OPTIMAL PLACEMENT OF PHASOR MEASUREMENT UNITS WITH LIMITED CHANNELS USING ANT COLONY OPTIMIZATION ALGORITHM

A. A. Abou El-Ela [(a)]   A. M. Kinawy [(a)]   R. A. El-Sehiemy [(b)]   M. T. Mouwafi [(a)]

a) Electrical Engineering Department, Faculty of Engineering, Shebin El-Kom,
Minoufiya University, Egypt
b) Electrical Engineering Department, Faculty of Engineering,
Kafrelsehiekh University, Egypt

ABSTRACT

This paper proposes an approach for optimal placement of phasor measurement units (PMUs) for complete observability at normal and emergency conditions such as any single line outage or any single PMU loss using ant colony optimization (ACO) algorithm. The objective function is to find the optimal locations of PMUs and the minimum number of PMU channels by optimizing the logic decision of control variables, under the observability constraint.

The ACO algorithm is applied to the standard IEEE 14-bus, 30-bus and New England 39-bus systems. Add to that, an application of the proposed algorithm to a real power system of the west Delta network (WDN) as a part of the Unified Egyptian Network (UEN) is presented. The results obtained are compared with those obtained using other techniques. Simulation results show that the proposed ACO algorithm is more accurate and efficient for obtaining the optimal placement of PMUs with minimum number of PMU channels, especially with increasing the system size.

يقدم هذا البحث طريقة مقترحة لإيجاد التوزيع الأمثل لوحدات قياس المتجهات للمراقبة الكاملة للنظام فى حالات التشغيل العادية والطارئة للنظم الكهربية مثل خروج أى خط نقل او وحدة قياس متجهة باستخدام خوارزم مستعمرة النمل (ACO). تم الأخذ فى الأعتبار دالة التوزيع الأمثل لوحدات قياس المتجهات، بالأضافة الى عدد قنوات القياس كدالة هدف حيث تم ايجاد المواقع المثلى لوحدات قياس المتجهات مع المراقبة الكاملة لمتغيرات معاملات النظام.
تم تطبيق الأسلوب المقترح بأستخدام خوارزم مستعمرة النمل على العديد من الأنظمة القياسية والحقيقية مثل نظام ١٤ قضيب، ٣٠ قضيب، ٣٩ قضيب. بالاضافة الى التطبيق على منظومة شبكة غرب الدلتا (WDN) كنظام حقيقى داخل الشبكة الموحدة لجمهورية مصر العربية، تم مقارنة نتائج الطريقة المقترحة مع نتائج طرق أخرى فى هذا المجال. حيث أوضحت النتائج أن أسلوب خوارزم مستعمرة النمل أكثر دقة وكفاءة فى إيجاد التوزيع الأمثل لوحدات قياس المتجهات خاصة عند زيادة حجم النظم الكهربية.

Keywords: Phasor measurement units, Ant colony optimization algorithm, Observability, Emergency condition.

1. ITRODUCTION

PMUs are one of the smart metering devices which measure in real time the positive, negative and zero sequence 50/60 HZ waveforms both magnitudes and phase angles of the bus voltages and/or the branch currents. PMUs can be carried out in many applications such as power system monitoring, protection and control schemes. In order to make the power system complete observable, the smart metering devices such as PMUs should be used. The power system is said to be smart grid if it operates with automatic way in monitoring, communication, controlling and protection in the transmission, distribution and appliances at customers employing smart meters such as PMUs.

Chakrabarti et al. [1] studied the placement of PMUs utilizing time-synchronized measurements of voltage and current phasors using an integer quadratic programming. Manousakis and Korres [2] solved this problem for complete system observability using unconstrained nonlinear weighted least squares (WLS) approach by performing the placement of PMUs as a quadratic minimization problem. Sodhi et al. [3] presented method utilizes a two-stage approach. In the first stage, an integer linear programming (ILP) based algorithm was used to determine the minimum number of PMUs. In the second stage, a sequential elimination algorithm (SEA) was used to determine the minimum additional PMU locations, if required. Chakrabarti and Kyriakides [4] presented an exhaustive binary search method to find the placement of PMUs solution for state estimation (SE), considering single branch outages. Xin et al. [5] introduced a modified ILP to find the optimal PMU placement in a power system suffering from random component outages (RCOs). Xu and Abur [6] introduced two procedures

