NON-EQUILIBRIUM TWO-PHASE FLOW OF WET STEAM THROUGH A NOZZLE

T.I.SABRY*, B.A.KHALIFA*, K.A.IBRAHIM**, and N.H.MAHMOUD***

ABSTRACT:

Flow of two-phase mixture in a nozzle is a complex process and problem of increasing technological importance. Two-phase flow, as the flow of such mixture is normally described, has been the subject of much study in recent years, and one aspect which is receiving increasing attention is two-phase flow at changes of flow section.

The present work is an analytical study of wet steam flows in a convergent nozzle, equations are developed for non-equilibrium flow conditions, and taking in our consideration the rate of condensation during the expansion. The principal items of concern in this investigation were the performance and establish design criteria for three nozzle profiles, leminscate, curvature and conical shape. The effect of nozzle profile and the disequilibrium conditions of flow prior to the nozzle on the flow parameters along the nozzle, such as static pressure, vapour velocity, vapour temperature, water saturation temperature, water droplet valocity, water droplet size distribution and the quality of steam were investigated.

1. INTRODUCTION:

In recent years, a study of gas dynamics of non-equilibrium supersaturated two-phase flow has received a considerable attention in various field of modern technology. Problems concerned with generation growth of condensed phase and an effect of non-equilibrium interphase heat transfer on the behaviour of supersaturated and moist vapor flows are important in wind tunnels turbines in a number of transport units and power plants, etc.

When steam turbines operate in a wet steam region, there are problems peculiar to wet steam such as additional losses due to wetness and erosion of rotating blade, which are cuased by droplets in wet steam. The additional losses due to wetness are a phenomenon in which the turbine efficiency is reduced by wetness, and tend to increase with wethness of the stage. These problems are particulary serious in those turbines, which are supplied with dry and saturated steam from nuclear steam generators.

Most of the previous studies on the calculations of water vapor expansion with homogeneous condensation are mainly depending on the; equations of conservation and thermodynamic picture of the condensing vapor; an expression for the nucleation rate and equations for droplets growth. The previous work in this area, may be classified into three topics;

---

* Ass. Prof., Mech. Power Eng. Dept., Menoufia University, Egypt.
** Lecturer, Mech. Power Eng. Dept., Menoufia University, Egypt.
*** Engineer, Mech. Power Eng. Dept., Menoufia University, Egypt.

1- Analytical studies of Bakhtar [1] , Bakhtar, et al. [2] and pakhorskiy, et al [3] .

2- Experimental studies of Gyarmathy [4] and Valha and Ruley [5] .

3- Comparative studies between the analytical and the experimental studies. Philip [6] , Saltanov, et. al [7] , Puzyrewski and Studzinski [8] , Bakhtar and Heaton [9] and Snoeck [10] , have discussed this topic.

In most of the above previous work, several approximations are usually involved in the calculations of the flow of condensing vapor through nozzles. These restrictions are neglecting the effect of droplet size; assuming that velocity and temperature equilibrium exists between the vapor phase and liquid phase throughout the nozzle, neglecting the effect of nozzle profile, neglecting the condensation rate through the expansion process,neglecting the variation of dryness fraction along the nozzle and assuming the physical properties of the phases are not changed during the flow process.

Thus, analysis basend on the absence of such restraints remained to be evaluated here, for three different nozzle profiles; leminscate; curvature and conical shape. The mechanism of the nucleation rate and droplet growth was considered also during the calculation.

NOMENCLATURE:

A	: cross-sectional area of the nozzle
a	: speed of sound
B	: second virial coefficient.
C	: Absolute velocity of gaseous phase.
C_{pg}	: Gas specific heat at constant pressure
C_{vf}	: Liquid specific heat at constant volume.
D	: Drag force on a droplet.
h	: Specific enthalpy.
L	: Nozzle length.
l	: Molecular mean free path length.
m	: mass of droplet.
N	: number of droplets per unit length.
n	: number of droplets per unit mass.
P	: Pressure.
R	: Gas constant.
r	: Droplet radius.
T	: Absolute temperature.
t	: time.
u	: droplet velocity.
v	: specific volume.
W	: Relative velocity = c − u
X	: Dryness fraction.
x	: Distance along the nozzle axis.
y	: Wetness fraction (by mass flow rate).
φ	: Condensation coefficient or value of X where droplet id born.
λ	: Thermal conductivity.
μ	: Dynamic viscosity.

ρ : Density

ΔT : Temperature difference = $T_s - T_g$

subscripts

C : critical value

f : Liquid phase.

fg : phase transition.

g : Gaseous phase.

i : Initial value ahead of nozzle.

S : saturation conditions.

2. ANALYSIS:

2.1- Assumptions:

The following assumptions apply to all the analytical work described below :-

1- The flow of mixture is treated as one-dimensional, steady and the effe ffect of wall boundary layers are neglected.

2- The vapor phase is considered to be a real gas with variable specific heats.

3- Liquid phase consists of spherical incompressibel droplets, occupies negligible volume and its enthalpy equals to of saturated liquid.

4- Number of droplets are assumed to be constant during the expansion, while the condensation effect will appear as an increase in the droplet diameter.

5- Subcritical droplets are not considered on the liquid phase.

2.2- Basic Equations:

The basic equations governing the motion may be written as in ref.[11].

