$$T = \psi(\alpha_i - \alpha_j)$$

Where α is considered in the direction of the joint (perpendicular to the direction i,j).

The shearing force at any joint is equal to the diffrence between the perpendicular to the surface displacements at the middle of the contacted edges multipled by the shear stiffness. For example, at a horizontal joint between elements i and j :

$$Q_{ij} = \xi^{n}[(Z_i + \alpha_{ix} \cdot \frac{b}{2}) - (Z_j - \alpha_{jx} \cdot \frac{b}{2})]$$

The reaction at any support is equal to the displacement at its point in its direction multiplied by the support stiffness.

Analysis of results:

Check of reactions:

Due to symmetry the reactions are 4R1 at the four outer corners,and 4R2 at the two middle supports (as stated before to keep the symmetry the reactions at the middle supports was assumed equal to 2R2).

For the four outer corner elements:

$Z = 0,193260638 . \frac{10E}{3}$, $\alpha_x = 0,438810508 . 10^{-2} . \frac{10E}{3}$,

$\alpha_y = 0,334106459 . 10^{-2} . \frac{10E}{3}$

For the four elements at the middle supports

$Z = 0,186566741 . \frac{10E}{3}$ $\alpha_x = 0,569560415 . 10^{-2} . \frac{10E}{3}$

$\alpha_y = 0,176508133 . 10^{-2} . \frac{10E}{3}$

$\Sigma R = 4R1+4R2= 4(\tau_1 z_1+\tau_2 z_2) = 4\tau(z_1+z_2)$ since $\tau_1=\tau_2$

$$\Sigma R = \frac{4}{1000}.50.\frac{10}{3}.2,0.10^5[[0,193260638-(0,438810508+0,334106459).10^{-2}.25]+[0,186566741-(0,176508133+0,569560415)10^{-2}.25]]$$

$\therefore \Sigma R = 10,8$ T, Where $25 = \frac{a}{2}=\frac{b}{2}$

Total load on roof= 0,6.3,0.3,0.2,0 = 10,8T= ΣR O.k

Check of bending moments:

Generally,to determine the bending moment at the joints the angles of rotation of the two elements in a direction perpendicular to the joint are considerd.

$M=(\alpha_i-\alpha_j)\,\omega$ for our example

$=(\alpha_i-\alpha_j).\frac{250}{3}.2,0.10^5.2,0.10^{-5}=(\alpha_i-\alpha_j)\frac{1000}{3}$ mt/m.

Where : 2,0 to change to 1m striup (width of the element 0,5m) and 10^{-5} to change to mt.

The total+ve bending moment in the y direction at the center line of the slab sec I-I can be calculated as follows:

Due to the symmetry $\alpha_i-\alpha_j = 2\alpha_i$

$\Sigma M=[1000/3\,\Sigma 2\alpha_i]\,0.5$ (0.5due to width of element=0,5m) (for onespan)

$$= [2,0.\frac{1000}{3}[1,06+0,8832+0,8190+0,9102+1,1091+1,2924]10^{-3}].0,5$$

$M=[0.707+0.589+0.546+0.607+0.759+0.862]0,5=2,025$

M(for one span)= $0,6.3,3^2/8 = 2,025$mt $=\Sigma M$ o.k

Check of Bending moments distribution:

The given example was also solved using the finite element method to compare the results.

In figure 3 the number given at the top left corner of the element is the element number used in the method of concentrated deformation and the number given in the bottom right corner is the number of element used in the method of finite elements.

Moments along section II - II fig.3.

$M = (\alpha_i - \alpha_j)\ 10^3/3 mt/m$

$M_{1,7} = (0,334106-0,205328).10^{-2}.10^3/3= 0,429$

$M_{7,13}= (0,205328-0,039897).10^{-2}.10^3/3=0,5514$

$M_{13,19}=(0,039897+0,116739).10^{-2}.10^3/3=0,522$

$M_{19,25}=(-0,116739+0,21727).10^{-2}.10^3/3=0,335$

$M_{25,31}=(-0,21727+0,176508).10^{-3}.10^3/3=-0,136$

$M_{31,37}=(-0,176508-0,176508).10^{-2}.10^3/3=-1,177$

The distribution of bending moments along section II-II are given in figure 3.