to find the placement of PMUs solution using the integer optimization problem and the topology-based method. Farsadi *et al.* [7] introduced depth first search (DeFS) to allocate the PMUs. In DeFS, the first PMU placed at the bus with the largest number of connected branches randomly chosen if there is more than one bus. Following PMUs placed with the same criterion, until the complete obervability obtained. Khavari *et al.* [8] used grenade explosion method (GEM) to find the placement of PMUs solution for rendering a power system topologically observable. Raju *et al.* [9] presented advanced particle swarm optimization (APSO) technique for finding the placement of PMUs solution for the complete system observability. Hurtgen and Maun [10] presented pagerank placement algorithm (PPA) and iterated local search (ILS) to find the placement of PMUs solution. Initially, suggested a PMU distribution which made the network observable using PPA by placing the measurements. Then, the ILS was used to minimize the size of the PMU configuration needed to observe the network. ACO algorithm was applied to find the optimal placement of PMUs on small size systems in [11-14]. While, the application of the proposed technique based on ACO algorithm on small and large scale power systems is presented in this paper.

In this paper, an approach is proposed to find the optimal placement of PMUs solution using the ACO algorithm. The optimal locations of PMUs and the minimum number of PMU channels are considered as an objective function with the observability constraint is satisfied.

2. PROBLEM FORMULATION

The solution of optimal placement of PMUs with minimum number of PMU channels, to make the system complete observability, can be expressed as a constrained optimization problem as:

$$Min \ F(x) = \sum_{i=1}^{N_b} w_i x_i \qquad (1)$$

Subject to:

$$g(x) = \sum_{i,j=1}^{N_b} A_{ij} x_i \geq b \qquad (2)$$

where, $F(x)$ is the objective function to be minimized to obtain the minimum number of PMUs with the minimum number of PMU channels. w_i refers to the weighting factor of the PMU located at the bus i based on the system configuration that is equal to the total number of branches incident to bus i plus one which is equivalent to total number of measurement channels. N_b is the number of system buses. x_i is the decision or control variables in a binary form [0 or

1], *e.g.* if x_i equals 0 refers to no existing of PMU at bus i, while 1 refers to installing of PMU at that bus, which is defined, as:

$$x_i = \begin{cases} 1 & \text{if a PMU installed at bus } i, \text{ otherwise} \\ 0 \end{cases} \qquad (3)$$

$g(x)$ is the observability constraint that must be verified at each bus in the system. On the other hand, if a PMU is installed at bus i, so all the buses connected to it don't need to install the PMUs, because phasors at these buses can be determined using the Ohm's law. A_{ij} in Eq. (2) is the connectivity matrix which can be constructed based on the line data of the system by replacing the values of line data with binary logic, so it can be expressed as:

$$A_{ij} = \begin{cases} 1 & \text{if} \quad Z_{ij} > 0 \\ 0 & \text{if} \quad Z_{ij} = \infty \end{cases} \qquad (4)$$

where Z_{ij} is the impedance between buses i and j. b in Eq. (2) is the minimum limit of measurement redundancy matrix of length N_b. In order to make the system complete topologically observable under normal condition, the value of b should be equal 1 which means that each bus is observed at least once, while at emergency conditions such as any single line outage or any single PMU loss, the value of b becomes 2 that means that each bus is observed at least twice to make the system completely observable at emergency conditions.

3. ACO ALGORITHM

ACO algorithm was first proposed by Dorigo and his related work [15,16].

3. 1. Mathematical Model of ACO Algorithm

A random amount of pheromone is deposited in each route after each ant completes it is tour, other ants attract to the shortest route according to the probabilistic transition rule. Therefore, the probabilistic transition rule of ant k to go from city i to city j can be expressed as in Traveling Salesman Problem (TSP) [16] as:

$$P_{ij}^k(t) = \frac{[\tau_{ij}(t)]^\alpha [\eta_{ij}(t)]^\beta}{\sum_q [\tau_{iq}(t)]^\alpha [\eta_{iq}(t)]^\beta} ; j, q \in N_i^k \qquad (5)$$

where, τ_{ij} is the pheromone trail deposited between city i and j by ant k, η_{ij} is the visibility equal to the inverse of the distance between city i and j ($\eta_{ij} = 1/d_{ij}$). α and β are two parameters that influence the relative weight of pheromone trail and heuristic guide function, respectively. q is the cities that will be visited after city i. While, N_i^k is a tabu list in memory of ant that recodes the cities are visited to avoid stagnations. After each tour is completed, a *local pheromone update* is determined by each ant as in