Conservation of mass for liquid phase :

$$u . \frac{dm}{dx} = \dot{m} \quad \text{.....} \quad (1)$$

Conservation of mass for the gaseous phase.

$$\frac{d}{dx} (\rho_g AC) + N\dot{m} = 0 \quad \text{.....} \quad (2)$$

Conservation of momentum for the liquid phase

$$D = mu \frac{du}{dx} - W \dot{m} \quad \text{.....} \quad (3)$$

Drag force D may also obtained from the following relation as in ref [11] .

$$D = \frac{6 \pi \mu r. \ W}{1 + 2.7 \ Kn} \quad \text{.....} \quad (4)$$

where, Kn is the Kundsen number

$$Kn = \frac{1.5 \mu}{2r \rho_g} \left(\frac{1}{RT_g} \right)^{\frac{1}{2}} \quad \text{.....} \quad (5)$$

Conservation of momentum for the gaseous phase

$$A . \frac{dP}{dx} + ND + A \rho_g C. \frac{dc}{dx} = 0 \quad \text{.....} \quad (6)$$

Energy Equation :

$$A \rho_g C^2 \frac{dc}{dx} + Nu.m.u. \frac{du}{dx} + A \rho_g C. \frac{dhg}{dx} + Nu.m \frac{dh_f}{dx} - Nu(h_{fg} + \frac{c^2-u^2}{2})\frac{dm}{dx} = 0 \quad \ldots\ldots (7)$$

Equation of state.

$$\frac{P}{\rho_g RT_g} = 1 + B. \rho_g \quad \ldots\ldots (8)$$

where, B is the second virial coefficient, and according to [2]:

$$B = C_1 - \frac{C_2 . 10^{[B1/(B2+T_g^2)]}}{T_g} \quad m^3/kg.$$

where,

$C_1 = 2.0624 \times 10^{-3}$, $C_2 = 2.61204$, $B_1 = 100800$, $B_2 = 34900$.

Solving Equs (1-6) yields the following relations ;

Droplet diameter variation along the nozzle axis

$$\frac{dr}{dx} = \frac{\dot{r}}{u} \quad \ldots\ldots (9)$$

where, $\dot{r}$ = droplet growth rate

$$\dot{r} = \frac{dr}{dt} = \frac{\lambda g}{\rho_f . h_{fg}} . \frac{1 - \frac{r_c}{r}}{(r+1.59) . 1} . \Delta T \quad \ldots\ldots (10)$$

Wetness gradiant along nozzle axis

$$\frac{dy}{dx} = \tfrac{1}{4} \pi r_c^3 . \rho_f . n + \frac{4\pi}{u} \rho_f \int_{x_c}^{X} r(x, \xi)^2 . \dot{r}(x, \xi) . N . d\xi \ldots\ldots (11)$$

where,

$$x_c = (1+2B \rho_g)/(1+B \rho_g)$$

Droplet velocity variation .

$$\frac{du}{dx} = \frac{3.D}{4.\pi . \rho_f r^3 . u} + \frac{3 . W}{r . u} . \frac{dr}{dt} \quad \ldots\ldots (12)$$

Gaseous phase velocity variation .

$$\frac{dc}{dx} = - \frac{2 \quad E \quad g}{T_g . K1 . K2} . \frac{1}{R} . \frac{dR}{dx} - \Big[- \Big(\frac{3 \ w.u}{r.T_g . C_{pg}} . \frac{Y}{1-y} \Big) + (hfg + \frac{c^2-u^2}{2}) .$$

$$\frac{4\pi r^2 . \rho_{f} . N.u}{T_g . C_{pg} . A. \rho_g . C} + \Big(\frac{E \rho_g . 4\pi r^2 . \rho_f . N.u}{T_g . K1.A. \rho_g . C} \Big) \Big] \times \frac{1}{K2} . \frac{dr}{dx} + \Big[\frac{1}{K1} +$$

$$\Big(\frac{3 \ mu}{4\pi \rho_f r^3 . C_{pg} . \rho_g . C} \Big) + \Big(\frac{m.N.u.R.T_s^2 . C_{vf}}{C_{pg} . A. \rho_g . C.P.h_{fg}} \Big) + \Big(\frac{V_g}{C_{pg}} (1 - \frac{Y_c}{X_c}) \Big) \Big] . \frac{N.D}{A.T_g . K2} \quad (13)$$

where,

$$E = (R^2 T_g^2 + 4 \ PBRT_g)^{\frac{1}{2}} , \ Y_c = 1 + \frac{T_g \rho_g}{1+B \rho_g} . \frac{dB}{dT_g} ,$$

$$K1 = - \frac{E}{2B \ T_g} + \frac{R}{2B} + \frac{P}{T_g} - \frac{E \ (E-RTg)}{2 \ BRT_g^2} + \Big[\frac{P}{B} - \frac{E \ (E-RTg)}{2B^2 RT_g} \Big] . \frac{dB}{dT_g}$$

and
$$K2 = \left(\frac{m.N.u.RT_s^2.C_{vf}}{A.T_g.C_{pg}.P.h_{fg}}\right) + \left(\frac{C}{T_g.C_{pg}}\left(1-\frac{y_c}{X_c}\right)\right) - \frac{C}{T_g\,C_{pg}} + \frac{E\,\rho_g}{C.T_g.K1} - \frac{\rho_g.C}{T_g.K1}$$

Pressure variation along the nozzle axis

$$\frac{dP}{dx} = -\frac{ND}{A} - \rho_g.C.\frac{dc}{dx} \quad \ldots\ldots (14)$$

Variation in wet-steam density.

$$\frac{d\rho}{dx} = -\frac{\rho}{A}\cdot\frac{dA}{dx} - \frac{\rho}{c}\cdot\frac{dc}{dx} - \frac{4\pi.r^2.\rho_f.N.\;.u}{A.\rho_g.\;C}\,\frac{dr}{dx} + \frac{\rho}{1-y}\cdot\frac{dy}{dx} \quad \ldots (15)$$

Variation in gaseous temperature

$$\frac{dT_g}{dx} = \frac{E\,(1-y)}{K1}\cdot\frac{d\rho}{dx} - \frac{E\rho}{K1}\cdot\frac{dy}{dx} + \frac{\rho_g.C}{K1}\cdot\frac{dc}{dx} + \frac{N.D}{A.K1} \quad \ldots\ldots (16)$$

Variation in saturation temperature of liquid phase

$$\frac{dT_s}{dx} = \frac{RT_s^2}{P.\;h_{fg}}\cdot\frac{dP}{dx} \quad \ldots\ldots (17)$$