Similarly the moments distribution along axes II.I is also given.

Along axes I-I the moments by the concentrated deformation method were given when the check of the bending moments was done . The finite element methed give the values at the middle of the element (at sec. I-I′), the values were increased by a coefficont 1,0285 to get the values of the moments at section I-I.

Figure 3 shown that the distribution of moments give good coincidence with other methods.

Conclusions:

1) For statically indeterminate slabs and beams the method of concentrated deformation give much easier approach for the solution compared with any other method, specially in case of existance of real joint which have stiffness different from that of the monolitic body.

2) The results obtained using the method of concentrated deformation are in good agreement with the results obtained by other known methods. Therefore, use of this method is advantageous due to its simplicity and lesser work.

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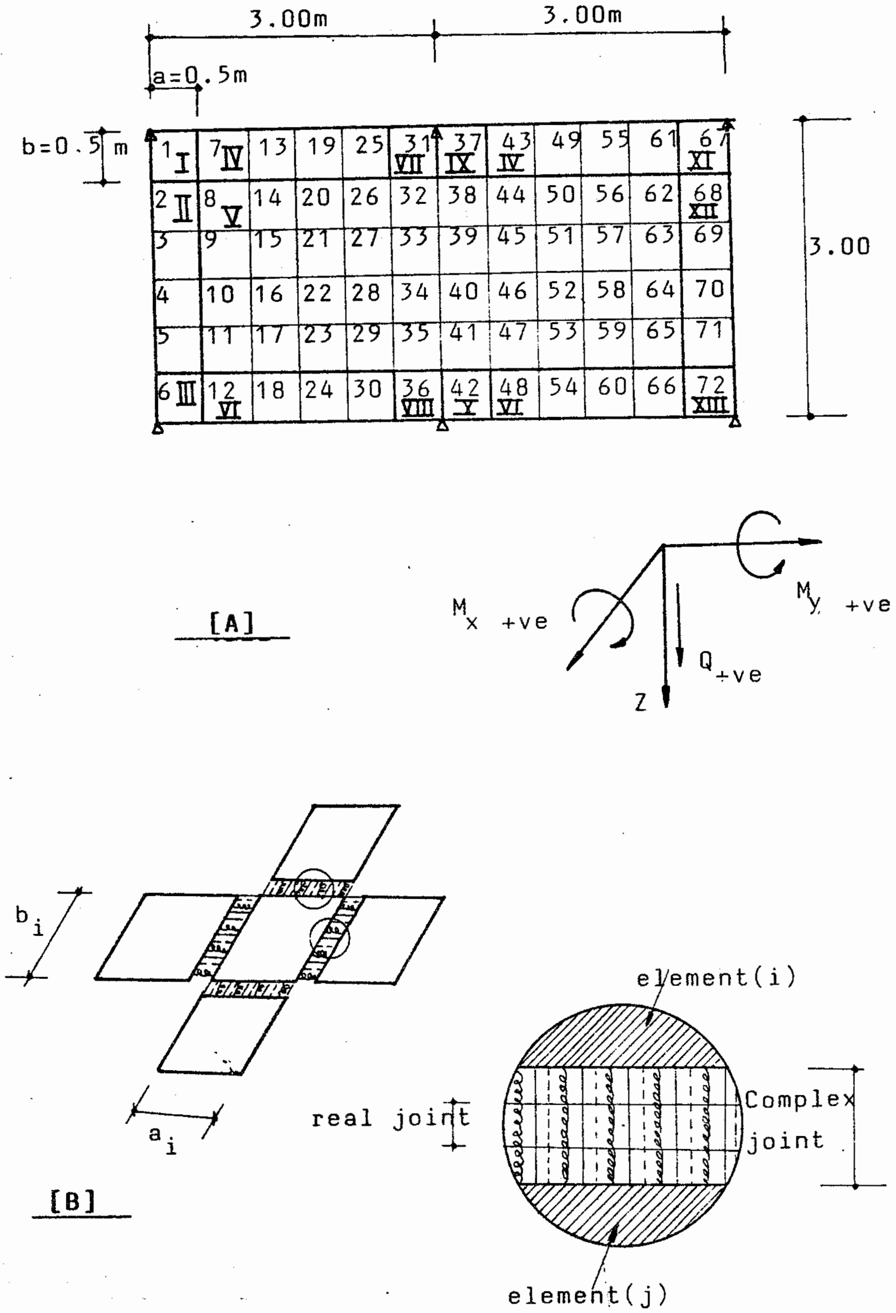
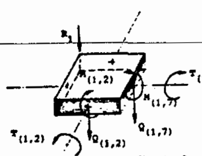
2- Sherif M.H.,Dadonou M.I. Analysis of composite beams by method of concentrated deformations. Express-Information Bulletin,All-Union scientefic-Research Institute of information on construction and Architecture, August 1986.