Eq. (6), after all ants attractive to the shortest route, a *global pheromone update* is considered to show the influence of the new addition deposits by the other ants that attractive to the best tour, as in Eq. (7).

$$\tau_{ij}(t+1) = (1-\rho)\tau_{ij}(t) + \rho\tau_\circ \qquad (6)$$

$$\tau_{ij}(t+1) = (1-\rho)\tau_{ij}(t) + \varepsilon\Delta\tau_{ij}(t) \qquad (7)$$

where, τ_{ij} *(t+1)* is the pheromone after one tour or iteration, ρ is the pheromone evaporation constant, ε is the elite path weighting constant, $\tau_o = 1 / d_{ij}$ is the incremental value of pheromone of each ant. While, $\Delta\tau_{ij}$ is the amount of pheromone for elite path as:

$$\Delta\tau_{ij}(t) = 1/d_{best} \qquad (8)$$

where, C is a large positive constant and d_{best} is the shortest tour distance found as in TSP.

4. PROPOSED OPTIMAL PROCEDURE

ACO algorithm is applied to find the optimal placement of PMUs solution as an optimization technique with observability constraint where artificial ants travel in search space to determine the shortest route that having the strongest pheromone trail and give the optimal solution. The proposed objective in this paper is to minimize the total number of PMUs, in addition the number of PMU channels with observability constraint in Eq. (2). The heuristic guide function of the problem is the inverse of the total objective function at iteration *t+1* as:

$$\eta(t+1) = 1/\sum_{i=1}^{N_b} F(x) \qquad (9)$$

In the ACO algorithm, a search space creates with dimensions of stages on number of control variables and states on the randomly distributed values of control variables that take random values between 0 and 1. Artificial ants leave colony to search randomly in the search space based on the probability as Eq. (5) to complete a tour matrix that consists of the positions of ants with the same dimension of the search space. Then, tour matrix is applied on the control variables for rearrangement. Therefore, the objective function is achieved to find the optimal solution using heuristic guide function. In addition, the local and global pheromones are updated to begin a next iteration. System parameters are adjusted by trail and error to find the best values of these parameters.

The ACO algorithm can be applied to find the optimal solution using the following steps:

Step 1: Initialization

Insert the control variables with random values between 0 and 1, system parameters to create the connectivity matrix A_{ij}, and create a search space with dimensions of number of decision variables. The

number of theses variables is equal to the number of system buses and the length of randomly distributed values with the same dimension of the initial pheromone that contains elements with very small values to give all ants with the same chance of searching.

Step 2: Provide first position

Each ant is positioned on the initial state randomly within the reasonable range of each control variable in a search space with one ant at each bus in the length of randomly distributed values.

Step 3: Evaluation

The merit of each initial control variable in the *d*-search space is found using corresponding objective function for each ant in each stage of the problem called individual evaluation function through the *d*-space. While, the whole initial objective function for each ant is obtained by summing of individual objective functions.

Step 4: Transition rule

Each ant decides to visit a next position in the range of other control variables according to the probability transition rule in Eq. (5) that depends on the amount of pheromone deposited and the visibility that is the inverse of objective function in equation (1). Where, the effect of pheromone and visibility on each other depends on the two parameters α and β.

Step 5: Local pheromone updating

Local updating pheromone is different from ant to other because each ant takes a different route. The initial pheromone of each ant is locally updated by using Eq (6).

Step 6: Fitness function

After all ants get attracted to the shortest path that having a strongest pheromone, the best solution of the objective function is obtained.

Step 7: Check the constraints

For each ant, check the observability constraint in equation (2) based on the values of control variables in the modified *d*-space. The objective function F_i, which corresponding to the index of violation constraints will be excluded from the optimal solution. Therefore, the global value of objective function can be obtained as the minimum value of the objective functions F_i which is satisfied the proposed constraint.

Step 8: Global pheromone updating

After all ants attractive to the minimum objective function that having a strongest pheromone, the mount of pheromone on the best tour becomes the strongest due to attractive of ants for this path. Moreover, the pheromone on the other paths is

evaporated in time. Finally, compares the current global best with the last previous global best to find F_{gbest}. Therefore, the global pheromone updated can be determined in the second term as Eq. (7).