Variation in the liquid phase enthalpy

$$\frac{d_{hf}}{dx} = C_{vf}\cdot\frac{dT_s}{dx} = C_{vf}\cdot\frac{RT_s^2}{P.\;h_{fg}}\cdot\frac{dP}{dx} \quad \ldots\ldots (18)$$

variation in the gaseous phase enthalpy

$$\frac{dh_g}{dx} = C_{pg}\cdot\frac{dT_g}{dx} + V_g\left(1-\frac{Y_c}{X_c}\right)\cdot\frac{dP}{dx} \quad \ldots\ldots (19)$$

Variation in the enthalpy of phase transition

$$\frac{dh_{fg}}{dx} = \frac{dh_g}{dx} - \frac{dh_f}{dx} \quad \ldots\ldots (20)$$

The above equations (9-20) are simultaneously integrated over a finite step length. The integration was carried out using a 4th order 5 stage Runge-Kutta Merson method. At each step, the vapour and the liquid parameters are known.

3. RESULTS AND DISCUSSION:

The work was mainly concerned with the behaviour of non-equilibrium wet-steam through three nozzle profiles, shown in Fig.(1). These profiles are, Leminscate (profile I), conical (Profile II) and Curvature (Profile III).

In order to establish this behaviour more clearly, all the calculation which carried out here; are for wet steam has the following conditions; $P = 7\ kg/cm^2$, saturation temperature $T_s = 438\ K^o$ and $X = 0.995$. The initial slip velocity or the initial disequilibrium ahead of each nozzle was assumed to be 0.98.

3.1- The Pressure Variations Along The Nozzle Axis:

The variations of pressure along the nozzle axis are shown in Fig.(2) Solid curve represents the result for the profile I and broken curve represents the result for the profile II, while the dashed and dotted curve represents the result for the profile III. From these curves it can be seen that the pressure along the nozzle axis, for each profile, decreases monotonically. The effect of changing the nozzle profile is clear also from the figure, where a ripid decrease is obtained from the nozzle which has a conical (profile II), as it compared with the other two profiles used here. This may be due to the effects of changing the flow area and the droplet growth which has an influence on the shape of pressure distribution as shown in the Fig. (2).

3.2- The Behaviour Of Gas And Droplet Velocities:

Fig. (3), shows the behaviour of the gas and droplet velocities through the nozzle. Solid curves indicate the change of the gas velocity along the nozzle axis and the broken curves are for the change of droplet velocity along the nozzle axis.

In all these results the gas velocity, C, is greater than the droplet velocity, u, and both of c and u are increase along the nozzle, for each nozzle profile used. The nozzle profiles have a significant effect on the result as shown in the figure. The effect of disequilibrium ahead of the nozzle is also clear in the figure. These results may be explained as the flow area decreases, the pressure distribution decreases and the momentum exchange between the gas phase and the liquid phase has a positive value; all of these and with the negative heat transfer between the two phases results an increase of gas velocity along the nozzle.

3.3- The Behaviour Of Droplet Growth:

The variation of the droplet radius along the nozzle has been calculated, through different nozzle profiles, and the results are shown in Fig. (4). Solid curve indicates the result for the nozzle profile I, and the broken curve indicates the result for the nozzle profile II, while the dashed and dotted curve indicates this result for the profile III. From this figure, it can be seen that the droplet size increases along the nozzle axis due to the condensation effect, since the number of droplets were assumed to be constant through the calculation. The nozzle profile has a significant effect on the droplet radius change along the nozzle axis as shown in this figure. Fig. (4), also indicates that, the droplet size distribution curve for the conical nozzle (profile II) has a smaller values than those obtained for the other profiles, i.e. the value of (r/r_i), at the nozzle exit, is 2.23, for profile II. While this value, for the other profiles I and III, is 2.66. This may be due to the rate of condensation through the conical nozzle is less than the other, since the number of droplets for all the profile were assumed to be constant through the calculation.

3.4- The Varation of Dryness Fraction:

Fig.(5) shows the variation of dryness fraction along the nozzle, for different nozzle profiles. Solid curve represents the variation of X through the nozzle profile I, broken curve represents the variation of X through the nozzle profile II and the dashed and dotted curve represents this variation through the nozzle profile III. From this figure, it can be seen that the dryness fraction X decreases along the nozzle axis and depends on the shape of nozzle profile. The rate decrease of X along the nozzle is greater for the profile I than the others. This is because the rate of condensation through the profile I is high as it compared with the other profiles.

3.5- The Variation Of Slip Velocity:

Fig.(6) shows the variation of slip velocity along the nozzle axis. Solid curve represents this variation along the nozzle I, broken curve represents this variation along the nozzle II, and dashed and dotted curve represents the result for the nozzle III. The nozzle profile shape has a significant effect on the result. The slip velocity decreases with the increase of distance along the nozzle axis. The values of final slip velocity were found to depend on the nozzle profile shape, for example this value for profile I is 0.77, for profile II is 0.645 and for profile III is 0.616. The discrepancy of the results would, of course , due to the rate of condensation and the change of flow area.

3.6- Variation Of Gas And Liquid Saturation Temperatures:

Fig.(7) shows the variation of gas and liquid saturation temperatures along the nozzle axis, for different nozzle profiles. Solid curves indicate the variation of liquid saturation temperature and broken curves indicate the variation of gas temperature along the nozzle axis. From this figure, it can be seen that both of Tg and Ts are decrease along the nozzle axis and their behaviour was found to depend on the nozzle profile shapes. In this figure Ts is greater than Tg through all the nozzles tested here. As the slip velocity (u/c), through the nozzle,becomes progressively smaller; if it compared with the initial value ahead of nozzle; the heat transfer increases in importance and eventually becomes the dominant factor and the temperatures are decreases through the expansion process.

4. CONCLUSIONS:

A one-dimensional approximation to steady non-equilibrium, two-phase flow of wet steam flow in a convergent nozzle has been shown to produce solutions which give a reasonably accurate prediction of condensing vapor and its behaviour in a real wet steam nozzle flow.