3.00m
3.00m
a=0.5m
b=0.5 m
3.00
[A]
M_x +ve
M_y +ve
Q +ve
Z
b_i
a_i
real joint
element(i)
Complex joint
element(j)
[B]

[FIGURE 1]

TYPE 1

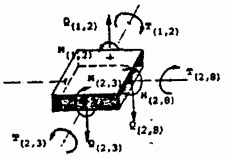


$\Sigma M_x=0\ ;\ b/2 \cdot Q_{(1,2)} - M_{(1,2)} - T_{(1,7)} - b/2 \cdot R_1 = 0$

$\Sigma M_y=0\ ;\ a/2 \cdot Q_{(1,7)} - M_{(1,7)} - T_{(1,2)} - a/2 \cdot R_1 = 0$

$\Sigma Q=0\ ;\quad Q_{(1,2)} + Q_{(1,7)} + q + R_1 = 0$

TYPE 2

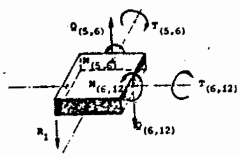


$\Sigma M_x=0\ ;\ b/2 \cdot Q_{(2,3)} - M_{(2,3)} + b/2 \cdot Q_{(1,2)} + M_{(1,2)} - T_{(2,8)} = 0$

$\Sigma M_y=0\ ;\ a/2 \cdot Q_{(2,8)} - M_{(2,8)} + T_{(1,2)} - T_{(2,3)} = 0$

$\Sigma Q=0\ ;\quad Q_{(2,3)} + Q_{(2,8)} - Q_{(1,2)} + q = 0$

TYPE 3

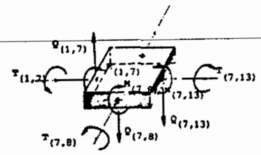


$\Sigma M_x=0\ ;\ b/2 \cdot Q_{(5,6)} + M_{(5,6)} - T_{(6,12)} + b/2 \cdot R_1 = 0$

$\Sigma M_y=0\ ;\ a/2 \cdot Q_{(6,12)} - M_{(6,12)} + T_{(5,6)} - a/2 \cdot R_1 = 0$

$\Sigma Q=0\ ;\quad Q_{(6,12)} - Q_{(5,6)} + q + R_1 = 0$

TYPE 4

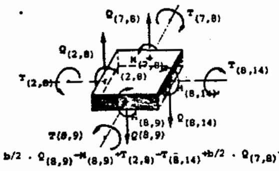


$\Sigma M_x=0\ ;\ b/2 \cdot Q_{(7,8)} - M_{(7,8)} + T_{(1,7)} - T_{(7,13)} = 0$

$\Sigma M_y=0\ ;\ a/2 \cdot Q_{(7,13)} + a/2 \cdot Q_{(1,7)} + M_{(1,7)} - M_{(7,13)} - T_{(7,8)} = 0$