Step 9: Program termination
If the number of iterations does not reach the maximum number of iterations, then go to Step 4. Otherwise, the ant that generates the last F_{gbest} is the best evaluation and is considered as the optimal solution.

Steps from 1 to 9 are repeated to obtain more than one solution with different acceptable candidate locations of PMUs, in addition minimum number of PMU channels.

5. APPLICATIONS

5. 1. Test Systems

Three standard test systems and a real power system are used to study the proposed algorithm to find the optimal placement of PMUs solution. The test systems are IEEE 14-bus, 30-bus and New England 39-bus test systems [17] as small size systems. The results are obtained and compared with those obtained using other techniques. In addition, the real power system of the west delta network (WDN) as a part of the Unified Egyptian Network (UEN) is used as large scale system which consists of 52-bus and 108 transmission lines with 6 single lines and the remaining lines are doubled. The single line diagram of the WDN is shown in Fig. 1. The best values of ACO algorithm parameters are assumed to be α =1, β=5, ρ=0.7, ε=5 and C=100. Two different cases are considered to obtain the minimum number of PMUs and minimum number of PMU channels with maximizing the measurement redundancy verifying complete network observability are normal condition and emergency conditions such as any single line outage or any single PMU loss.
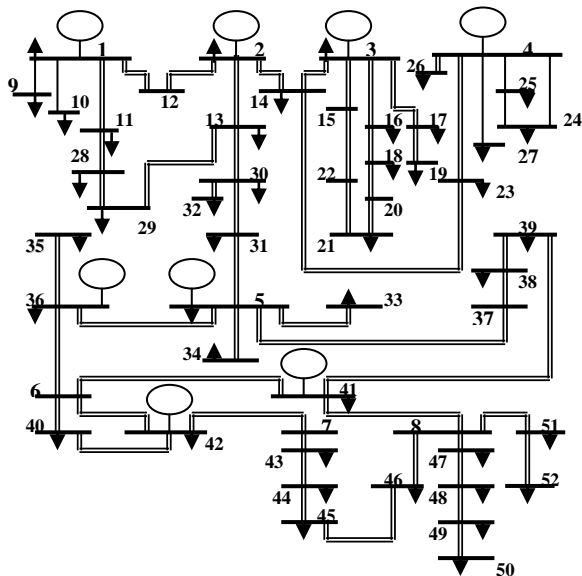

Fig. 1 Single line diagram of the WDN

5. 2. Results and Comments

The ACO algorithm is used to obtain the optimal results that were processed using MATLAB code version 7.1 on a Pentium 4, 3.0 GHz PC, 1 GB of RAM memory.

5.2.1 Normal conditions

Table 1 shows the minimum number of PMUs at different candidate locations with the minimum number of each PMU channels required to make a complete observability for the test systems at normal condition. e.g., for IEEE 14-bus test system, the optimal number of PMUs is 4 at buses {2,8,10,13}, while the corresponding number of each PMU channels is {5,2,3,4} as shown in Table 1. It is clear that, the number of PMUs is increased as increasing of the system buses. The optimal number of PMU channels is divided into the current measurement channels for each line connected to the bus and one channel for the voltage measurement at that bus. Therefore, the total cost of PMUs will be reduced.

Figure 2 shows the optimal placement of PMUs required to make a complete topological observability for the test systems at normal condition for 50-running for proposed technique. From this figure, the number of different candidate solutions is {6,5,4} for IEEE 14-bus, {13,12,11,10} for IEEE 30-bus, {16,15,14,13} for New-England 39-bus and {19,18,17,16} for the WDN through 50-running of the proposed technique, respectively. Therefore, the ACO algorithm is capable to obtain the global optimal solution with minimum locations of PMUs.
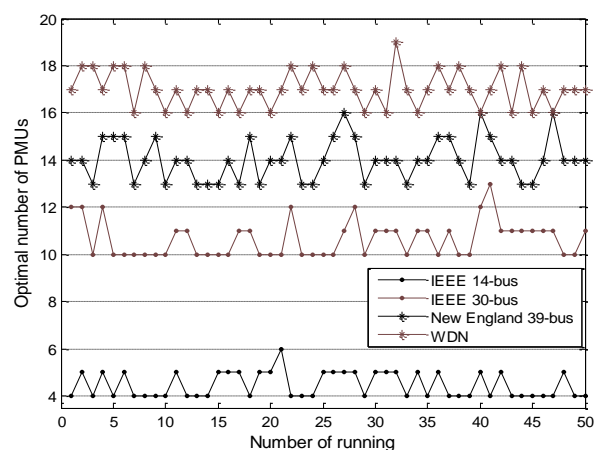