An analysis based on the Runge-Kutta-Merson method was used in the calculations. The disequilibrium flow conditions between the phases have been shown to have an important influence on the flow properties along the nozzle. The variation of droplet size, dryness fraction, pressure distribution, gas velocity, liquid velocity, gas temperature and liquid saturation temperature, were found to depend on the both of the nozzle profile shape and the rate of condensation.

REFERENCES:

1- Bakhter, F.: "Supersaturation in flow of steam through nozzles", Report of a Meting at NEL "Two-phase flow through Orifices and Nozzles", NEL Report No. 549, pp.93-101, 1972.

2- Baktar, F., Ryley, D.J., Tubman, K.A. and Young, J.B.: "Nucleation studies in flowing high-pressure steam", Proc. Inst. Mech. Engrs, Vol. 189, 41-74, 1975.

3- Pakhorskiy, V.A., Aladyev, I.T. and Pishuk, R.A.: "A method for determining the properties of a two-phase, single-component flow along a nozzle", Fluid Mechanics, Sov. Res., Vol. 4, No 5, 1975.

4- Gyarmathy, G., Burkhard, H.B., Lesch, F. and Siegenthaler, A.: "Spontaneous condensation of steam at higher pressure", Inst. Mech. Eng. Conference on Heat and Fluid Flow in Steam and Gas Turbine Plant, University of Warwick, pp. 182-186, 1974.

5- Valha, J. and Ryley, D.J.:" Optical studies of nucleation in high pressure expanding steam", Applied Mechanics Fluid Eng. and Bioengineering Conference, Yale University, 15-17 June 1977.

6- Philip G. Hill: "Condensation of water vapour during supersonic expansion in nozzles", J. Fluid Mech., Vol. 25, part 3, pp. 593-620, 1966.

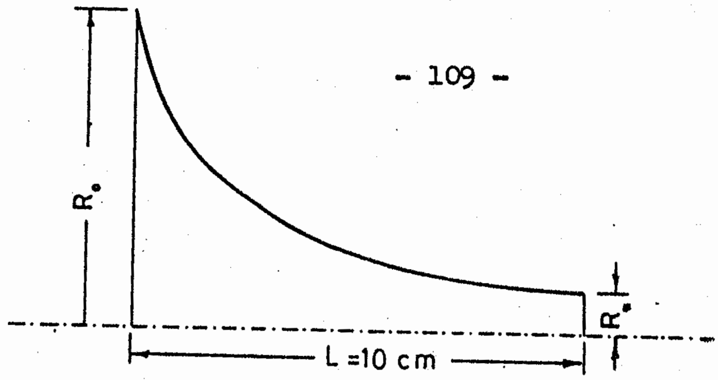
7- Saltanov, G.A., Seleznev, L.I. and Tsiklauri, G.V.: "Generation and growth of condensed phase in high-velocity flows", Int. J. Heat Mass Transfer, Vol. 16, pp. 1577-1587, 1973.

8- Puzyrewski, R. and Studzinski, W.: "One-dimensional water vapor expansion with condensation at higher pressures", Int. J. Multiphase Flow, Vol. 6, pp. 425-439, 1980.

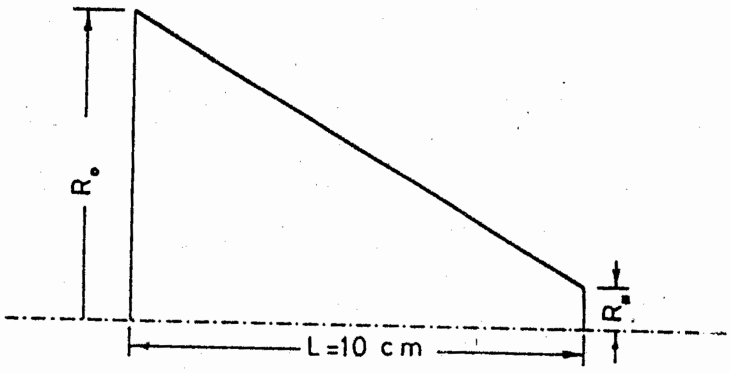
9- Bakhtar, F. and Heaton, A.V.: "A theoretical comparative study of wetness problems in a model and full scal turbine", Aero-Thermodynamics of steam Turbines, The winter annual meeting of the Am. Soc. Mech. Engrs, Washington, pp. 29-35, 1981.

10- Snoeck, J.: "Calculation of mixed flows with condensation in one dimensional nozzles", Aero-Thermodynamics of Steam Turbines, the winter annual meeting of the Am. Soc. Mech. Engrs, Washington, pp. 29-35, 1981.

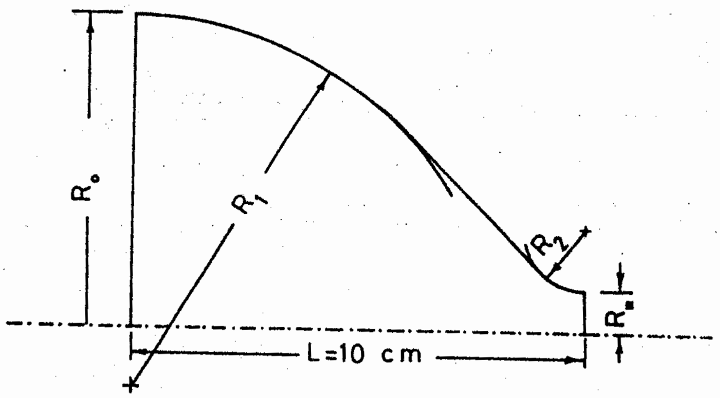
11- Moore, M.J. and Sieverding, C.H., "Two-phase steam flow in turbines and separators" McGraw-Hill Book Company, 1976.



I - Leminscate.