$\Sigma Q=0\ ;\quad Q_{(7,8)} + Q_{(7,13)} - Q_{(1,7)} + q = 0$

TYPE 5

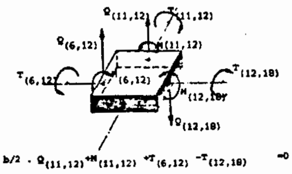


$\Sigma M_x=0\ ;\ b/2 \cdot Q_{(8,9)} - M_{(8,9)} + T_{(2,8)} - T_{(8,14)} + b/2 \cdot Q_{(7,8)} + M_{(7,8)} = 0$

$\Sigma M_y=0\ ;\ a/2 \cdot Q_{(8,14)} - M_{(8,14)} + a/2 \cdot Q_{(2,8)} + M_{(2,8)} + T_{(7,8)} - T_{(8,9)} = 0$

$\Sigma Q=0\ ;\quad Q_{(8,9)} + Q_{(8,14)} - Q_{(7,8)} - Q_{(2,8)} + q = 0$

TYPE 6

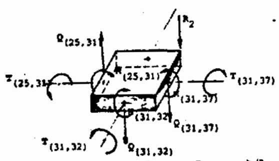


$\Sigma M_x=0\ ;\ b/2 \cdot Q_{(11,12)} + M_{(11,12)} + T_{(6,12)} - T_{(12,18)} = 0$

$\Sigma M_y=0\ ;\ a/2 \cdot Q_{(12,18)} - M_{(12,18)} + a/2 \cdot Q_{(6,12)} + T_{(11,12)} = 0$

$\Sigma Q=0\ ;\quad Q_{(12,18)} - Q_{(6,12)} - Q_{(11,12)} + q = 0$

TYPE 7

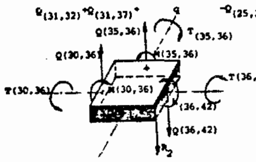


$\Sigma M_x=0\ ;\ b/2 \cdot Q_{(31,32)} - M_{(31,32)} + T_{(25,31)} - T_{(31,37)} - b/2 \cdot R_2 = 0$

$\Sigma M_y=0\ ;\ a/2 \cdot Q_{(31,37)} - M_{(31,37)} + a/2 \cdot Q_{(25,31)} + M_{(25,31)} - T_{(31,32)} + a/2 \cdot R_2 = 0$

$\Sigma Q=0\quad Q_{(31,32)} + Q_{(31,37)} + q - Q_{(25,31)} + R_2 = 0$

TYPE 8

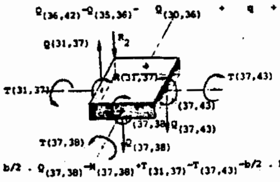


$\Sigma M_x=0\ ;\ b/2 \cdot Q_{(35,36)} + M_{(35,36)} + T_{(30,36)} - T_{(36,42)} + b/2 \cdot R_2 = 0$

$\Sigma M_y=0\ ;\ a/2 \cdot Q_{(36,42)} - M_{(36,42)} + a/2 \cdot Q_{(30,36)} + M_{(30,36)} + T_{(35,36)} + a/2 \cdot R_2 = 0$

$\Sigma Q=0\ ;\quad Q_{(36,42)} - Q_{(35,36)} - Q_{(30,36)} + q + R_2 = 0$

TYPE 9

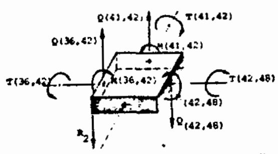


$\Sigma M_x=0\ ;\ b/2 \cdot Q_{(37,38)} - M_{(37,38)} + T_{(31,37)} - T_{(37,43)} - b/2 \cdot R_2 = 0$

$\Sigma M_y=0\ ;\ a/2 \cdot Q_{(37,43)} - M_{(37,43)} + a/2 \cdot Q_{(31,37)} + M_{(31,37)} - T_{(37,38)} - a/2 \cdot R_2 = 0$