Fig. 2 Optimal placement of PMUs at 50-running of the proposed technique for the test systems at normal condition

Table 2 shows a comparison between the minimum number of PMUs that is obtained from the proposed technique and the other methods to make a complete

observability for the test systems at normal condition. For IEEE 14-bus system, the proposed technique gives 4 PMUs which is the same minimum number of PMUs as well as other methods except pagerank placement algorithm (PPA) [10] that gives 5 PMUs and DeFS [7] that gives 6 PMUs which have greater number of PMUs than the proposed technique. For IEEE 30-bus and New-England 39-bus systems, the proposed technique gives the same optimal number

of PMUs corresponding to the other methods to make the test systems complete observability except GEM [8] which gives 9 PMUs that leads to unobservable for the bus number 11 in IEEE 30-bus system. So, this solution is not perfect solution. In addition, ILP [3] gives 15 PMUs which is grater the optimal number of PMUs than the proposed technique for New-England 39-bus system.

Table 1 Optimal number of PMUs with minimum number of PMU channels at normal condition

System	N_{PMU}	Optimal placement of PMUs	Minimum number of PMU channels	Total channels
IEEE 14-bus	4	2,8,10,13	5,2,3,4	14
IEEE 30-bus	10	1,5,10,11,12,19,23,26,28,29	3,3,7,2,6,3,3,2,4,3	36
NE 39-bus	13	2,6,9,10,13,14,17,22,23,25,29,33,34	5,5,3,4,4,4,4,4,4,4,4,2,2	49
		2,6,9,10,11,14,17,22,23,25,29,33,34	5,5,3,4,4,4,4,4,4,4,4,2,2	49
WND	16	1,4,5,8,14,18,19,22,29,32,36,39,42,44,49,52	5,6,6,5,4,3,2,3,3,2,4,3,4,3,3,2	58

NE denotes the New-England.

Table 2 A comparison between the proposed technique and other methods for three test systems at normal condition

Methods	IEEE 14-bus	IEEE 30-bus	NE 39-bus
Proposed technique	**4**	**10**	**13**
Binary search [4]	4	10	13
WLS [2]	4	10	N/A
IP [6]	4	10	N/A
GEM [8]	4	9[*]	13
ILP [3]	4	N/A	15
Modified ILP [5]	4	N/A	13
APSO [9]	4	N/A	13
Integer quadratic programming [1]	N/A	10	N/A
DeFS [7]	6	N/A	N/A
PPA [10]	5	N/A	N/A
ILS [10]	4	N/A	N/A

[*] : denotes that the bus number 11 is unobservable bus.

5.2.2 Emergency conditions

Table 3 shows the minimum number of PMUs at different candidate locations with the minimum number of each PMU channels required to make a complete observability for the test systems at emergency conditions such as any single line outage or any single PMU loss. It is clear that, the optimal number of PMUs are more than those of the normal condition to make each bus observed at least twice to come over the emergency conditions. In addition, IEEE 30-bus test system gives only one optimal solution that leads to the minimum number of PMU channels. Therefore, the results in Table 3 are the optimal solutions which give the minimum total number of PMU channels. Hence, the total cost of PMUs with the same complete system observability will be reduced.

Figure 3 shows the optimal placement of PMUs to make a complete observability for the test systems at emergency conditions such as any single line outage or any PMU loss for 50-running for proposed