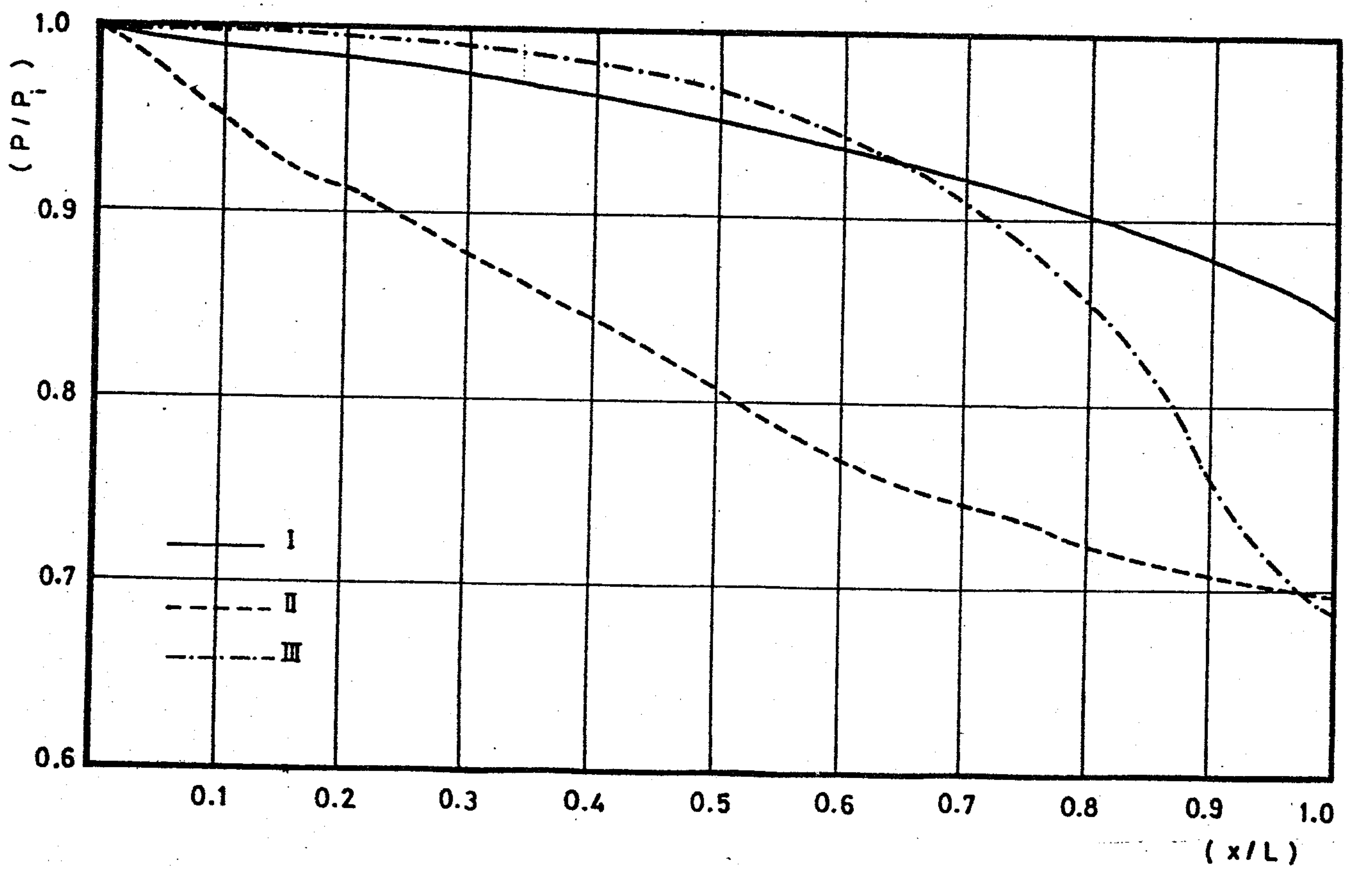


II - Conical.