$\Sigma Q=0\ ;\quad Q_{(37,38)} + Q_{(37,43)} - Q_{(31,37)} + q + R_2 = 0$

TYPE 10

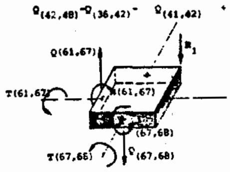


$\Sigma M_x=0\ ;\ b/2 \cdot Q_{(41,42)} + M_{(41,42)} + T_{(36,42)} - T_{(42,48)} + b/2 \cdot R_2 = 0$

$\Sigma M_y=0\ ;\ a/2 \cdot Q_{(42,48)} - M_{(42,48)} + a/2 \cdot Q_{(36,42)} + M_{(36,42)} + T_{(41,42)} - a/2 \cdot R_2 = 0$

$\Sigma Q=0\ .\quad Q_{(42,48)} - Q_{(36,42)} - Q_{(41,42)} + q + R_2 = 0$

TYPE 11

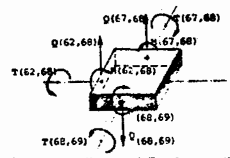


$\Sigma M_x=0\ ;\ b/2 \cdot Q_{(67,68)} - M_{(67,68)} + T_{(61,67)} - b/2 \cdot R_1 = 0$

$\Sigma M_y=0\ ;\ a/2 \cdot Q_{(61,67)} + M_{(61,67)} - T_{(67,68)} + a/2 \cdot R_1 = 0$

$\Sigma Q=0\ ;\quad Q_{(67,68)} - Q_{(61,67)} + q + R_1 = 0$

TYPE 12



$\Sigma M_x=0\ ;\ b/2 \cdot Q_{(68,69)} - M_{(68,69)} + b/2 \cdot Q_{(67,68)} + M_{(67,68)} + T_{(62,68)} = 0$

$\Sigma M_y=0\ ;\ a/2 \cdot Q_{(62,68)} + M_{(62,68)} + T_{(67,68)} - T_{(68,69)} = 0$

$\Sigma Q=0\ ;\quad Q_{(68,69)} - Q_{(62,68)} - Q_{(67,68)} + q = 0$

[FIGURE 2]

ele.			M 1,2	T 1,2	Q 1,2	M 1,7	T 1,7	Q 1,7	M 2,3	T 2,3	Q 2,3	M 2,8	T 2,8	Q 2,8	M 3,4	T 3,4	Q 3,4	M 3,9	T 3,9	Q 3,9	M	T	Q	M 71,72	T 71,72	Q 71,72	R 1	R 1	R 2	R 2	R 2	R 2	R 1	R 1
NO.			n $\omega_{1,2}$	n $\psi_{1,2}$	n $\xi_{1,2}$	m $\omega_{1,7}$	m $\psi_{1,7}$	m $\xi_{1,7}$	n $\omega_{2,3}$	n $\psi_{2,3}$	n $\xi_{2,3}$	m $\omega_{2,8}$	m $\psi_{2,8}$	m $\xi_{2,8}$	n $\omega_{3,4}$	n $\psi_{3,4}$	n $\xi_{3,4}$	m $\omega_{3,9}$	m $\psi_{3,9}$	m $\xi_{3,9}$	ω	ψ	ξ	m $\omega_{71,72}$	m $\psi_{71,72}$	m $\xi_{71,72}$	η_1	η_1	η_2	η_2	η_2	η_2	η_1	η_1
1	Σ Mx	1	-1		b/2		-1																				-b/2							
	Σ MY	2		-1		-1		a/2																			-a/2							
	Σ Q	3			1			1																			1							
2	Σ Mx	4	1		b/2				-1		b/2		-1																					
	Σ MY	5		1						-1		-1		a/2																				
	Σ Q	6			-1						1			1																				
3	Σ Mx	7													-1		b/2		-1															
	Σ MY	8														-1		-1		a/2														
	Σ Q	9															1			1														
4	Σ Mx	10													1		b/2											b/2						
	Σ MY	11														1												-a/2						
	Σ Q	12															-1											1						
5	Σ Mx	13																																
	Σ MY	14																																
	Σ Q	15																																
6	Σ Mx	16																																
	Σ MY	17																																
	Σ Q	18																																
7	Σ Mx	19																																
	Σ MY	20																																
	Σ Q	21																																
71	Σ Mx	211																						-1		b/2								
	Σ MY	212																							-1									
	Σ Q	213																								1								
72	Σ Mx	214																						1		b/2								b/2
	Σ MY	215																							1									a/2
	Σ Q	216																								-1								1

NOTE n index for horizontal joint , m index for vertical joint.