technique. From this figure, the number of different candidate solutions is {12,11,10,9} for IEEE 14-bus, {25,24,23,22,21} for IEEE 30-bus, {31,30,29,28} for New-England 39-bus and {42,41,40,39,38} for the WDN through 50-running of the proposed technique. Therefore, the proposed technique is capable to reach the global optimal solution with different locations of PMUs.
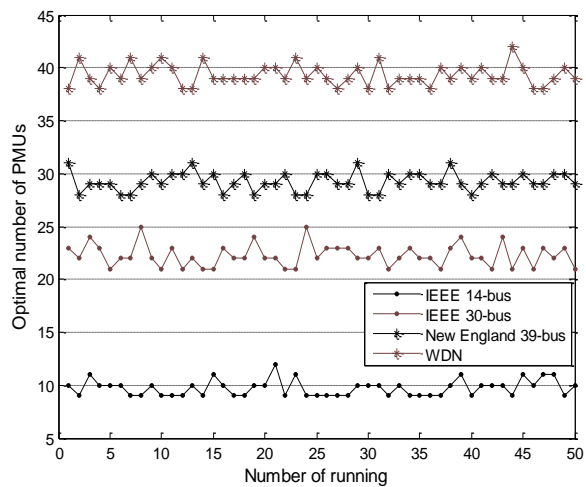

Fig. 3 Optimal placement of PMUs at 50-running of the proposed technique for test systems at emergency conditions

Table 4 shows a comparison between the proposed technique and other methods at emergency conditions. The proposed technique gives the same optimal number of PMUs as well as the integer quadratic programming [1] for IEEE 30-bus system. In addition, the proposed technique gives the same optimal number of PMUs compared to APSO [9] for IEEE 14-bus and New-England 39-bus systems and less than the number of PMUs obtained using IP [6] for IEEE 30-bus system. The APSO and IP are used to make a complete observability for the test systems

for any single PMU loss condition. The results obtained using the proposed technique are suitable to keep a complete observability in case of any single line outage or any single PMU loss conditions for the test systems.

Integer quadratic programming [1]	N/A	21	N/A
Binary search* [4]	7	15	17
IP** [6]	9	22	N/A
APSO** [9]	9	N/A	28

* : The optimal number of PMUs in case of single line outage condition.

** : The optimal number of PMUs in case of single PMU loss condition.

Table 4 A comparison between the proposed technique and other methods at emergency conditions

Methods	IEEE 14-bus	IEEE 30-bus	NE 39-bus
Proposed technique	9	21	28

Table 3 Optimal number of PMUs with minimum number of PMU channels at emergency condition

System	N_{PMU}	Optimal placement of PMUs	Minimum number of PMU channels	Total channels
IEEE 14-bus	9	1,2,3,6,7,8,9,10,13	3,5,3,5,4,2,5,3,4	34
		1,2,3,6,7,8,9,11,13	3,5,3,5,4,2,5,3,4	34
IEEE 30-bus	21	1,2,3,6,7,9,10,11,12,13,15,16,18,20,21, 24,25,26,28,29,30	3,5,3,8,3,4,7,2,6,2,5,3,3,3,3,4,4,2,4,3,3	80
NE 39-bus	28	1,2,3,6,8,9,10,12,13,14,16,17,19,20,22,23, 25,26,29,30,31,32,33,34,35,36,37,38	3,5,4,5,4,3,4,3,4,4,6,4,4,3,4,4,4,5, 4,2,2,2,2,2,2,2,2,2	95
		2,3,6,8,9,10,12,13,14,16,17,19,20,22,23, 25,26,29,30,31,32,33,34,35,36,37,38,39	5,4,5,4,3,4,3,4,4,6,4,4,3,4,4,4,5,4, 2,2,2,2,2,2,2,2,2,3	95
WND	38	1,2,4,5,7,8,9,10,14,15,16,17,18,19,20,22,25, 26,27,28,29,30,32,33,34,35,36,38,39,40,42, 44,45,47,49,50,51,52	5,4,6,6,3,5,2,2,4,3,3,3,3,2,3,3,3,2,2,3, 3,4,2,2,2,2,4,3,3,3,4,3,3,3,3,2,3,2	118
		1,2,4,5,8,9,10,14,15,16,17,18,19,21,22,24,26, 27,28,29,30,32,33,34,35,36,38,39,40,42,43, 44,46,47,49,50,51,52	5,4,6,6,5,2,2,4,3,3,3,3,2,3,3,3,2,2,3,3, 4,2,2,2,2,4,3,3,3,4,3,3,3,3,3,2,3,2	118

6. CONCLUSION

This paper presents a proposed approach using the ACO algorithm to find the optimal placement of PMUs with minimum number of measurement channels while the observability constraint is satisfied at normal and emergency conditions. The proposed technique has been tested on a three test systems and an actual system which is the WDN system as a part of the Unified Egyptian Network (UEN). The results obtained have been compared with that obtained using other methods. The results show that, the proposed technique leads to minimum number of PMUs with minimum number of PMU channels to make complete systems observability. So, the proposed technique gives more accurate and efficiently solution to reduce the number of PMUs because it gives the optimal solutions through 50-running without any violations of the system constraints.