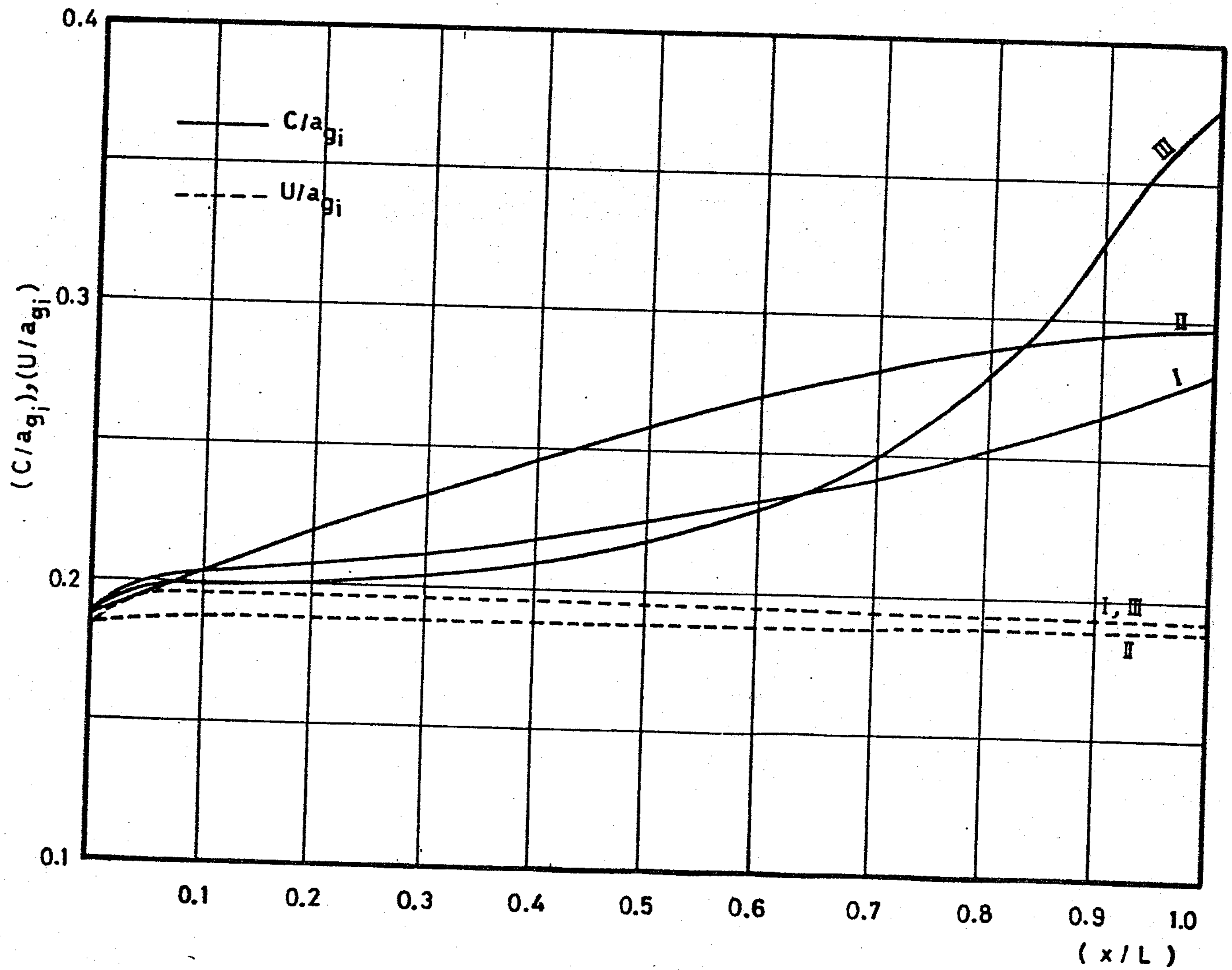


III - Curvature.

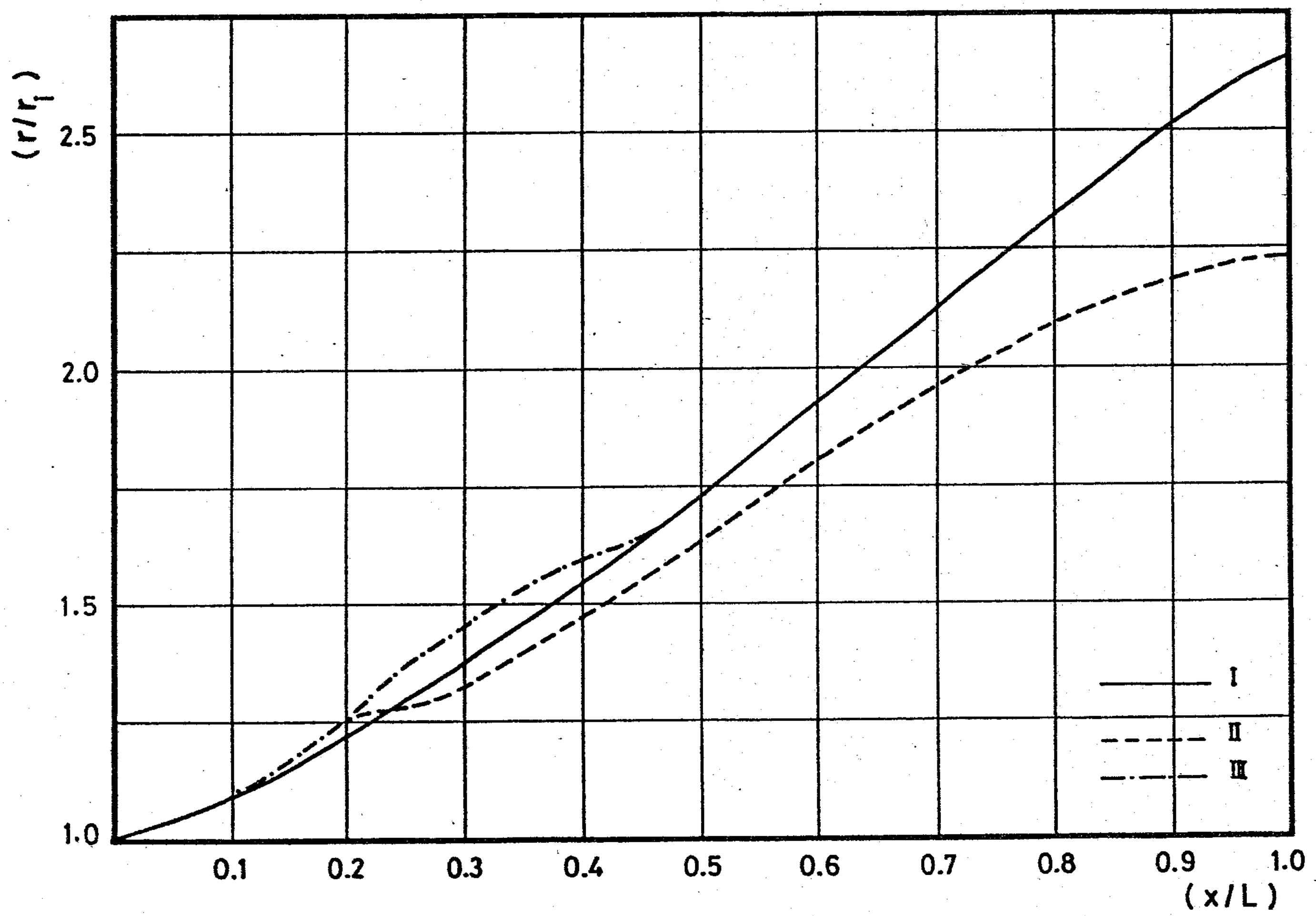
FIG(1): Nozzle Shape.