TABLE 1

TYPE I ELEMENT NO. 1

elem. NO			$\omega^n_{1.2}$	$\psi^n_{1.2}$	$\xi^n_{1.2}$	$\omega^m_{1.7}$	$\psi^m_{1.7}$	$\xi^m_{1.7}$	η_1
1	Σ Mx	1	-1		b/2		-1		-b/2
	Σ My	2		-1		-1		a/2	-a/2
	Σ Q	3			1			1	1
2	Σ Mx	4	1		b/2				
	Σ My	5		1					
	Σ Q	6			-1				
7	Σ Mx	19					1		
	Σ My	20				1		a/2	
	Σ Q	21						-1	

(1) (20)

	1	2	3	4	5	6 ←	12	→	19	20	
(1)	$\psi+\omega+\frac{b^2}{4}(\xi+\eta)$	$\frac{ab}{4}\eta$	$\frac{b}{2}(\xi-\eta)$	$-\omega+\frac{b^2}{4}\xi$	0	$-\frac{b}{2}\xi$			$-\psi$	0	(21)
(2)		$\omega+2\psi+\frac{a^2}{4}(\xi+\eta)$	$\frac{a}{2}(\xi-\eta)$	0	$-\psi$	0			0	$-\omega+\frac{a^2}{4}\xi$	$-\frac{a}{2}\xi$
(3)			$2\xi+\eta$	$\frac{b}{2}\xi$	0	$-\xi$			0	$\frac{a}{2}\xi$	$-\xi$

TYPE II ELEMENT NO. 2

elem. NO.			$\omega^n_{1.2}$	$\psi^n_{1.2}$	$\xi^n_{1.2}$	$\omega^n_{2.3}$	$\psi^n_{2.3}$	$\xi^n_{2.3}$	$\omega^m_{2.8}$	$\psi^m_{2.8}$	$\xi^m_{2.8}$
2	ΣMx	4	1		b/2	-1		b/2		-1	
	ΣMy	5		1			-1		-1		a/2
	ΣQ	6			-1			1			1
3	ΣMx	7				1		b/2			
	ΣMy	8					1				
	ΣQ	9						-1			
8	ΣMx	22								1	
	ΣMy	23							1		a/2
	ΣQ	24									-1

(4) (23)

	1	2	3	4	5	6 ←	12	→	19	20	
(4)	$\psi+2\omega+\frac{b^2}{4}\xi$	0	0	$-\omega+\frac{b^2}{4}\xi$	0	$-\frac{b}{2}\xi$			$-\psi$	0	(24)
(5)		$\omega+2\psi+\frac{b^2}{4}\xi$	$\frac{a}{2}\xi$	0	$-\psi$	0			0	$-\omega+\frac{a^2}{2}\xi$	$-\frac{a}{2}\xi$
(6)			3ξ	$\frac{b}{2}\xi$	0	$-\xi$			0	$\frac{a}{2}\xi$	$-\xi$

TYPE 5 ELEMENT NO. 8

ele. NO.			$\omega^n_{7.8}$	$\psi^n_{7.8}$	$\xi^n_{7.8}$	$\omega^n_{8.9}$	$\psi^n_{8.9}$	$\xi^n_{8.9}$	$\omega^m_{2.8}$	$\psi^m_{2.8}$	$\xi^m_{2.8}$	$\omega^m_{8.14}$	$\psi^m_{8.14}$	$\xi^m_{8.14}$
8	ΣMx		1		b/2	-1		b/2		1			-1	
	ΣMy			1			-1		1		a/2	-1		a/2
	ΣQ				-1			1			-1			1
9	ΣMx					-1		b/2						
	ΣMy						-1							
	ΣQ							-1						
14	ΣMx												1	
	ΣMy											1		a/2
	ΣQ													-1