7. REFERENCES

[1] S. Chakrabarti, E. Kyriakides and D. G. Eliades, "Placement of Synchronized Measurements for Power System Observability", *IEEE Trans. on Power Delivery*, Vol. 24, No. 4, pp. 12-19, January 2009.

[2] N. M. Manousakis and G. N. Korres, "A Weighted Least Squares Algorithm for Optimal PMU Placement", *IEEE Trans. on Power Systems*, Vol. 28, No. 3, pp. 1-2, August 2013.

[3] R. Sodhi, S. C. Srivastava and S. N. Singh, "Optimal PMU Placement Method for Complete Topological and Numerical Observability of Power System", *Electric Power Systems Research*, Vol. 80, No. 9, pp. 1154-1159, September 2010.

[4] S. Chakrabarti and E. Kyriakides, "Optimal Placement of Phasor Measurement Units for Power System Obervability", *IEEE Trans. on Power Systems*, Vol. 23, No. 3, pp. 1433-1440, August 2008.

[5] X. Tai, D. Marelli, E. Rohr and M. Fu, "Optimal PMU Placement for Power System State Estimation with Random Component Outages", *International Journal of Electrical Power and Energy Systems*, Vol. 51, No. 1, pp. 35-42, October 2013.

[6] B. Xu and A. Abur, "Optimal Placement of Phasor Measurement Units for State Estimation", Final Project Report, *Power System Engineering Research Center (PSERC)*. Texas A&M University. October 2005.

[7] M. Farsadi, H. Golahmadi and H. Shojaei, "Phasor Measurement Unit (PMU) Allocation in Power System with Different Algorithms", *International Conf. on Electrical and Electronics Engineering*, pp. 396-400, 5-8 November 2009.

[8] F. Khavari, V. Naseri and Ali. H. Naghshbandy, "Optimal PMUs Placement for Power System Observability Using Grenade Explosion Algorithm", *International Review of Electrical Engineering (I.R.E.E)*, Vol. 6, No. 3, pp. 1332-1338, June 2011.

[9] V. B. Raju, T. TejaSwaroop and R. K. Rao, "Optimal Placement of Phasor Measurement Units Against PMU Outage and Line Outage Using Advanced Particle Swarm Optimization Techniques", *World Congress on Information and Communication Technologies (WICT)*, pp. 501-506, December 11-14, 2011.

[10] M. Hurtgen and J.-C. Maun, "Optimal PMU Placement Using Iterated Local Search", *International Journal of Electrical Power and Energy Systems*, Vol. 32, No. 8, pp. 857-860, October 2010.

[11] M. R. Mosavi, A. A. Akhyani and A. Rahmati, "A PMU Placement Optimal Method in Power Systems Using Modified ACO Algorithm and GPS Timing", Iran University of Science and Technology, Przeglad Elektrotechniczny (Electrical Review), ISSN 0033-2097, pp. 346-349, R. 88 NR 8/2012.

[12] M. Beykverdi, A. Ashouri and A. Bagheri, "Optimal Location of PMUs in Distribution Network for Best State estimation Based on ACO-NM Hybrid Algorithm", Journal of Artificial Intelligence in Electrical Engineering, Vol. 1, No. 1, pp. 66-73, June 2012.

[13] B. Wang, Dichen Liu and L. Xiong, "Advance ACO system in Optimizing Power System PMU Placement Problem", *Power Electronics and Motion Control Conference*, IPEMC'09, 17-20 May, 2009.

[14] B. Wang, Dichen Liu and L. Xiong, "An Improved Ant Colony System in Optimizing Power System PMU Placement Problem", *Asia Pacific Conf. on Power and Energy Engineering*, pp. 1-3, 2009.

[15] Dorigo M. and C. Blum, "Ant Colony Optimization Theory: A Survey", *Theoretical Computer Science,* Vol. 344, No. (2-3), pp. 243-278, November 2005.

[16] Dorigo M. and L. M. Gambardella, "Ant Colonies for the Traveling Salesman Problem", *BioSystems*, Vol. 43, No. 2, pp. 73-81, July 1997.

[17] Washington University Website:
      www.ee.washington.edu/research/pstca/