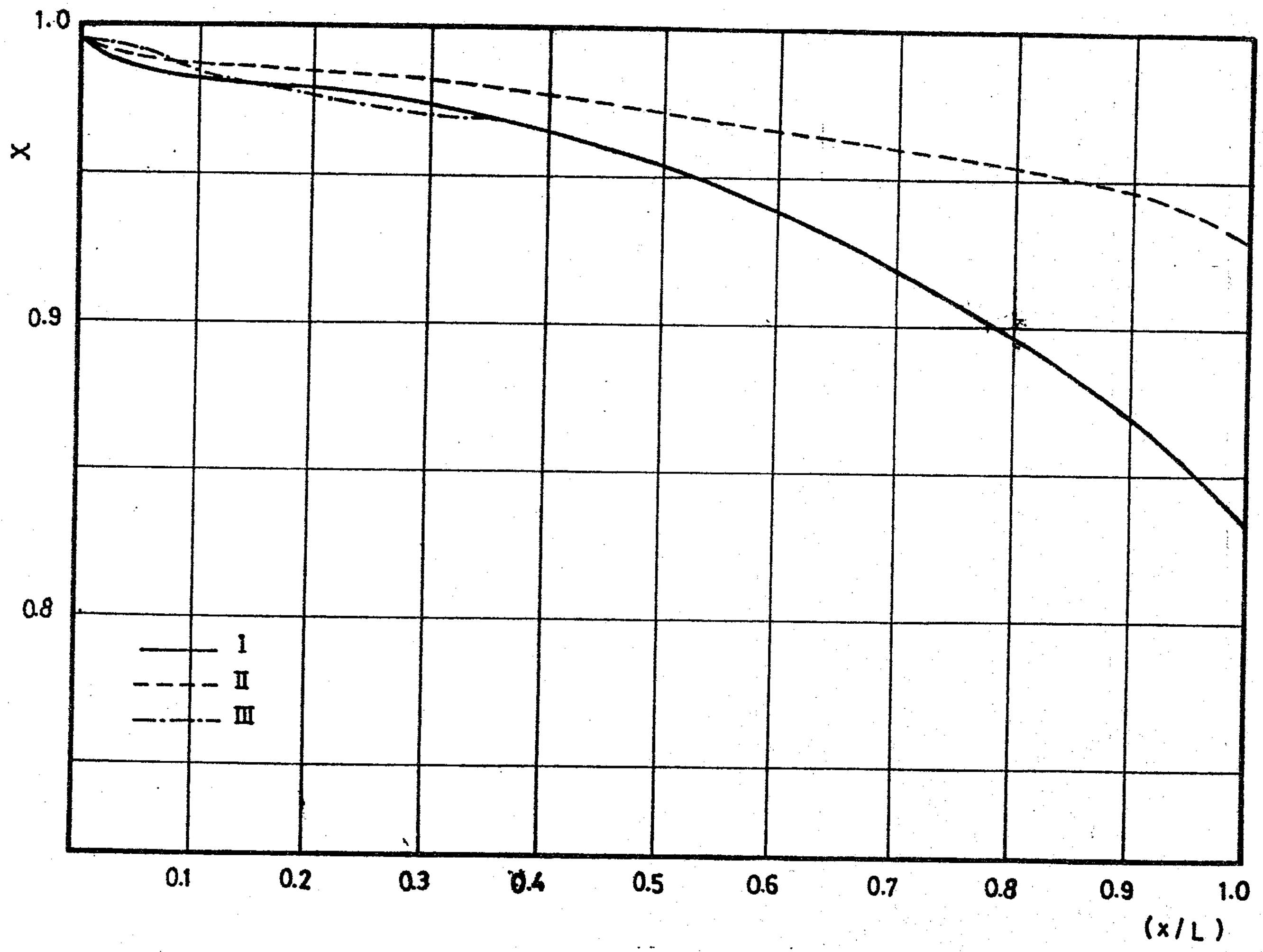
FIG(2): Variation of Pressure Along the Nozzle Axis.



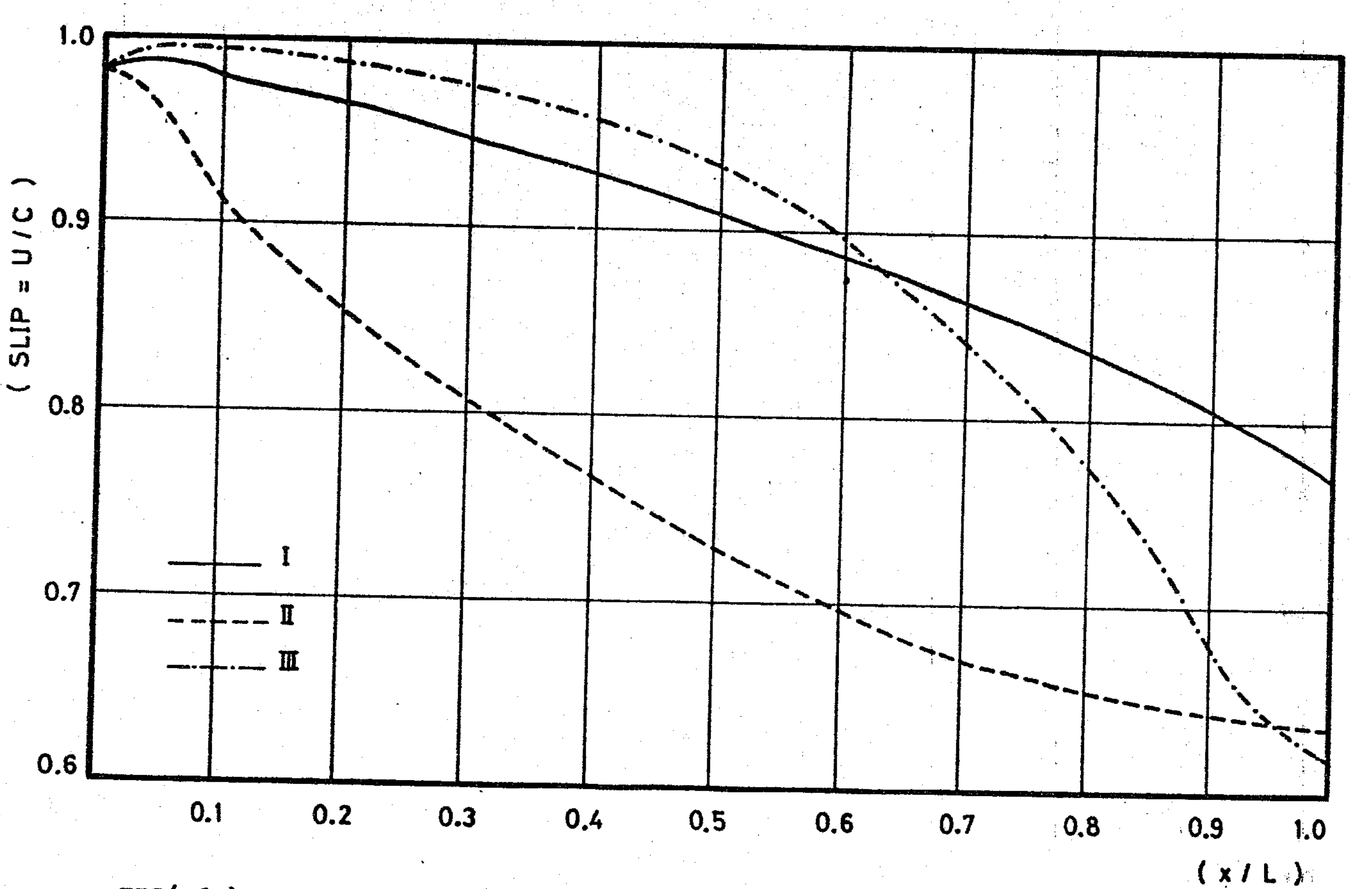
FIG(3): Variation of Gas and Droplet Velocities Along the Nozzle Axis.