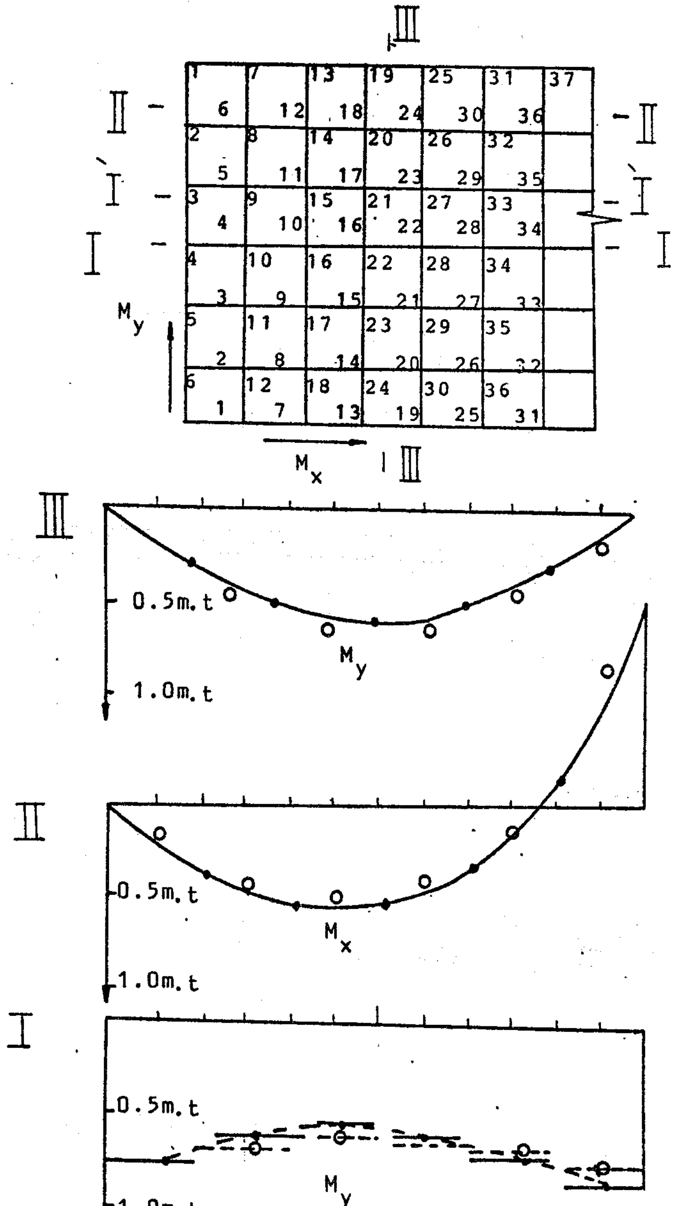
(22) (41)

	1	2	3	4	5	6 ←	12	→	19	20	
(22)	$2\psi+2\omega+\frac{b^2}{4}\xi$	0	0	$-\omega+\frac{b^2}{4}\xi$	0	$-\frac{b}{2}\xi$			$-\psi$	0	(42)
(23)		$2\psi+2\omega+\frac{a^2}{4}\xi$	0	0	0	$-\psi$			0	$-\omega+\frac{a^2}{4}\xi$	$-\frac{a}{2}\xi$
(24)			4ξ	$\frac{b}{2}\xi$	0	$-\xi$			0	$\frac{a}{2}\xi$	$-\xi$

[TABLE 2]

NOTES All ω, ψ, ξ, η are equal in the given example.

Number in ◯ is the number in the matrix.



• For concentrated deformation method

o For finite element method

[figure 3]