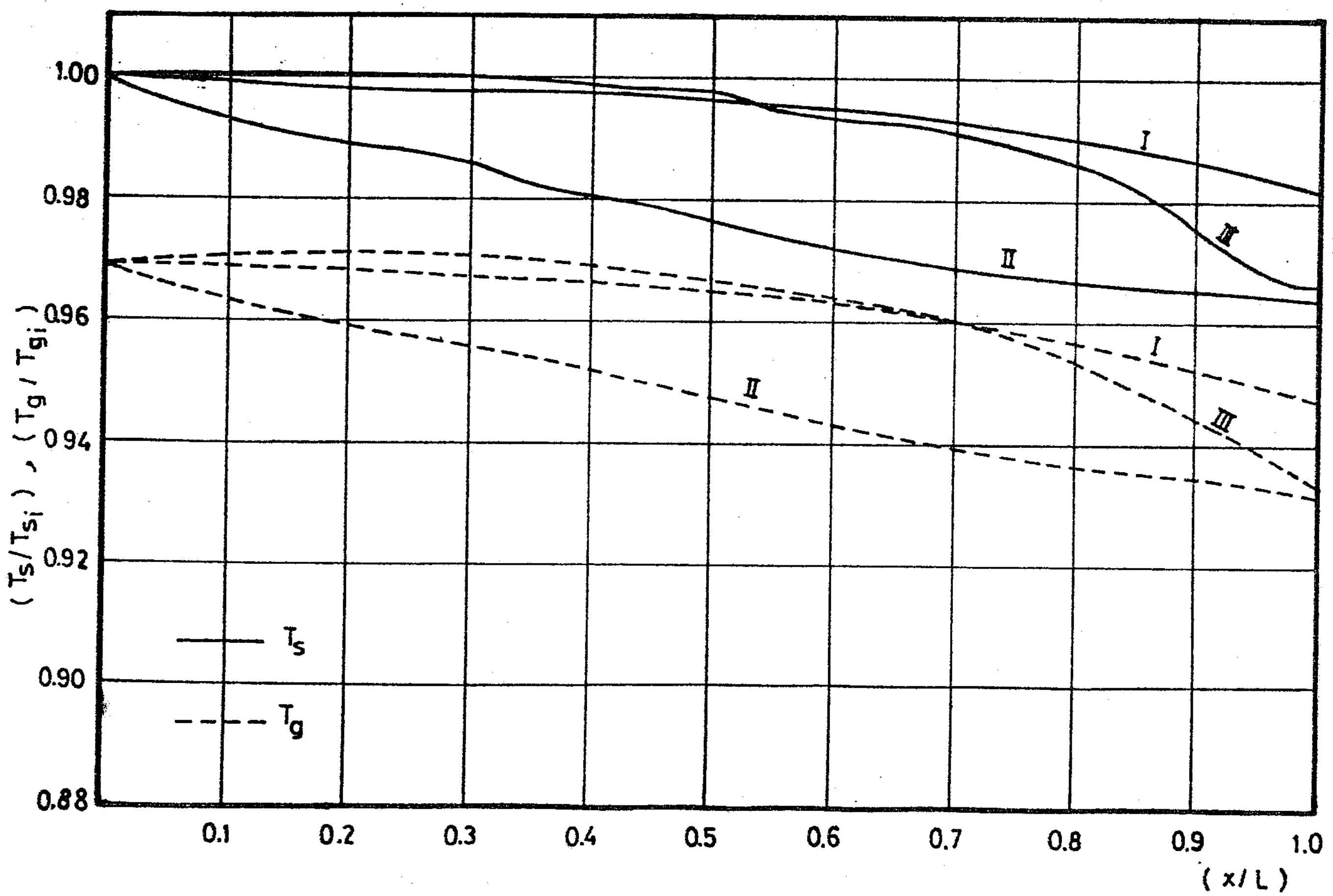
FIG(4): Variation of Droplet Radius Along the Nozzle Axis.



FIG(5): Variation of Steam Dryness Fraction Along the Nozzle Axis.



FIG(6): Variation of Slip Velocity Along the Nozzle Axis.



FIG(7): Variation of Gas and Liquid Saturation Temperatures Along the Nozzle Axis.